

**INSIGHT** Trial Exam Paper

# 2010

# **FURTHER MATHEMATICS**

# Written examination 1 MULTIPLE-CHOICE QUESTION BOOK

Reading time: 15 minutes Writing time: 1 hour 30 minutes

#### Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
А	13	13			13
В	54	27	6	3	27
				]	Cotal 40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference that may be annotated (can be typed, handwritten or a textbook), one approved graphics calculator (memory DOES NOT have to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring blank sheets of paper or white out liquid/tape into the examination.

#### **Materials provided**

- The question and answer book of 39 pages, with an answer sheet for the multiple-choice questions.
- A separate sheet with miscellaneous formulas.
- Working space is provided throughout the question book.

#### Instructions

- Write your name in the box provided on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, diagrams in this book are not drawn to scale.

#### At the end of the examination

• You may keep this question book.

# Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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### **Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is **correct** for the question. A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. No marks will be given if more than one answer is completed for any question.

# **Core: Data analysis**

The following information relates to Questions 1 to 3.

The histogram below shows the distribution of secondary school attendance rates (in percentages) for 200 countries between the years 2000 and 2007.



Secondary school attendance rates

#### **Question 1**

The percentage of countries with secondary school attendance rates of 55% or smaller was closest to

- A. 29%
- **B.** 35%
- **C.** 46%
- **D.** 54%
- E. 92%

For these 200 countries, secondary school attendance rates were most frequently

- A. less than 25%
- **B.** between 25% and 55%
- C. between 55% and 65%
- **D.** between 75% and 95%
- **E.** greater than 75%

#### **Question 3**

Which one of the boxplots below could best be used to represent the same secondary school attendance rate data as displayed in the histogram?



Secondary school attendance rates (in percentages)

#### The following information relates to Questions 4 to 6.

The ordered, back-to-back stemplot below shows exclusive breastfeeding rates for six months, expressed as a percentage, in 22 developed and developing countries.

#### Exclusive breastfeeding rates for 6 months (%)

developed countries								deve	eloping	; cour	tries				
9	8 7	7 7	9 2 5	9 2 3 5	7 1 3 5 4	4 0 2 1 2	1 2 3 4 5 6 7 8	7 4 0 1 1 3 2	8 7 1 2 3	1 3 4	2 3 6	6	7	8	9

#### **Question 4**

For these 22 developed countries, the highest exclusive breastfeeding rates for 6 months is

- A. 28%
- **B.** 45%
- **C.** 54%
- **D.** 73%
- E. 82%

#### **Question 5**

For these 22 developing countries, the interquartile range of exclusive breastfeeding rates for 6 months is

- **A.** 18
- **B.** 20
- **C.** 21.5
- **D.** 25
- **E.** 28

#### **Question 6**

For these 22 developing countries, exclusive breastfeeding rates for 6 months are generally

- A. higher and more variable than the exclusive breastfeeding rates of the developed countries
- **B.** higher and less variable than the exclusive breastfeeding rates of the developed countries
- C. about the same as the exclusive breastfeeding rates of the developed countries
- **D.** lower and more variable than the exclusive breastfeeding rates of the developed countries
- **E.** lower and less variable than the exclusive breastfeeding rates of the developed countries

Consider the graph below. The gradient of the 3- median regression line is



- **A.** 3
- **B.** 3.7
- **C.** 4
- **D.** 4.5
- **E.** 5

# **Question 8**

17 students sat a geometry exam and their teacher calculated some statistics for the scores of the students who sat for this exam, noting that no two of the students had the same score. Sarah scored the lowest mark in the class. The teacher then recalculated the statistics excluding Sarah's score.

Which one of the following statistics would not change when Sarah's score is excluded?

- A. the median
- **B.** the mean
- **C.** the standard deviation
- **D.** the lower quartile
- **E.** the upper quartile

#### The following information relates to Questions 9 and 10.

The distribution of times for the 500 metres running championship amongst the high school students is bell shaped with a mean of 92 seconds and a standard deviation of 4.5 seconds.

#### **Question 9**

The slowest 2.5% of the athletes would have run 500 metres in

- A. less than 83 seconds
- **B.** more than 101 seconds
- C. between 83 and 101 seconds
- **D.** more than 96.5 seconds
- **E.** less than 78.5 seconds

#### Question 10

Jordan is a high school student who attended the 500 metres running championship. If she ran 500 metres in 85.25 seconds, her standardised score (z-score) is closest to

- **A.** -1
- **B.** -1.5
- **C.** -1.75
- **D.** 1.75
- **E.** 1.5

#### Question 11

The dam levels and average rainfall (in cm) at several different locations in a particular region of Australia is shown below.

Dam level (%)	25	67	48	85	13	55	61	70	34
Average rainfall (in cm)	2.3	4.5	3.6	8.8	1.1	3.3	4.1	4.5	2.6

Using *dam level* as the dependent variable, a least squares regression line is fitted to the data. The equation of the least squares regression line is closest to

- A. Average rainfall =  $0.083 0.3653 \times dam \, level$
- **B.** Average rainfall =  $-0.3653 + 0.083 \times dam \, level$
- C.  $dam \, level = -0.3653 + 0.083 \times average \, rainfall$
- **D.**  $dam \, level = 13.76 + 9.6 \times average \, rainfall$
- **E.**  $dam \, level = 9.6 + 13.76 \times average \, rainfall$

The relationship between two variables *number of accidents made* and *the experience level of the drivers (in years)*, as shown on the scatterplot below, is nonlinear.

Which one of the following would be the most likely transformation to linearise the data?



- A. *number of accidents made* against *the experience level of the drivers*.
- **B.**  $(number of accidents made)^2$  against the experience level of the drivers.
- **C.** *number of accidents made against log (the experience level of the drivers).*
- **D.** number of accidents made against  $\frac{1}{\text{the experience level of the drivers}}$ .
- E. log (number of accidents made) against the experience level of the drivers.

# Question 13

A line of best fit for deseasonalised data was given as

*Deseasonalised Monthly Sales* =  $2500 - 1.2 \times timecode$  where for December 2009, t = 1.

Predict the actual expected sales figures for January 2012 if the January seasonal index is 1.15.

- **A.** 2470
- **B.** 2468.8
- **C.** 2839.12
- **D.** 2840.5
- **E.** 2146.78

# **Instructions for Section B**

Select **three** modules and answer **all** questions within the modules selected on the answer sheet provided.

Indicate the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

**One** mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.

Marks are not deducted for incorrect answers.

No marks will be awarded if more than one answer is completed for any question.

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#### Module 1: Number Patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

#### Question 1

The second and fourth terms of an arithmetic sequence are 24 and 42 respectively. The sum of the first 10 terms of the sequence is

- A. 9
- **B.** 15
- **C.** 555
- **D.** 890
- **E.** 1550

#### **Question 2**

For the sequence 53, 49, 45, 41,...the first term that will be negative will be the term

- **A.** 23
- **B.** 10
- **C.** 14
- **D.** 15
- **E.** -3

# **Question 3**

A certain geometric sequence has common ratio of 0.85. The sum of the infinite number of terms of this sequence is 20. The fourth term of this sequence is closest to

- A. 3B. 2.55
- **C.** 2.17
- **D.** 1.84
- **E.** 1.95

# Question 4

The first four terms of a geometric sequence are 7 200,  $t_2$ ,16 200,-24 300. The value of the second term is

- **A.** 8 100
- **B.** -8 100
- **C.** -10 800
- **D.** 10 800
- **E.** None of the above

In a Fibonacci sequence,  $t_{n+2} = t_n + t_{n+1}$ . If  $t_{16} = 221\,250$  and  $t_{18} = 579\,240$ , the  $17^{\text{th}}$  number in this Fibonacci sequence is

- A. 84 510
- **B.** 125 670
- **C.** 569 140
- **D.** 456 000
- E. 357 990

#### **Question 6**

Members of a gymnastic club have been decreasing by 20% every year since 2007. In order to reduce the loss, the club management decided to increase the yearly membership fees by \$200 every year. The club had 8 000 members and membership fee was \$540 in 2007. How much less total fees will the club collect in 2010 than 2008?

- A. \$85 560
- **B.** \$47 000
- **C.** \$66 560
- **D.** \$123 450
- E. \$150 000

#### **Question 7**

In a chicken farm, the number of chickens is naturally increasing by 42% per year. The farm started with 400 chickens and at the end of each year 120 chickens are sold.

The difference equation that represents the number of chickens  $C_n$  at the start of  $n^{th}$  year is

- A.  $C_{n+1} = 1.42 \times (C_n 120)$   $C_1 = 400$
- **B.**  $C_{n+1} = 1.42 \times C_n 120$   $C_1 = 400$
- C.  $C_{n+1} = 0.58 \times C_n 120$   $C_1 = 400$
- **D.**  $C_{n+1} = 0.58 \times (C_n 400)$   $C_1 = 120$
- E.  $C_{n+1} = 400 \times C_n 120$   $C_1 = 42$

A cactus was 32 cm high initially. It grew 1.2 cm during the first week, 0.96 cm during the second week and 0.768 cm during the third week. If this cactus continues to grow in this manner, it will reach the maximum height of

- **A.** 6
- **B.** 35
- **C.** 37
- **D.** 38
- **E.** 41

# **Question 9**

The first four terms of a sequence are: 3, 5, 9, 17. The difference equation of the sequence could be

A.  $t_{n+2} = 3t_n - 2t_{n+1}$   $t_1 = 3$ ,  $t_2 = 5$ B.  $t_{n+1} = 3t_n - 4$   $t_1 = 3$ C.  $t_{n+1} = t_n + 2$   $t_1 = 3$ D.  $t_{n+1} = 4t_n - 7$   $t_1 = 3$ E.  $t_{n+2} = 3t_{n+1} - 2t_n$   $t_1 = 3$ ,  $t_2 = 5$ 

#### Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

#### **Question 1**



DE || BC. If AE= 11 cm, EC= 5.5 cm and BC= 13.5 cm, the length of DE, in cm, is

- **A.** 9
- **B.** 6.75
- **C.** 20.5
- **D.** 13.5
- **E.** 27

#### **Question 2**



In the triangle above, the angle LMK is closest to

- **A.** 45°23'
- **B.** 49°41'
- **C.** 59°5'
- **D.** 63°9'
- **E.** 73°51'



The locations of three houses, Jann's, Veronica's and Richard's houses are shown in the diagram above. Jann's house is due north of Veronica's house. The bearings of Jann's and Veronica's houses from Richard's house are

- A.  $303^{\circ}$  and  $180^{\circ}$  respectively
- **B.**  $123^{\circ}$  and  $0^{\circ}$  respectively
- C.  $33^{\circ}$  and  $0^{\circ}$  respectively
- **D.**  $47^{\circ}$  and  $180^{\circ}$  respectively
- **E.**  $303^{\circ}$  and  $270^{\circ}$  respectively



A contour map of a region is shown above. The contour interval is 20 m. The actual horizontal distance between points A and B is 40 m. The angle of depression of B from A is closest to

- **A.** 27°
- **B.** 63°
- **C.** 72°
- **D.** 47°
- **E.** 85°

#### The following information relates to the questions 5 and 6

A 60 cm high wedding cake has yellow, pink, violet and purple icing as shown in the diagram below



If the volume of the cake is found to be  $625 \ cm^3$ , then the volume of the part covered in purple icing is closest to

- A. 305 cm<sup>3</sup>
- **B.** 105 *cm*<sup>3</sup>
- C.  $5 cm^3$
- **D.** 425 *cm*<sup>3</sup>
- **E.** 610 *cm*<sup>3</sup>

#### **Question 6**

The area of the base, in  $cm^2$ , is closest to

- **A.** 25
- **B.** 31.25
- **C.** 45.5
- **D.** 62.5
- **E.** 125

#### **Question 7**



A cylindrical vase is 70 centimetres long. The outside diameter of the vase is 30 centimetres. Its walls are 3 centimetres thick. One litre of water occupies 1000 cm<sup>3</sup>. When the vase is full of water, the volume it holds is closest to

- **A.** 5 L
- **B.** 15 L
- **C.** 18 L
- **D.** 22 L
- **E.** 32 L



A tree is 6 metres tall and stands near a house. The base of the tree is 10 metres from both ends of the nearest wall of the house which is also 10 metres long.

The minimum distance between the top of the tree and the house when the tree falls is closest to

- **B.** 8.66 m
- **C.** 4 m
- **D.** 2.66 m
- **E.** 1.26 m

# **Question 9**



A rectangular prism has a volume of 240 cm<sup>3</sup>. A second rectangular prism is made with the same length, half the width and four times the height of the prism shown.

The volume of the second prism is

- A. 240 cm<sup>3</sup>
- **B.** 480 cm<sup>3</sup>
- C. 120 cm<sup>3</sup>
- **D.** 360 cm<sup>3</sup>
- **E.** 540 cm<sup>3</sup>

# Working Space

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#### Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

#### **Question 1**



The equation of the straight line shown is

- A. x = 8
- **B.** y = 4
- C. 2y x = 8
- **D.** 4x + 8y = 2
- **E.** x + 2y = 8

Alice's father Danny manufactures LCD televisions in his electronic company's factory. The graph below shows the cost (in hundreds of dollars) of producing the LCD televisions in Danny's company.



The increase in cost that occurs from producing one LCD television is cost of producing each LCD television is:

- **A.** 22
- **B.** 23
- **C.** 2 200
- **D.** 23 000
- **E.** 254



Which of the following rules would best describe the graph above?

А.	$\begin{cases} y = x - 3, & 0 \le x < 3 \\ y = 2, & 3 \le x < 6 \\ y = -x + 9, & 6 \le x \le 9 \end{cases}$
B.	$\begin{cases} y = x + 3, & 0 \le x < 3 \\ y = 2, & 3 \le x < 6 \\ y = -x + 9, & 6 \le x < 9 \end{cases}$
C.	$\begin{cases} y = x - 3, & 0 \le x < 3 \\ y = 2, & 3 \le x \le 6 \\ y = x + 9, & 6 \le x \le 9 \end{cases}$
D.	$\begin{cases} y = x - 3, & 0 < x \le 3 \\ y = 2, & 3 < x \le 6 \\ y = -x + 9, & 6 < x < 9 \end{cases}$
E.	$\begin{cases} y = x - 3, & 0 \le x < 2\\ y = 3, & 2 \le x < 6\\ y = -x + 9, & 6 \le x \le 9 \end{cases}$

The cost of hiring 7 overnight and 6 weekly DVDs is \$106.60. At the same shop, the cost of hiring 5 overnight and 2 weekly DVDs is \$70.20.

Let x be the cost of hiring a weekly DVD and y be the cost of hiring an overnight DVD. The set of simultaneous equations that can be solved to find the cost of hiring a weekly and an overnight DVD is

- A. 6x + 7y = 70.202x + 5y = 106.60
- **B.** 7x + 6y = 106.605x + 2y = 70.20
- C. 7x + 5y = 106.606x + 2y = 70.20
- **D.** 6x + 5y = 70.207x + 2y = 106.60
- E. 2x + 5y = 70.206x + 7y = 106.60

The salt concentration (in g/L) in a particular pond is graphed over a six-month period.



For this 6-month period, the salt concentration of the pond was between 2 g/L and 4 g/L for

- A. between two and three months
- **B.** two months
- C. three months
- **D.** between three and four months
- **E.** four months

#### **Question 6**

"Tasty world" chocolate company produces two different types of handmade chocolates in five factories. Each factory has the capacity to produce 500 boxes of each type of chocolates every day. The factories will produce chocolates for spring festival over a 10 day period. The company had to spend \$20 000 as fixed cost of the preparation for the manufacture of the chocolate and the cost of producing one box of chocolate is \$5. To break even, the selling price of one box of chocolate should be

- **A.** \$5.40
- **B.** \$6.20
- **C.** \$5.80
- **D.** \$10.50
- **E.** \$70

The graph of y versus x<sup>3</sup> is shown



The graph of y versus x is



Amber is a year 12 student who spends at least 14 hours a week studying further mathematics and at least 9 hours a week studying chemistry. She can devote a maximum of 42 hours in total to studying further mathematics and chemistry in a week. She always makes sure that she spends at least three times as many hours studying further mathematics than studying chemistry.

Let x be the number of hours per week Amber studies further mathematics.

Let *y* be the number of hours per week Amber studies chemistry.

The constraints that apply to Amber's studying further mathematics and chemistry are described by

- A.  $x \le 14, y \le 9, x + y \ge 42, x \ge 3y$
- **B.**  $x \ge 14, y \ge 9, x + y \le 42, y \ge 3x$
- C.  $x \ge 14, y \ge 9, x + y \le 42, 3x \ge y$
- **D.**  $x \le 14, y \le 9, x + y \ge 42, y \ge 3x$
- E.  $x \ge 14, y \ge 9, x + y \le 42, x \ge 3y$

Two car production companies, "Fast Car" and "Luxury Car" decided to merge in order to decrease the production costs. Let x be the number of cars produced weekly by the "Fast Car" company and y be the number of cars produced weekly by the "Luxury Car" company.

In which one of the following graphs does the point M, the intersection point of two lines, minimise the cost function, C = 3x + 2y for the given shaded region?



#### **Module 4:Business-related mathematics**

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

#### Question 1

Jane invested her money in an account earning simple interest at a rate of 5.2% per annum in January 2005. When she withdrew all her money in January 2010 to buy a car, she had earned \$2600 in interest. The total amount of money she withdrew to buy a car is

- A. 10 000
- **B.** 11 600
- **C.** 12 600
- **D.** 14 400
- **E.** 15 000

#### **Question 2**

Anthony wants to invest \$300 000 in a bank for a period of 4 years. He can either invest his money in an account earning compound interest at a rate of 9% per annum compounding monthly or in an account earning simple interest at a rate of 10.7% per annum. The best option and amount extra in interest earned is given by

- A. Simple interest is the best option by \$15 650.20
- **B.** Simple interest is the best option by \$128 400
- C. Compound interest is the best option by \$301 021.60
- **D.** Simple interest is the best option by \$1248.80
- **E.** Compound interest is the best option by \$1021.60

#### **Question 3**

Peter and Mary-Ellen borrowed \$428 000 to purchase a house and land package. The loan was due to run for 30 years and attract interest at 6.5% per annum, debited quarterly on the outstanding balance. After 12 years the rate changed to 9.5% per annum. Peter and Mary-Ellen decided to change their payment method to monthly payments where interest is calculated monthly on the reducing balance of the loan. Their new repayments were \$3100 per month. The total time it will take Peter and Mary-Ellen to pay off the loan completely is closest to

- A. 30 years
- **B.** 28 years
- C. 36 years
- **D.** 34 years
- E. 32 years

Rob and Jane bought their cars at the same time from different car dealers. John's car costs \$53 760 and depreciates by the flat rate method. The depreciation rate of John's car is 12% of the prime cost price each year and its scrap value is \$2150.40. Jane's car costs \$32 000 and depreciates by the reducing balance method. The depreciation rate of Jane's car is 12% and it has a scrap value of \$1309. Which car will be written off first and how much later would the other car need to be replaced?

- A. Rob's car will be written off first and Jane's car will need to be replaced 17 years later
- **B.** Rob's car will be written off first and Jane's car will need to be replaced 25 years later
- C. Both Rob's and Jane's cars will be written off at the same time
- **D.** Jane's car will be written off first and Rob's car will need to be replaced 2 years later
- E. Jane's car will be written off first and Rob's car will need to be replaced 8 years later

# **Question 5**

Dr. Susan Lane is aged 47 and is planning to retire at 68 years of age. She estimates that she needs \$1 200 000 to provide for her retirement. Her current superannuation fund has a balance of \$150 000 and is delivering 8.4% per annum compounding monthly. The monthly contributions needed to meet her retirement lump sum target is closest to

- **A.** \$153
- **B.** \$277
- **C.** \$346
- **D.** \$481
- **E.** \$785

# **Question 6**

Jasmine saw a laptop in an electronic shop and wanted to buy it. Although its marked price was \$3200, after Jasmine's negotiation the shop owner offered her a discount of 12%. She paid \$316 deposit and signed a hire-purchase agreement that she would pay the balance of the laptop's cost at 22% per annum flat rate with 24 equal monthly instalments.

The total price that Jasmine paid for the laptop is closest to

A.	\$3200
1 10	$\psi J L 0 0$

- **B.** \$3916
- **C.** \$3366
- **D.** \$4055
- **E.** \$4469

Serra invests \$290 000 in an annuity that earns 4.9% per annum interest compounded quarterly and pays her a quarterly sum of \$5890. The number of years for which this annuity continues is closest to

- **A.** 10
- **B.** 17
- **C.** 19
- **D.** 21
- **E.** 22

# **Question 8**

At the start of 2009 the price of a basketball was \$45.60 while Josh's annual salary was \$46 000. At the start of each subsequent year his annual salary increases by 3.42%. The predicted annual inflation rate between the years 2009 and 2013 is 6.4%.

In 2013 the predicted price of the basketball and Josh's annual salary will be closest to

- A. \$58.40 and \$52 623 respectively
- **B.** \$54.90 and \$50 883 respectively
- C. \$57.30 and \$52 293 respectively
- **D.** \$54.40 and \$50 720 respectively
- E. \$52.20 and \$58 956 respectively

John receives the following statement from his bank. Due to a technical error the interest and balances were not calculated.

Date	Debit	Credit	Balance
1 January			25 156.70
5 January		2156.80	
14 January		175.45	
19 January	218.65		
21 January		612.15	
8 February	4231.70		
15 February	105.20	55.20	
27 February	67.90		
3 March		125.50	
7 March	762.00		
18 March		235.10	
24 March		120.00	
31 March	105.50		

Interest is accrued at the end of March at a rate of 5.25% per annum calculated monthly on her minimum monthly balance. The total interest that John received in this statement is closest to

- **A.** \$252.50
- **B.** \$146.60
- **C.** \$453.40
- **D.** \$510.90
- **E.** \$313.20

#### Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Network and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

#### **Question 1**



The number of vertices with an odd degree in the graph above is

- **A.** 1
- **B.** 2
- **C.** 3
- **D.** 4
- **E.** 5

#### **Question 2**

The bipartite graph below shows VCE subject preferences of five VCE students in a government school.



Which one of the following statements is not supported by the graph?

- A. All five students have equal number of VCE subject preferences
- **B.** English is the most popular subject
- C. Further mathematics is the second mostly preferred subject
- **D.** Mathematical methods and psychology are equally preferred by students
- E. Michelle and Adrian both preferred to study chemistry

# SECTION B - Module 5: Networks and decision mathematics - continued



For the network shown above, a Hamiltonian circuit can be created by

- A. adding an edge between A and B
- **B.** adding an edge between F and G
- **C.** removing the edge between B and C
- **D.** removing the edge between D and G
- E. not changing the network, it is already a Hamiltonian circuit

#### Question 4

A connected planar graph has 12 edges and 8 vertices. A further 3 vertices and 5 edges were added to the graph. The number of faces

- A. increased by 2
- **B.** decreased by 3
- **C.** increased by 5
- **D.** decreased by 8
- **E.** remained the same





The minimum spanning tree for the above network is









On the directed graph above the numbers on the edges give the maximum flow possible through each of those edges. The maximum flow possible through the network is 27.

The value of x is

A. 12
B. 18
C. 7.5
D. 9
E. 15

# **Question 7**

The network below shows 11 activities required to complete a project, together with the time it takes in days to complete each activity.



The critical path for this project has a length of

- **A.** 54 days
- **B.** 50 days
- **C.** 44 days
- **D.** 58 days
- **E.** 66 days

Four different tutors have different hourly prices for tutoring different subjects which is shown on the table below.

	Tutors					
Subjects	June	Sharon	Sydney	Max		
further mathematics	32	25	12	27		
English	45	26	30	50		
specialist mathematics	22	34	41	23		
chemistry	33	30	42	45		

Amy wants to get tutoring from all these subjects.

If she chooses to get each subject from a different tutor, which of the following allocations provides Amy's minimum cost for getting tutoring from all four subjects?

- A. June  $\rightarrow$  further maths, Sharon  $\rightarrow$  English, Sydney  $\rightarrow$  specialist maths, Max  $\rightarrow$  chemistry
- **B.** June  $\rightarrow$  English, Sharon  $\rightarrow$  further maths, Sydney  $\rightarrow$  specialist maths, Max  $\rightarrow$  chemistry
- C. June  $\rightarrow$  English, Sharon  $\rightarrow$  specialist maths, Sydney  $\rightarrow$  further maths, Max  $\rightarrow$  chemistry
- **D.** June  $\rightarrow$  specialist maths, Sharon  $\rightarrow$  chemistry, Sydney  $\rightarrow$  further maths, Max  $\rightarrow$  English
- **E.** June  $\rightarrow$  chemistry, Sharon  $\rightarrow$  English, Sydney  $\rightarrow$  further maths, Max  $\rightarrow$  specialist maths

Six teams, B, C, D, E, F, G and H have played in a round-robin tournament where each team plays each other team only once. The results of the games are represented by the dominance matrix M, below, where a "1" in the matrix represents a win and a "0" represents a loss.

$$A = \begin{bmatrix} B & C & D & E & F & G \\ B & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using A and A<sup>2</sup> to rank the teams, the ranking, first to last will be

- A. BCEGDF
- **B.** CBEGFD
- C. GBCFDE
- **D.** BEDCGF
- **E.** CBDFGE

#### **Module 6: Matrices**

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

#### **Question 1**

If  $5\begin{bmatrix} -4x & -6\\ 6 & 2y \end{bmatrix} - 3\begin{bmatrix} y & 5\\ 7 & 4x \end{bmatrix} = \begin{bmatrix} -3 & -45\\ 9 & 10 \end{bmatrix}$ , then the values of x and y are **A.** x = 2, y = 1 **B.** x = 0, y = 1 **C.** x = 2, y = -1 **D.** x = -1, y = 2**E.** x = 0, y = 0

# **Question 2**

If 
$$A = \begin{bmatrix} 1 & -3 & 2 & 5 & 0 \\ 0 & 2 & 6 & 7 & 2 \\ 3 & 4 & 5 & 1 & 0 \end{bmatrix}$$
 and  $B = \begin{bmatrix} 5 \\ 7 \\ 9 \\ 1 \\ 8 \end{bmatrix}$ , then the order of the matrix product A×B is

- **A.** (3×5)
- **B.** (5×3)
- **C.** (3×1)
- **D.** (1×3)
- E. Undefined

#### **Question 3**

Given = 
$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 1 & 5 \end{bmatrix}$$
,  $B = \begin{bmatrix} 3 & -1 \\ 4 & -3 \end{bmatrix}$  and  $C = \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 2 & 6 \\ 2 & 2 \end{bmatrix}$ 

The evaluation of  $3(A \times C-B)$  is

А.	[48  98	70 81	В.	[42  90	66 69]
C.	[48  98	68] 75]	D.	[42  90	72 87]

E. cannot be evaluated

The determinant of  $A = \begin{bmatrix} -2 & -5 \\ x & 7 \end{bmatrix}$  is equal to 1. The determinant of  $B = \begin{bmatrix} 6 & -2y \\ -2 & 3 \end{bmatrix}$  is equal to 2. The value of  $\frac{8x}{y}$  is **A.** -4

- A. -4
- B. 3C. 1.2
- **D.** 6
- **E.** 4.16

# Question 5

Joan, Brian and Harry went to the vegetable market for their weekly shopping. Joan bought 3 kg of cucumber, 4 kg of broccoli and 1 kg of tomatoes and paid \$19.60. Brian bought 5 kg of cucumber, 2 kg of broccoli and 1.5 kg of tomatoes and paid \$21.80. Harry bought 2 kg of cucumber, 2 kg of broccoli and 4.5 kg of tomatoes and paid \$23.50. They all shopped from the same stall. Let x, y, z represent the cost of one kg of tomatoes, cucumber and broccoli respectively. A matrix equation to calculate the cost of each item (per kg) is

А.	$ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left( \begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix} $	В.	$\begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix}$
C.	$\begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23.50 \\ 21.80 \\ 19.60 \end{bmatrix}$	D.	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 3 & 1 & 4 \\ 5 & 1.5 & 2 \\ 2 & 4.5 & 2 \end{bmatrix} )^{-1} \times \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix}$
E.	$ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left( \begin{bmatrix} 1.5 & 5 & 2 \\ 1 & 3 & 4 \\ 4.5 & 2 & 2 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 21.80 \\ 19.60 \\ 23.50 \end{bmatrix} $		

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#### The following information relates to Questions 6 to 7.

An ice cream factory conducted a survey in December 2009 on the ice cream sales of their three shops. The following information relates to this survey.

- •15% of shop A customers will move to shop B the following month.
- •37% of shop A customers will move to shop C the following month.
- •48% of shop B customers will move to shop C the following month.
- •42% of shop B customers will remain at shop B the following month.
- •5% of shop C customers will move to shop B the following month.
- •10% of shop C customers will move to shop A the following month.

The market share at the time of the survey showed that shop A had 1800 customers, shop B had 1000 customers and shop C had 1300 customers.

#### **Question 6**

The transition matrix showing the way customers move between the ice cream shops is given by

٨	This month	D	This month			
А.	A B C	D,	А	B C		
Next month	$\begin{bmatrix} 0.48 & 0.1 & 0.1 \\ 0.15 & 0.42 & 0.05 \\ 0.37 & 0.48 & 0.85 \end{bmatrix}$	Next month	$ \begin{array}{c} A \\ B \\ C \\ 0.37 \end{array} \left[ \begin{array}{c} 0.48 \\ 0.15 \\ 0.37 \end{array} \right] \left( \begin{array}{c} 0 \\ 0.37 \end{array} \right) \right] $	.42 0.05 .01 0.85 ).48 0.1		
C	This month	D	This month			
С.	A B C	<b>D</b> .	A B	С		
Next month	$\begin{bmatrix} 0.15 & 0.48 & 0.05 \\ 0.37 & 0.42 & 0.1 \\ 0.48 & 0.1 & 0.85 \end{bmatrix}$	Next month	A [15 48 B 37 42 C 48 10	5 10 85		
Е.	This month					
Next month	$\begin{array}{cccc}     A & B & C \\     A & \begin{bmatrix} 48 & 1 & 10 \\     B & 15 & 42 & 5 \\     C & 37 & 48 & 85 \end{bmatrix}$					

The number of customers expected to remain at each shop in September 2010 is

- A. Shop A: 1094 customers, Shop B: 755 customers, Shop C: 2251 customers
- B. Shop A: 661 customers, Shop B: 431 customers, Shop C: 3008 customers
- C. Shop A: 1024 customers, Shop B: 1071 customers, Shop C: 1027 customers
- **D.** Shop A: 638 customers, Shop B: 857 customers, Shop C: 2606 customers
- E. Shop A: 537 customers, Shop B: 1220 customers, Shop C: 2056 customers

# **Question 8**

The long-term share of customers shopping at each shop is expected to be

- A. Shop A: 755 customers, Shop B: 896 customers, Shop C: 2341 customers
- B. Shop A: 645 customers, Shop B: 428 customers, Shop C: 3003 customers
- C. Shop A: 1065 customers, Shop B: 1056 customers, Shop C: 1035 customers
- D. Shop A: 661 customers, Shop B: 430 customers, Shop C: 3008 customers
- E. Shop A: 587 customers, Shop B: 1343 customers, Shop C: 2234 customers

# **Question 9**

The movement of university students doing biomedical science course between two universities, university A and university B can be predicted by the transition matrix T where

		this year	
		А	В
next year	А	[0.3	0.1]
	В	0.7	0.9

. .

The number of biomedical science students in university A was twice as much as the number of biomedical science students in university B in 2009. In 2010 there were 115 students doing biomedical science in university B. The number of biomedical science students in university A in 2009 was

- A. 50
- **B.** 65
- **C.** 82
- **D.** 112
- **E.** 100

# END OF QUESTION AND ANSWER PAPER