2010 Further Mathematics Trial Examination 2 Suggested Solutions

2010

VCE Further Mathematics Trial Examination 2

Suggested Solutions

© Kilbaha Multimedia Publishing 2010



Kilbaha Multimedia Publishing ABN 47 065 111 373 PO Box 2227 Kew Vic 3101 Australia Tel: (03) 9817 5374 Fax: (03) 9817 4334 kilbaha@gmail.com http://kilbaha.com.au

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from **Copyright Agency Limited**. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000 Tel: (02) 9394 7600 Fax: (02) 9394 7601 Email: info@copyright.com.au

- Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
- Schools which purchase a licence to use this material may distribute this electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
- Schools which purchase a licence to use this material may distribute this printed file to the students at the school for their exclusive use.
- The Word file is for use ONLY within the school
- It may be modified to suit the school syllabus and for teaching purposes.
- All modified versions of the file must carry this copyright notice
- Commercial use of this material is expressly prohibited

Core



Core

b.

Question 2 a.



(1 mark) for correct point

On calculator press stat edit and enter temperature in L₁ and number of drinks sold in L₂. Go to stat calc LinReg (ax + b) L₁L₂. This gives $r^2 = 91.6\%$, to one decimal place

91.6% of the variation in the number of drinks sold can be explained by the variation in the temperature.

1 mark for each of the three answers.

(3 marks)

c. Using the same screen as for b. a = 17.25 and b = -93 So equation is

Number of drinks sold = $-93 + 17.25 \times temperature$

1 mark for each of the two values.

(2 marks)

Core

Question 2 (continued)

d.

$$440 = -93 + 17.25 \times t$$

 $533 = 17.25 \times t$
 $t = \frac{533}{17.25} = 30.9^{\circ}C$ (1 mark)

a. $\frac{1}{26} = 0.04$	b. On calculator press stat edit. Put the temperature values in L_1 , and the number of hot dogs sold in L_2
(1 mark)	In L ₃ put $L_3 = \frac{1}{L_1}$ Go to stat calc LinReg($ax + b$) L ₃ ,L ₂ This gives $a = 2290$ and $b = 15$ Number of hot dogs sold = $2290 \times \frac{1}{temperature} + 15$ (1 mark)
c . Number of hot dogs sold = 22	$90 \times \frac{1}{15} + 15 = 168$
	(1 mark)

a(i)	a(ii)
25 + 12 + 12 = 49	This is an arithmetic sequence with $a = 25$ and $d = 12$
	$t_n = a + (n-1)d$
	$t_n = 25 + (n-1)12$
	$t_n = 25 + 12n - 12$
	$t_n = 13 + 12n$
(1 mark)	(1 mark)
b.	
This is an arithmetic sequence	
$S_n = \frac{n}{2} [2a + (n-1)d]$	
$S_{16} = \frac{16}{2} [2 \times 25 + 15 \times 12]$	
$S_{16} = 8 \times (50 + 180) = 1840$ people.	
(1 mark)	

Question 2 a. i. a. ii. This is a geometric sequence with a = 5 and r = 1.4 $5 \times (1 + 0.4) = 7$ $t_n = ar^{n-1}$ $t_n = 5(1.4)^{n-1}$ (1 mark) (1 mark) b. This is a geometric sequence with a = 5, n=10 and r = 1.4 $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{10} = \frac{5(1.4^{10} - 1)}{1.4 - 1} = 349$ (1 mark) c. Enter two equations into calculator in sequence mode. Press y =nMin = 1 $\mu(n) = 2n + 13$ $\mu(nMin) = 25$ $v(n) = 5(1.4)\Lambda(n-1)$ v(nMin) = 5Press 2nd table When $n = 11, \mu(n) > v(n)$ But when n = 12, $v(n) > \mu(n)$ Hence it will take 12 days.

(1 mark)

Question 2 (continued)

d. Number infected = $25 + 40 + 1.2 \times 40 + (1.2)^2 \times 40 +$ $(1.2)^{18} \times 40$ Number infected = 65 + sum of geometric sequence where $a = 1.2 \times 40, r = 1.2$ and n = 18Number infected = $65 + \frac{1.2 \times 40(1.2^{18} - 1)}{1.2 - 1}$ Number infected = 65 + 6150 = 6215(1 mark)

a. i.	a. ii.
In one lot of 20 minutes, Number bacteria = double 4 = 8	3 lots of 20 minutes in 1 hour. 4,8,16,32 32 bacteria after one hour.
(1 mark)	(1 mark)
b.	с.
32 is 8 times greater than 4, so 8 times.	$B_t = 8B_{t-1} \qquad B_0 = 4$
(1 mark)	(1 mark)

Question 3 (continued)

d.	e.
Use calculator in sequence mode	Using the same computer table as in d, the answer can be seen to be 4 hours.
Press $y =$	
nMin = 1	
$\mu(n) = 8 \times \mu(n-1)$	
$\mu(nMin) = 4$	
Press 2nd table and go to $n = 6$.	
This is 5 hours later.	
$\mu(n) = 131,072$	
(1 mark)	(1 mark)
f.	
3600 killed in 1 hour. From table for d, when time is 4 hours, number bacteria = 16384 Number of hours to eliminate = $\frac{16384}{3600}$ = 4.6 hour	s to 1 d.p.
	(1 mark)

Module 2 Geometry and trigonometry



Module 2 Geometry and trigonometry

Question 1 (continued)

Gradient = $\tan \theta = \tan 23.57817848 = 0.44$

(1 mark)

Question 2

d.



Question 3

a.	b.
$\frac{1}{2} \times 18 = 9 \text{ m}$	Surface area = $\frac{1}{2}(2\pi r^2 + 2\pi rh)$ Surface area = $\frac{1}{2}(2\pi \times 9^2 + 2\pi \times 9 \times 22)$
(1 mark)	Surface area = 876.5 m^2 (1 mark)
c. V = Volume of cuboid + volume of half cylinder $V=18 \times 22 \times 12 + \frac{1}{2} \times \pi \times 9^2 \times 22$ $V = 7551.16 \text{ m}^3$	d. Heating cost/hour = $7551.16 \times \frac{0.02}{100}$ Heating cost/hour = $$1.51$ Heating cost/day = $$36.25$
(1 mark)	(1 mark)

		b.
a.		<i>AO</i> 20
There are 8 equal angles at the centre		$\frac{1}{\sin 67.5^{\circ}} = \frac{1}{\sin 45^{\circ}}$
that all add to 360° .		$AO = \frac{20}{1000} \times \sin 67.5^{\circ} = 26.13 \text{ m}$
Each angle = $\frac{360}{8} = 45^{\circ}$		sin 45°
$\angle AOB = 45^{\circ}$		
ΔAOB is isosceles		
$\therefore \angle OAB = \frac{1}{2}(180 - 45) = 67.5^{\circ}$		
	(1 mark)	(1 mark)
	(1	
c.		
Area of Piazza = $8 \times$ Area of triangle <i>AOB</i>		
Area of Piazza = $8 \times \frac{1}{2} \times 26.13^2 \times \sin 45^\circ$		
Area of Piazza = 1931.18 m^2		
		(1 mark)

a. i.	a. ii.
b.	
$65 \times 10 = 650	$65 \times x = \$65x$
(1 mark)	(1 mark)
b.	с.
$C = \$(20 x \pm 37, 890)$	65 r = 20 r + 37,800
$C = \Im(20x + 57, 850)$	0.5x - 20x + 57,890
	45x = 37,890
(1	x = 842
(1 mark)	(1 mark)
d.	е.
37890 - 3789 = \$34,101	$SP = 65 \times 30000 = \$1,950,000$
	$CP = 40 \times 30000 + 3789 = \$1,203,789$
(1 mark)	CP < SP
	: Profit
	Profit = 1,950,000 - 1,203,789
	Profit = \$746211
	Profit (1 mark)
	Actual profit (1 mark)





Question 2 (continued)

e. Profit = 30 p + 21q (1 mark) f. The corner points are (0,0), (0,10), (30,70) and (75,25) Profit at (0,0) = \$0 Profit at (0,10) = $21 \times 10 = 210 Profit at (30,70) = $30 \times 30 + 21 \times 70 = 2370 Profit at (75,25) = $30 \times 75 + 21 \times 25 = 2775 ∴ make 75 units of glasswork and 25 units of plastic. (1 mark)

Module 4 Business-related mathematics

Question 1

a.	b.
$\frac{48}{25} = \$1.92$ (1 ma	rk) Increase = $57 - 48 = \$9$ % increase = $\frac{9}{48} \times 100 = 18.75\%$ (1 mark)

Question 2

a.	b.
Total earned = $280 + \frac{12}{100} \times 1300 = 436	Commission = $464.50 - 150 = 314.50$ Let percentage be $x\%$
	$\frac{x}{100} \times 1850 = 314.50$
	$x = 314.50 \div 1850 \times 100 = 17\%$
(1 mark)	(1 mark)

a.	b. i.	
Stamp duty = $60 + \frac{4}{100}(45000 - 3000) = 1740	There are 60 months in 5 years. Instalments paid = $950 \times 60 = $57,000$	
(1 mark)		(1 mark)
b. ii.	b. iii.	
Total amount paid = 57,000 + 5000 = \$62,000	Interest = $57,000 - 45,000 = $12,000$	
(1 mark)		(1 mark)
b. iv.		
$R = \frac{100I}{Pt} = \frac{100 \times 12000}{45000 \times 5} = 5.3\%$		(1 mark)

Module 4 Business-related mathematics

Question 4

a.	b.
$\frac{6}{12} = 0.5\%$	$\frac{0.5}{100} \times 50000 = 250
(1 mark)	(1 mark)
c.	d.
50000(1.005) ⁶⁰ = \$67442.51	Press Apps Finance Enter TVM Solver Enter N = 60 I = 6 PV = 50000 PMT = FV = 0 P/Y = 12 C/Y = 12 End Put cursor on <i>PMT</i> and press alpha solve. PMT = \$966.64
(1 mark)	(1 mark)

a.	b.
Amount depreciation = $45000 - 18800$ Amount depreciation = 26200 Cost/km = $\frac{26200}{42000}$ = 0.62 = 62 cents.	Depreciation = $14000 \times \frac{62}{100} = 8733.33$ Value after two years = $18,800 - 8733.33$ Value after two years = $$10,066.67$
(1 mark)	(1 mark)

Module 5 Networks and decision mathematics





Module 5 Networks and decision mathematics

Question 1 (continued)





Module 5 Networks and decision mathematics

Question 2 (continued)

e	f	
c.	1.	
$D^{2} = \begin{bmatrix} A & B & C & D & E \\ B & B & \\ D & C & \\ D & E & \end{bmatrix}$ $\begin{pmatrix} A & B & C & D & 2 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$	Yacht Alpha Star Battering Ram Challenger Defending Champ Earnest Endeavour	Two – Step Dominance 4 4 1 5 2
(1 mark)	(1 (2	mark) for any two correct. 2 marks) for all five correct
g.		
$D^{r} + D^{r} \text{ gives}$ $A = 2 + 4 = 6$ $B = 3 + 4 = 7$ $C = 1 + 1 = 2$ $D = 3 + 5 = 8$ $E = 1 + 2 = 3$ Place	Yacht	
1	Defending Cham	
1	Defending Cham	<u>p</u>
2	Battering Ram	
3	Alpha Star	
4	Earnest Endeavo	ur
5	Challenger	
		(1 mark) for 2 correct (2 marks) for all correct.
h.		
If B defeated A, then A defeated C, D and E. This means that E defeated B, C and D. Hence, D defeated B and C. So C defeated B i.e. <i>Battering Ram</i> .		

(1 mark)

Module 6 Matrices

a.		b. i.
$A = \left[\begin{array}{cc} 8 & 2 \\ 3 & 12 \end{array} \right]$	(1 mark)	$\begin{bmatrix} 8 & 2 \\ 3 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ (1 mark)
b. ii.		с.
$\left[\begin{array}{rrr} 20 & 8\\ 13 & 25 \end{array}\right]$		8 is the number in the first row, second column. It represents the number of behinds obtained by Hawood.
	(1 mark)	(1 mark)
$ \begin{bmatrix} 6 \\ 1 \end{bmatrix} $	(1 mark)	e. $\begin{bmatrix} 20 & 8 \\ 13 & 25 \end{bmatrix} \times \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ (1 mark) for correct matrices (1 mark) for multiplication.
f. i.		f. ii.
$\begin{bmatrix} 120+8\\78+25 \end{bmatrix} = \begin{bmatrix} 128\\103 \end{bmatrix}$	(1 mark)	2×1
		(1 mark)

Module 6 Matrices

a. i. 58% H 15% 42%				
	(1 mark)			
a. ii. This Week <i>H I</i> $\begin{bmatrix} 0.58 & 0.85 \end{bmatrix} H$	b. $S_0 = \begin{bmatrix} 24\\ 10 \end{bmatrix}$			
$\begin{bmatrix} 0.30 & 0.05 \\ 0.42 & 0.15 \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix}$ Next Week (1 mark)	(1 mark)			
c. $S_2 = T^2 S_0$ $S_2 = \begin{bmatrix} 0.58 & 0.85 \\ 0.42 & 0.15 \end{bmatrix}^2 \begin{bmatrix} 24 \\ 10 \end{bmatrix} = \begin{bmatrix} 22.8466 \\ 11.1534 \end{bmatrix}$ Expect 11 to be injured.	d. $S_{100} = T^{100}S_{0}$ $S_{100} = \begin{bmatrix} 22.76\\ 11.24 \end{bmatrix}$ Expect about 23 players to be healthy in the long term.			
(1 mark)	(1 mark)			

Module 6 Matrices

Question 3

Let number of people who bought beef, chicken, vegetable and curry pies last week be x, 3x, x and x, respectively.

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} x \\ 3x \\ x \\ x \end{bmatrix} = \begin{bmatrix} 0.6x + 0.3x + 0.1x + 0.2x \end{bmatrix}$$
$$= \begin{bmatrix} 1.2x \end{bmatrix}$$
$$1.2x = 12,000$$
$$x = 10,000$$
$$6x = 60,000$$

(1 mark)

End of Suggested Solutions 2010 Further Mathematics VCE Trial Examination 2

Kilbaha Multimedia Publishing	Tel: (03) 9817 5374
PO Box 2227	Fax: (03) 9817 4334
Kew Vic 3101	kilbaha@gmail.com
Australia	http://kilbaha.com.au