Neap:

Trial Examination 2011

VCE Further Mathematics Units 3 & 4

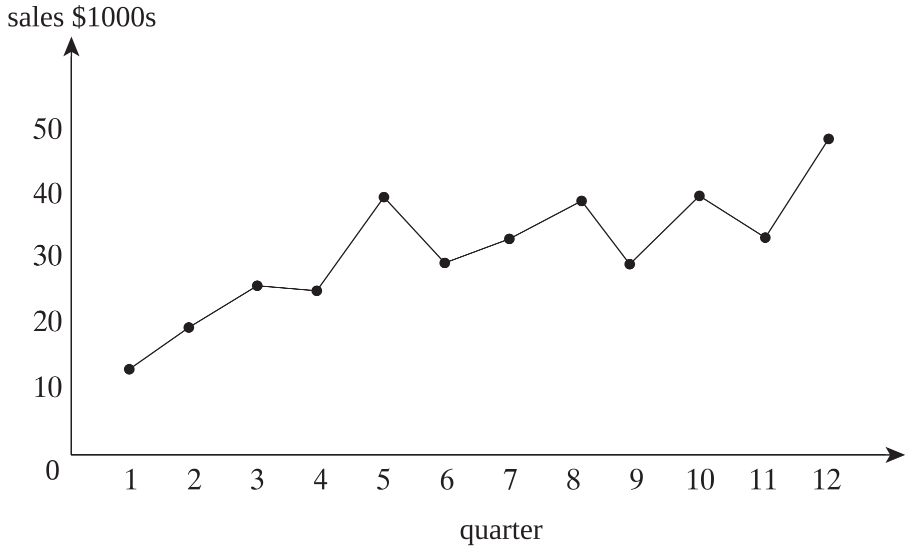
Written Examination 2

Suggested Solutions

Core

Question 1

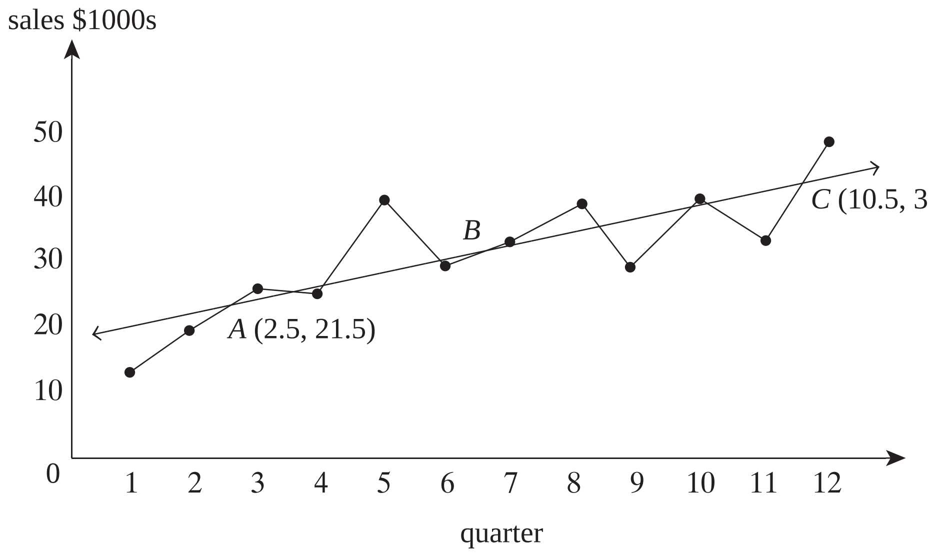
a.



A2

b. There is a moderate positive relationship. A1

c.



A1

The gradient of $AC = \frac{\text{rise}}{\text{run}} = \frac{37 - 21.5}{10.5 - 2.5}$ M1

$= \frac{16.5}{8}$

$= 2.1$

Sales ($1000s) = A + 2.1 × Quarter

Reading from the graph the intercept is approximately 18

Equation in Sales($1000s) = 18 + 2.1 × Quarter A1

As the 18 is read from the graph, it is only approximate so answers between 17.5 and 18.5 are acceptable.

Question 2

a. $\bar{x} = \dfrac{14 \times 0.5 + 2 \times 1.5 + 1 \times 2.5 + 1 \times 4.5}{18}$ A2

$= \dfrac{17}{18} = 0.94$

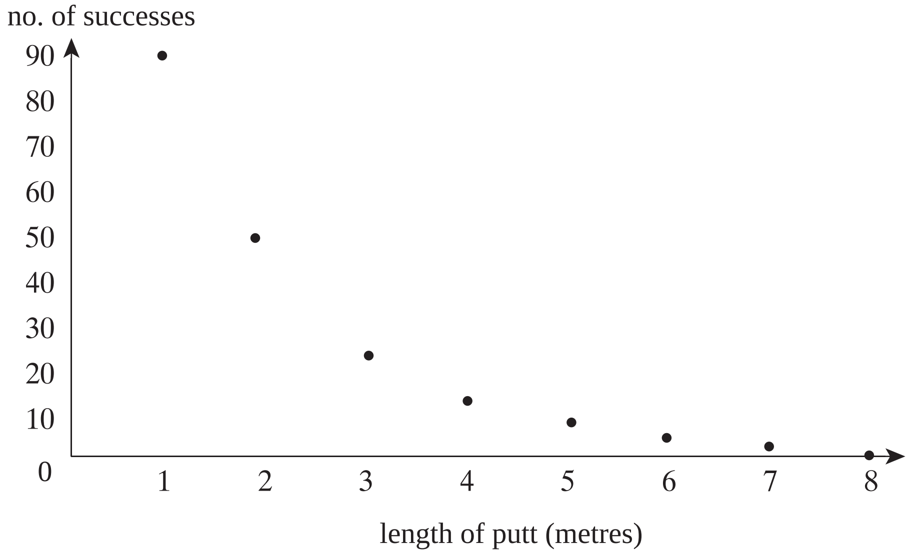
b. The modal class is $3 - < 4$ metres. A1

Accept mode = 3.5 m

c.

i. Length of putt is the independent variable. A1

ii.



M1 A1

iii. The dataset is not linear and Pearson's product-moment correlation coefficient should not be used. A1

iv. Using the calculator $r = 0.99$ A1

Number of successful putts per 100 putts = $-11.7 + 105.7 \times \dfrac{1}{\text{Length of putt (m)}}$ A1

Module 1: Number patterns

Question 1

a. There is a common difference and thus it is an arithmetic sequence. A1

b. We need term 21 (20 terms after 2010)

$a = 600$

$r = 20$

$t_{21} = a + (n-1)d$

$= 600 + 20(20)$

$= 1000$ A1

$S_{21} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{21}{2}[1200 + 400] = 16\ 800$ A1

c. $S_{21} = \frac{n}{2}[2a + (n-1)d] = 400\ 000$ M1

$\frac{n}{2}[1200 + 20n - 20] = 400\ 000$

$n(n + 59) = 40\ 000$

The value of n can be found in various ways from this point but most would use the graphing calculator. Graphs of $n(n + 59)$ and $y = 40\ 000$ could be compared to find where they cross.

Alternatively the sequence $t_n = n(n + 59)$ could be graphed on the calculator and traced until it exceeded 400 000 for the first time. Those familiar with quadratics could use the quadratic formula to obtain the result.

All methods should obtain the result term 173. Thus the required year is 2182. A1

d. $a = 600$ A1

$r = 1.02$

$t_{21} = 600(1.02)^{20} = 891.57$

e. This is another question for which calculator use is essential. Two sequences, those of Shouad and Jill should be entered separately and compared. M1

Jill's is lower for the first few predictions but eventually catches up to and exceeds that of Shouad at term 50. Thus the required year is 2059. A1

f. $P_{n+1} = 1.01P_n + 10$ M1

Thus $a = 1.01$ and $b = 10$. A1

g. $P_2 = 1.01(600) + 10 = 616$ M1

$P_3 = 1.01(616) + 10 = 632.16$

$P_4 = 1.01(632.16) + 10 = 648.4816$

$P_5 = 1.01(648.4816) + 10 = 664.9664$

$P_6 = 1.01(664.9664) + 10 = 681.62$

Thus the premium is \$682. A1

h. $P_{n+2} = 1.025P_{n+1} - 0.4(P_{n+1} - P_n)$

$P_{n+2} = 0.625P_{n+1} + 0.4P_n$

Thus $a = 0.625$ and $b = 0.4$ A1

i. $P_{n+2} = 0.625P_{n+1} + 0.4P_n$ M1

$P_3 = 0.625(650) + 0.4(600) = 646.25$

$P_4 = 0.625(646.25) + 0.4(650) = 663.91$

The premium is predicted to be $663.91 A1

Module 2: Geometry and trigonometry

Question 1

a. $\frac{14.4}{21.4} = \tan\theta$ A1

$33.9365...° = \theta$

$34° \cong \theta$

b. $21.4 + EF + 10.0 = 46.8$

$EF = 15.4$ m

A1

c. $h^2 = a^2 + b^2$

$h^2 = 21.4^2 + 14.4^2$

$h = 25.7938...$

$h \cong 25.8$ m

A1

d. For DEJ Area $= \frac{1}{2} \times 21.4 \times 14.4$

$= 154.08$ m^2

For $FGHI$ Area $= \frac{10 + 31.4}{2} \times 14.4$ M1

$= 298.08$ m^2

Total $= 154.08 + 298.08 \cong 452.2$ m^2 A1

e. area of walkway = total area – (triangle DEJ + trapezium $FGHI$) M1

$= (44.8 \times 14.4) - 452.16$

$= 221.76$ cm^2

Area = base × height (where height = width of path)

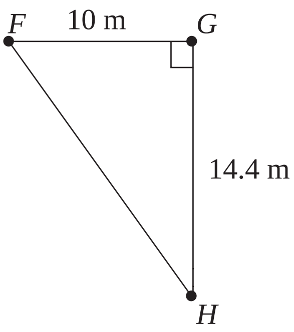
$221.76 = 25.7938 \times$ height

$8.5974 =$ height

width of path $\cong 8.6$ m A1

Question 2

a.

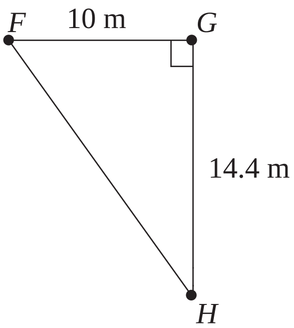


$$\frac{10}{14.4} = \tan\theta$$ M1

$34.8° = \theta$

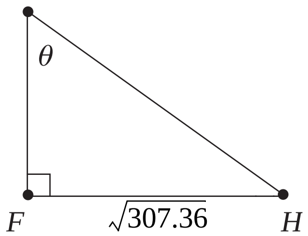
Thus bearing $HF = 360° - 34.8° \cong 325.2°$. A1

b.



$FH^2 = 10.0^2 + 14.4^2$

$FH^2 = 307.36$



$$\frac{\sqrt{307.36}}{13.5} = \tan\theta$$

$52.4° = \theta$ M1

angle of depression = $90° - 52.4° = 37.6°$ A1

Question 3

a. $\frac{4}{x} = 0.25$ A1

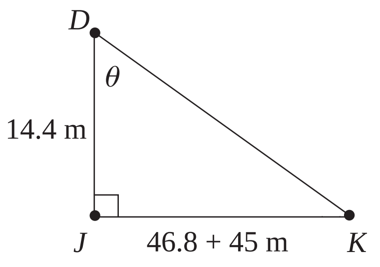
$\frac{4}{0.25} = x$

$16 \text{ m} = x$

b. $135 \text{ m} = 13\ 500 \text{ cm}$ A1

$\frac{13\ 500}{300} = 45 \text{ cm}$

Question 4



$DK^2 = 14.4 \ + (46.8 + 45)^2$

$DK = 92.9225 \text{ m}$ M1

Distance *KD* is less than the height of the building so point *D* will be hit by the falling building. A1

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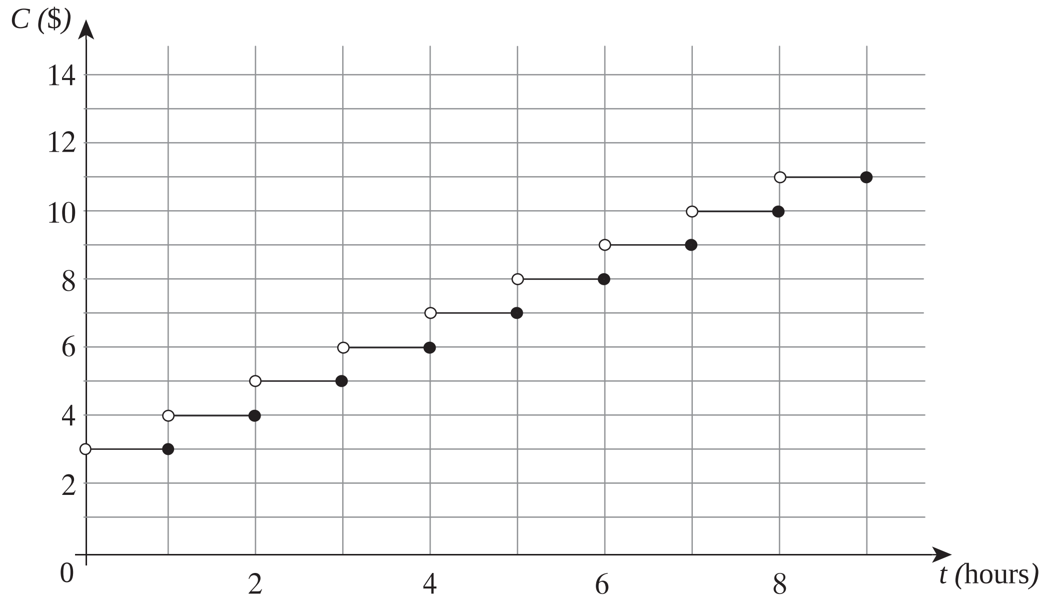
Module 3: Graphs and relations

Question 1

a. He will be charged for four hours.

Cost = 2 + 4(1) = 6
\$6 A1

b.



Required region unshaded.

1 mark for correct line locations, 1 for correct endpoints A2

c. $C = 500 + 0.5x$ A1

d. 9 hours parking costs \$11. Thus $R = 11x$. A1

e. When Bridget breaks even: $11x = 500 + 0.5x$

$$10.5x = 500$$

$$x = \frac{500}{10.5} = 47.6$$

Thus Bridget requires a minimum of 48 cars to make a profit. A1

Question 2

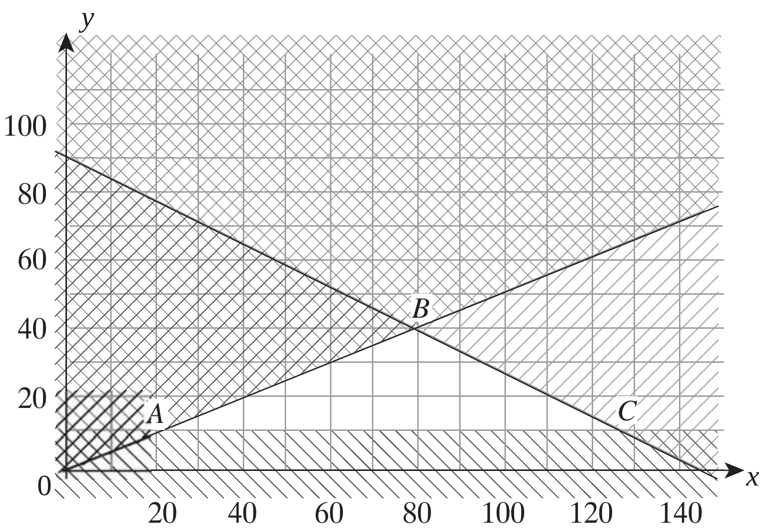
a. The inequation relates to the use of the 720 square metres of space. If x small car spaces exist they will occupy $5x$ square metres. If y large car spaces exist they will occupy $8y$ square metres. Thus total space must be $5x + 8y$ and can't exceed the total of 720. A1

b. $x \geq 2y$

$y \geq 10$

One mark lost for each incorrect or missing inequation A2

c.



A2

d. $P = 20x + 30y$ A1

e. Find coordinates of all points and determine value of P at each.

$x = 2y$	$x = 2y$	$5x + 8y = 720$
$y = 10$	$5x + 8y = 720$	$y = 10$
A: $\therefore x = 20$	B: $\therefore 10y + 8y = 720$	B: $\therefore 5x = 640$
$A(20, 10)$	$y = 40, x = 80$	$x = 128$
$P = 400 + 300 = 700$	$P = (1600 + 1200) = 2800$	$P = 2560 + 300 = 2860$

Calculation of coordinates A1

Thus maximum profit is \$2860 per day. A1

f. The maximum profit was achieved at point C on the graph: 128 small and 10 large car spaces. A1

Module 4: Business-related mathematics

Question 1

a. The saving is \$16.50, so % saving is $\frac{16.50}{31.50} \times 100 = 52.4\%$ A1

b. **i.** deposit = 0.2 × 199 = 39.80 + 18 payments of \$12 = total of \$255.80 A1

ii. Interest is \$56.80 so interest rate = $\frac{56.80 \times 100}{159.20 \times 1.5} = 23.8\%$ A1

iii. $r_e = \frac{2n \times r_f}{n+1} = \frac{2 \times 18 \times 23.8}{18+1}$ A1

$= 45.1\%$

c. The decrease in value is \$159.

Rearranging $I = \frac{Prt}{100}$ gives r (% depreciation) = $\frac{154 \times 100}{199 \times 2} = 38.7\%$ A1

Question 2

a. Using your CAS calculator (or similar), enter $N = 12$, $R = 8$, *Initial Value* is 0, *Future Value* is \$40 000, compounding periods and number of payments are both 12. The value of the monthly deposit is \$3212.87, or to the nearest dollar \$3213. A1

b. Using your CAS calculator (or similar), enter $N = 25 \times 12 = 300$, $R = 7.75$, *Initial Value* is \$300 000, *Future Value* is 0, compounding periods and number of payments are both 12. The monthly repayment is \$2266. A1

c. Using your CAS calculator (or similar), keep the settings from **part b** and change only the value of N to 20 × 12 = 240. The monthly repayment is \$2462.85. Then calculate the total repayments by 240 × 2462.85 = 591 084.

The total repayments over 25 years is 25 × 12 × 2266 = \$679 800.

The money saved is 679 800 – 591 084 = \$88 716. A1

d. Using your CAS calculator (or similar), enter repayments are 2766, $R = 7.75$, *Initial Value* is 300 000, *Future Value* is 0, compounding periods and number of payments are both 12. The loan is paid off in 187.3 months which, to the nearest month, is 187 months. A1

Question 3

a. The decrease in book value is \$81 000 for 20 million caps. Depreciation per cap is:

$\frac{8\ 100\ 000 \text{ cents}}{20\ 000\ 000} = 0.405$ cents per cap A1

b. 3 000 000 × 0.405 cents = \$12 150.00
Book value = 85 000 – 12 150 = 72 850 A2

c. $\frac{20\ 000\ 000}{3\ 500\ 000} = 69$ months A1

d. 69 months at \$1220 each month is \$84 180, which is dearer than the total of \$85 000 for purchase minus the \$4000 for scrap, which give a total of \$81 000. A1

Module 5: Networks and decision mathematics

Question 1

a. Cut A = 100 seats. A1
Cut B = 100 seats. A1

b. Cut C does not prevent all flow of traffic from Warsaw to Dublin. A1

c. Minimum cut = $20 + 40 = 60$ seats A1

Question 2

a. Lily must complete Task A.
Harly must complete Task E.
This means that James must complete Task D. A1

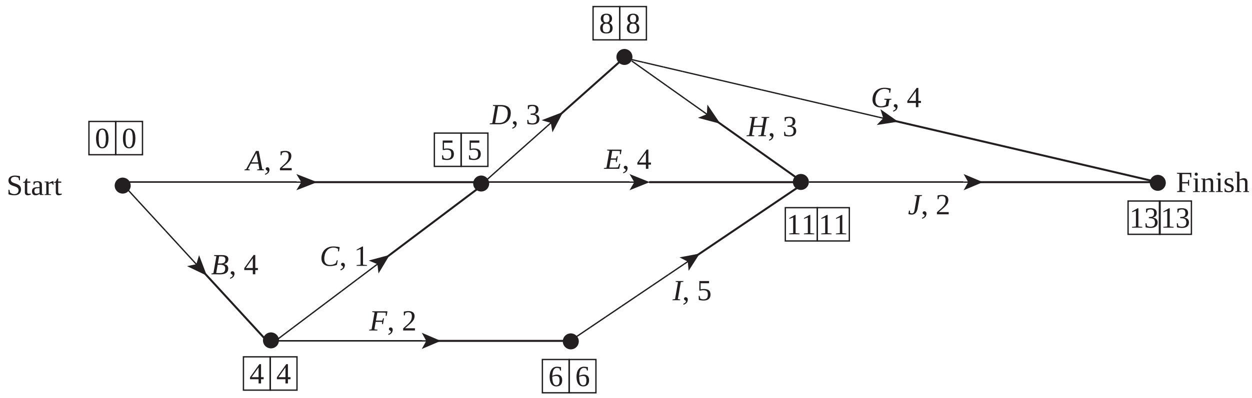
b.

Employee	**Task**
Kristy	C
Harley	E
Charlie	B
Lily	A

A1

A1

Question 3



a.

Activity	Immediate Predecessor	Earliest Start Time	Latest Start Time
A	–	0	3
B	–	0	0
C	*B*	4	4
D	*A*, *C*	5	5
E	*A*, *C*	5	7
F	*B*	4	4
G	*D*	8	9
H	*D*	8	8
I	*F*	6	6
J	*E*, *H*, *I*	11	11

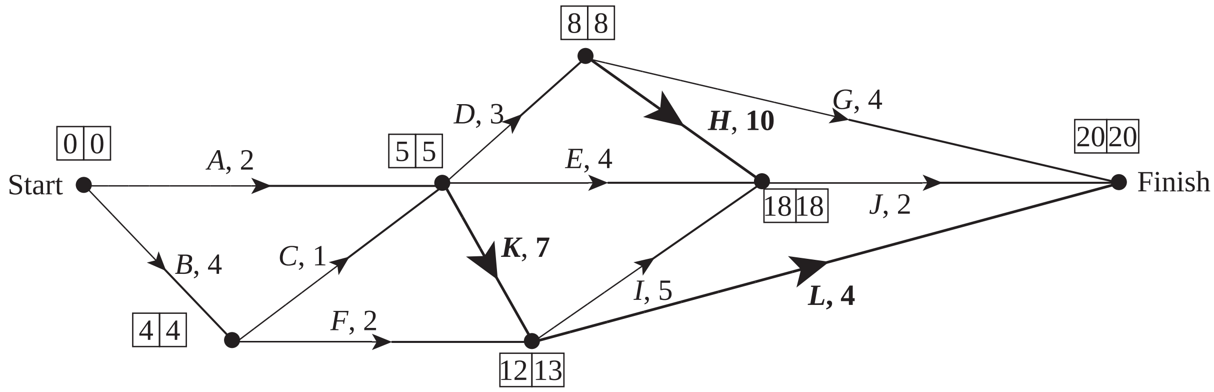
A3

b. There are two critical paths:

$B \rightarrow F \rightarrow I \rightarrow J$

$B \rightarrow C \rightarrow D \rightarrow H \rightarrow J$ A1

Question 4



a. The non-critical activities are *A*, *E*, *F*, *G*, *I*, *K* and *L* A1

b. The earliest start time for *Activity H* is 8 minutes. A1

c. Original slack time for *Activity E*: $LFT - EST -$ duration $= 11 - 5 - 4$

$= 2$ minutes

Revised slack time for *Activity E*: $LFT - EST -$ duration $= 18 - 5 - 4$ M1

$= 9$ minutes

Difference = 9 – 2 = 7. A1

Module 6: Matrices

Question 1

a. $NP = \begin{bmatrix} 54 & 21 & 18 \end{bmatrix} \begin{bmatrix} 20 \\ 23 \\ 28 \end{bmatrix} = \begin{bmatrix} 2067 \end{bmatrix}$

This is the total value of shirt sales. A1

$NP = \begin{bmatrix} 20 \\ 23 \\ 28 \end{bmatrix} \begin{bmatrix} 54 & 21 & 18 \end{bmatrix} = \begin{bmatrix} 1080 & 420 & 360 \\ 1242 & 483 & 414 \\ 1512 & 588 & 504 \end{bmatrix}$ A1

This matrix has no physical significance. The leading diagonal involves values in each matrix multiplied by corresponding values in the other (i.e. corresponding types of shirts) but the other values do not. For example, the value 1242 in row 2, column 1, results from multiplying the cost of business shirts by the number of standard shirts sold – a process with no physical meaning.

A1

b. $R = \begin{bmatrix} 1.06 & 0 & 0 \\ 0 & 1.08 & 0 \\ 0 & 0 & 1.10 \end{bmatrix}$

All values are zero except those on leading diagonal because leading diagonal describes how each new price is affected by that of the previous price of that same item. It is only the previous price of each item that affects the new price – not the previous price of other items.

Thus:

$x = 6\%$ A1

$y = 8\%$ A1

$z = 10\%$ A1

c. *deal* 1 = 0.8 (2 × standard + 1 business)

deal 2 = 1 formal + 0.5 × standard

deal 3 = 2 formal

equivalent justification/working M1

$C = \begin{bmatrix} 1.6 & 0.8 & 0 \\ 0.5 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ A1

d. The price of *deal* 2 after the price change is:

$$CRP = \begin{bmatrix} 1.60 & 0.80 & 0 \\ 0.50 & 0 & 1.00 \\ 0 & 0 & 2.00 \end{bmatrix} \begin{bmatrix} 1.06 & 0 & 0 \\ 0 & 1.08 & 0 \\ 0 & 0 & 1.10 \end{bmatrix} \begin{bmatrix} 20 \\ 23 \\ 28 \end{bmatrix} = \begin{bmatrix} 1.60 & 0.80 & 0 \\ 0.50 & 0 & 1.00 \\ 0 & 0 & 2.00 \end{bmatrix} \begin{bmatrix} 21.20 \\ 24.84 \\ 30.80 \end{bmatrix} = \begin{bmatrix} 53.79 \\ 41.40 \\ 61.60 \end{bmatrix}$$

A business shirt will now cost \$24.84 and a standard shirt will cost \$21.20.

Therefore *deal* 2 now costs \$41.40. A1

Question 2

a. $\begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix} = \begin{bmatrix} a & 1-a \\ 1-a & a \end{bmatrix} \begin{bmatrix} 0.40 \\ 0.60 \end{bmatrix}$ *original matrix equation* M1

$0.52 = 0.4a + 0.6 - 0.6a$

$-0.08 = -0.2a$ M1

$a = 0.40$

b. $S_3 = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix} = \begin{bmatrix} 0.496 \\ 0.504 \end{bmatrix}$

The proportions are 0.496 Manton's and 0.504 Greatfit in 2011. A1

c. $T^{50} = \begin{bmatrix} 0.500 & 0.500 \\ 0.500 & 0.500 \end{bmatrix}$

In the long term, the proportions are 0.5 for each. A1

d. We would require the inverse of the transition matrix, T^{-1}.

$T^{-1} = \dfrac{1}{-0.2} \begin{bmatrix} 0.4 & -0.6 \\ -0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix}$ M1

$S_0 = T^{-1} S_1$

$= \begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Manton's had a proportion of 1.00 of sales in 2008. A1