VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2011

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

FURTHER MATHEMATICS

Written examination 2

Monday 7 November 2011

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 38 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Core

Question 1

The stemplot in Figure 1 shows the distribution of the average age, in years, at which women first marry in 17 countries.

average age, in years, of women at first marriage

key: 27|3 represents 27.3 years

24	
25	0
26	6
27	1 1 3 4 7
28	2 2 2 3 3 6
29	1 1
30	1 4
31	

Figure 1

a. For these countries, determine

i. the lowest average age of women at first marriage

ii. the median average age of women at first marriage.

1 + 1 = 2 marks

The stemplot in Figure 2 shows the distribution of the average age, in years, at which men first marry in 17 countries.

average age, in years, of men at first marriage

key: 32|5 represents 32.5 years

25	
26	0
27	
28	9
29	0 9 9
30	0 0 3 5 6 7 9
31	0 0 2
32	5 9
33	

Figure 2

b. For these countries, determine the interquartile range (IQR) for the average age of men at first marriage.

1 mark

Core – Question 1 – continued

c. If the data values displayed in Figure 2 were used to construct a boxplot with outliers, then the country for which the average age of men at first marriage is 26.0 years would be shown as an outlier.

Explain why this is so. Show an appropriate calculation to support your explanation.

2 marks

Question 2

Table 1 shows information about a particular country. It shows the percentage of women, by age at first marriage, for the years 1986, 1996 and 2006.

Table 1

Age of women at first marriage	**Year of marriage**		
	1986	**1996**	**2006**
19 years and under	8.5%	3.7%	2.0%
20 to 24 years	42.1%	31.3%	21.5%
25 to 29 years	23.4%	31.7%	34.5%
30 years and over	26.0%	33.3%	42.0%

a. Of the women who first married in 1986, what percentage were aged 20 to 29 years inclusive?

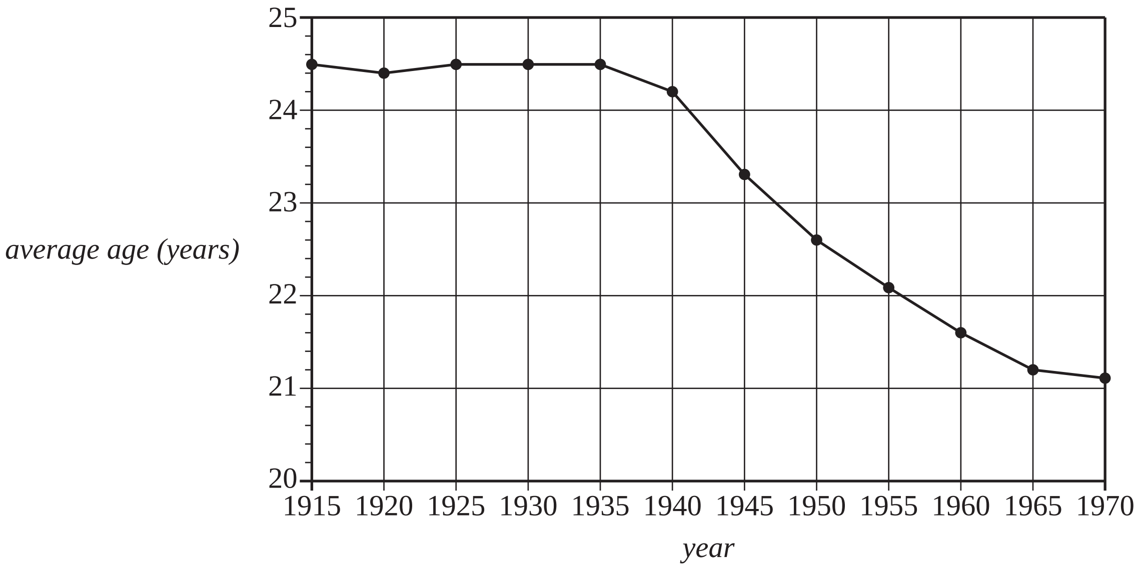
1 mark

b. Does the information in Table 1 support the opinion that, for the years 1986, 1996 and 2006, the age of women at first marriage was associated with year of marriage?
Justify your answer by quoting appropriate percentages. It is sufficient to consider one age group only when justifying your answer.

2 marks

Question 3

The following time series plot shows the average age of women at first marriage in a particular country during the period 1915 to 1970.

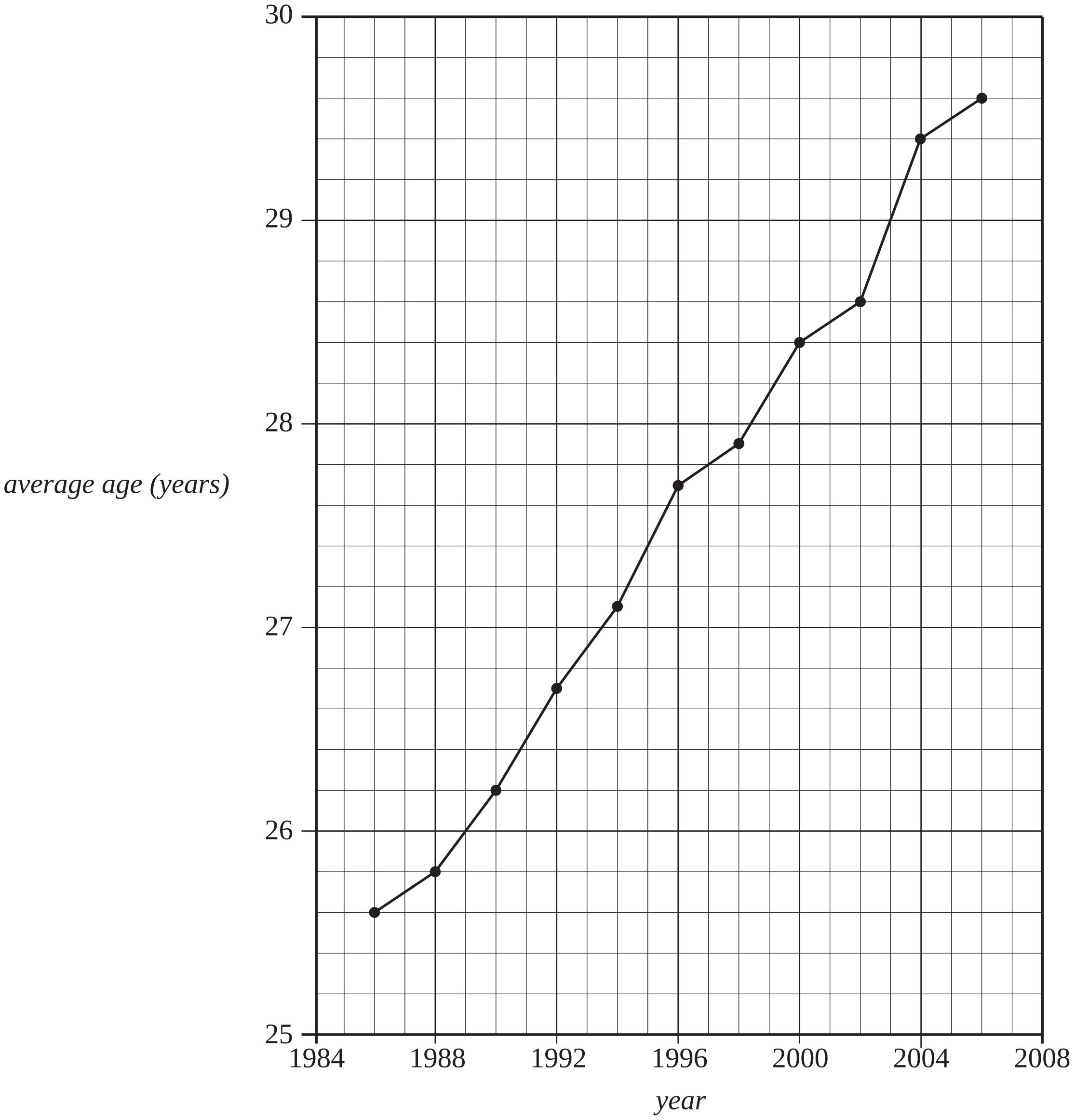


a. Use this plot to describe, in general terms, the way in which the average age of women at first marriage in this country has changed during the period 1915 to 1970.

1 mark

Core – Question 3 – continued

During the period 1986 to 2006, the average age of men at first marriage in a particular country indicated an increasing linear trend, as shown in the time series plot below.



A three-median line could be used to model this trend.

b. On the graph above

- **i.** clearly mark with a cross (×) the three points that would be used to fit a three-median line to this time series plot
- **ii.** draw in the three-median line.

2 + 1 = 3 marks

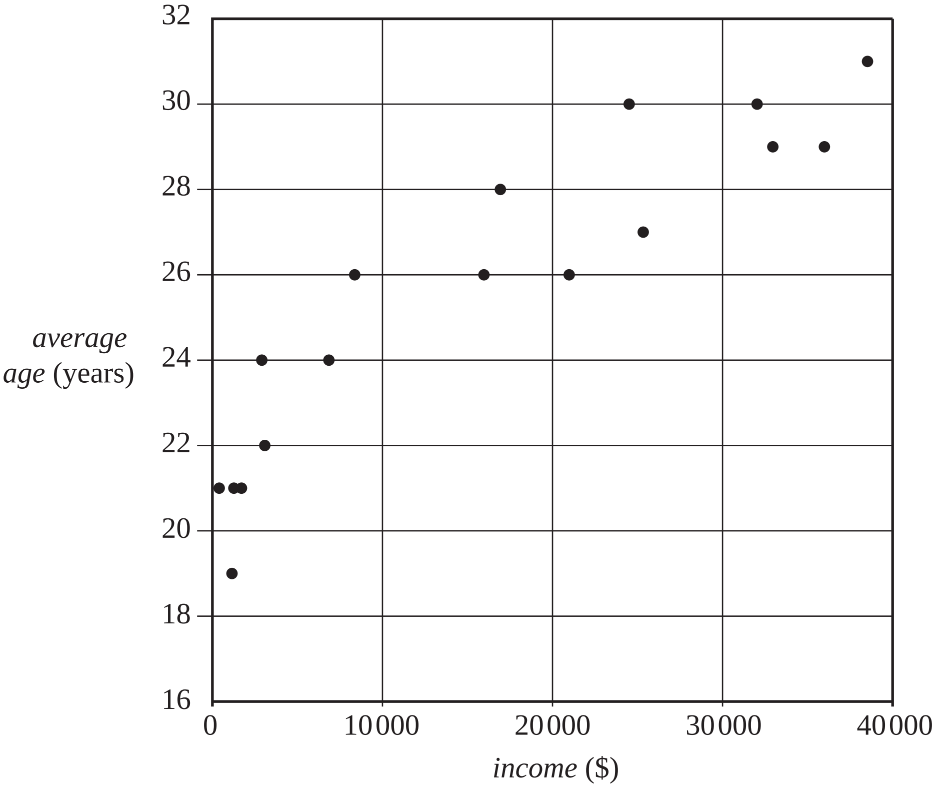
Question 4

The average age of women at first marriage in years (*average age*) and average yearly income in dollars per person (*income*) were recorded for a group of 17 countries.

The results are displayed in Table 2. A scatterplot of the data is also shown.

Table 2

average age (years)	*income* ($)
21	1750
22	3200
26	8600
26	16000
28	17000
26	21000
30	24500
30	32000
31	38500
29	33000
27	25500
29	36000
19	1300
21	600
24	3050
24	6900
21	1400



The relationship between *average age* and *income* is nonlinear.

A **log transformation** can be applied to the variable *income* and used to linearise the scatterplot.

a. Apply this log transformation to the data and determine the equation of the least squares regression line that allows *average age* to be predicted from log (*income*).

Write the coefficients for this equation, correct to two decimal places, in the spaces provided.

average age = ☐ + ☐ × log (*income*)

2 marks

b. Use this equation to predict the average age of women at first marriage in a country with an average yearly income of $20 000 per person.

Write your answer correct to one decimal place.

1 mark

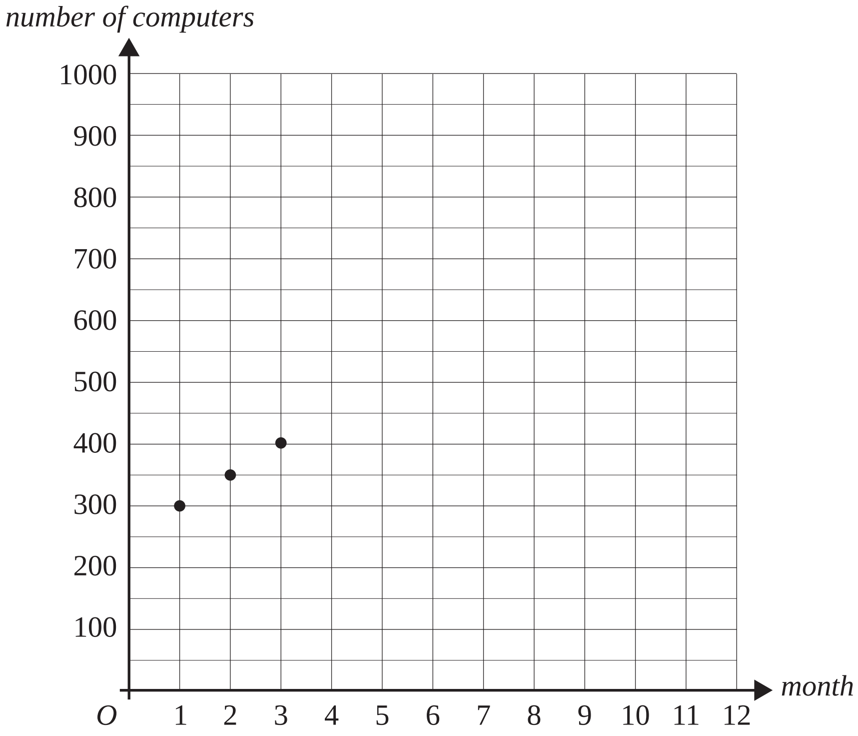
Total 15 marks

END OF CORE

Module 1: Number patterns

Question 1

The number of computers produced by a company each month forms the terms of an arithmetic sequence.
The graph below shows the number of computers produced in each of the first three months of the year.



a. On the graph above, **plot** the point for the number of computers produced in month 7.

1 mark

b. What is the common difference for this sequence of terms?

1 mark

c. How many more computers will the company produce in month 12 than they did in month 9?

1 mark

Question 2

Streaming Media is a company that provides Internet access to its customers.

Customers are charged for data transfer during each calendar month (Jan, Feb, . . .) as follows.

- The first gigabyte (GB) of data transfer costs $4.
- Each GB of data transfer after this costs 20 cents less than the previous one.
- Once the cost of a GB of data transfer reaches zero, any additional data transfer during that month is also free.

a. How much does the fifth GB of data transfer cost?

1 mark

b. What is the cost of transferring a total of eight GB of data during a calendar month?

1 mark

A customer paid the maximum charge for data transfer in a month.

c. What is the minimum number of GB of data that this customer could have transferred?

1 mark

Let D_n be the number of gigabytes (GB) that Danny transfers in his *n*th month with Streaming Media.
Danny finds that his data transfer each month can be determined using the difference equation

$$D_{n+1} = 1.5D_n - 1 \quad \text{where} \quad D_1 = 6$$

d. **i.** How many GB of data does Danny transfer in his third month?

ii. Determine how much Danny will be charged in his third month.

1 + 1 = 2 marks

e. According to the difference equation above, in which month will Danny's transfer first exceed 100 GB of data?

1 mark

Question 3

Streaming Media also offers a 12-month Internet access contract.
The charge for the first month is $50.
Each month after this, customers will be charged 95% of the previous month's charge.

a. What is the charge for the second month of the contract?

1 mark

b. Determine the difference between the charges for the first month and the sixth month.
Write your answer to the nearest cent.

1 mark

c. Determine the total of all payments made for a 12-month contract.
Write your answer correct to the nearest dollar.

1 mark

Question 4

The number of customers on contract with Streaming Media is decreasing.
At the start of each month, 1% of the previous month's customers do not renew their contracts and 100 new customers sign a contract.

At the start of January, there were 200 000 customers on contract with Streaming Media.

a. Determine the number of customers on contract with Streaming Media at the start of February.

1 mark

Let C_n be the number of customers on contract with Streaming Media at the start of the *n*th month. Let January be month 1.

b. Write a difference equation, in terms of C_n and C_{n+1}, that can be used to generate the number of customers on contract with Streaming Media at the start of each month from February.

1 mark

c. How many more customers, in addition to the regular 100 new customers, must be attracted at the start of March to bring the total number of customers on contract to 200 000?

1 mark

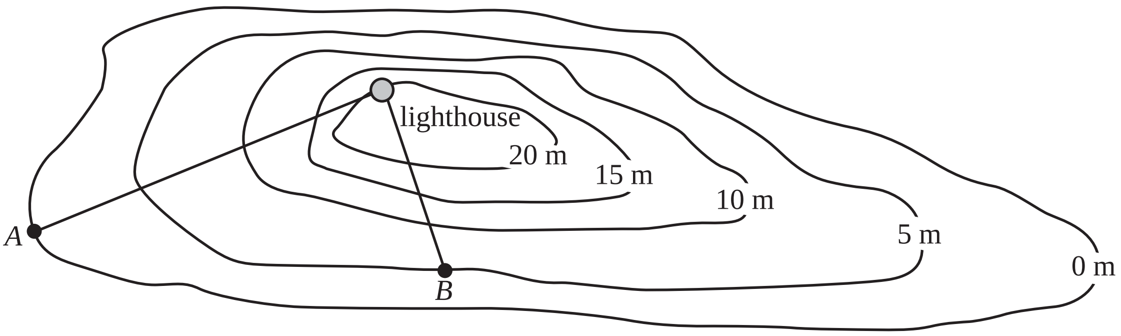
Total 15 marks

Module 2: Geometry and trigonometry

Question 1

A lighthouse is located on a hill overlooking the sea. A contour map of the hill is shown below. The lighthouse is located at an altitude of 20 metres.

Two points, *A* and *B*, are shown on the contour map.



On the contour map, 1 centimetre represents 30 metres on the **horizontal** level.
On the contour map, the length of the line from point *A* to the lighthouse is 5 centimetres.

a. Determine the horizontal distance, in metres, from point *A* to the lighthouse.

1 mark

The horizontal distance from point *B* to the lighthouse is 75 metres.

b. Calculate the average slope between point *B* and the lighthouse.

1 mark

Question 2

Ship A and Ship B can both be seen from the lighthouse.
Ship A is 5 kilometres from the lighthouse, on a bearing of 028°.
Ship B is 5 kilometres from Ship A, on a bearing of 130°.
This information is shown in Figure 1 below.

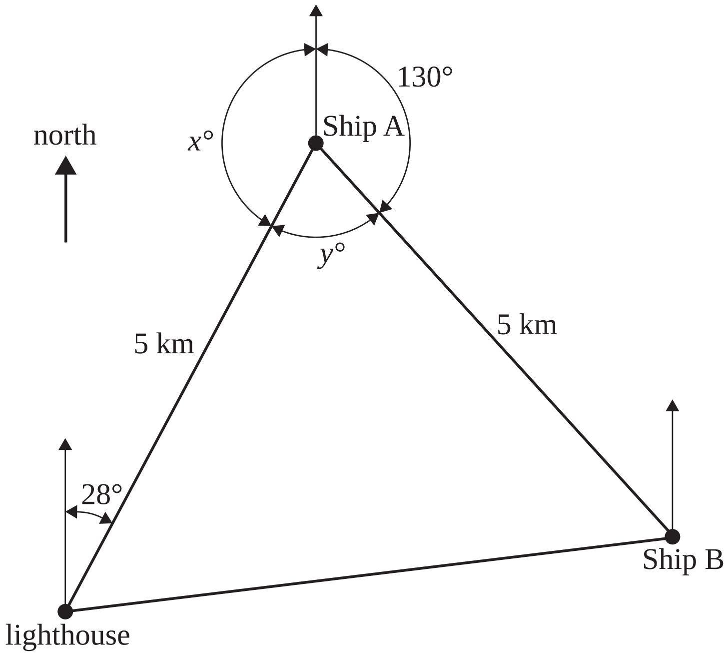


Figure 1

a. Two angles, x and y, are shown in Figure 1 above.

i. Determine the size of the angle x in degrees.

ii. Determine the size of the angle y in degrees.

1 + 1 = 2 marks

b. Determine the bearing of the lighthouse from Ship A.

1 mark

c. Determine the bearing of Ship B from the lighthouse.

1 mark

Question 3

The lighthouse has a lightroom, shown shaded in Figure 2 below.
The floor of the lightroom is in the shape of a regular octagon.
The longest distance across the floor is 4 metres.

The lightroom floor and $\angle POQ = \theta°$ are shown in Figure 3 below.

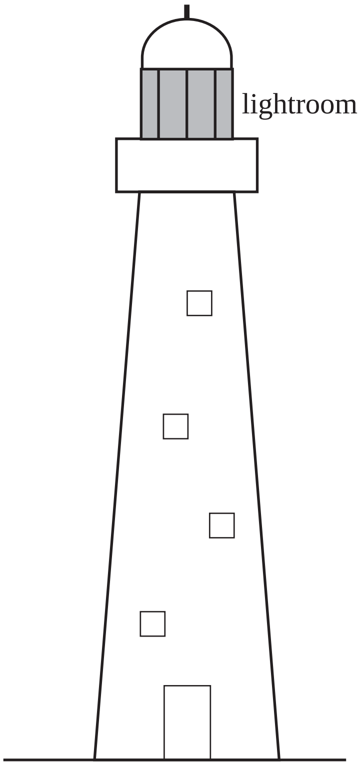


Figure 2

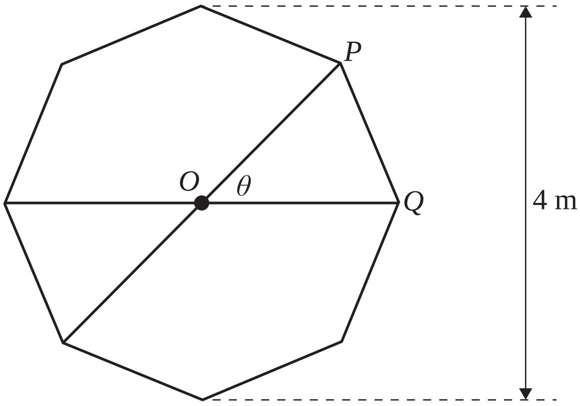


Figure 3

a. Show that the size of the angle θ is 45°.

1 mark

b. Determine the area of triangle POQ.
Write your answer in square metres correct to one decimal place.

1 mark

The lightroom is surrounded by a walkway of diameter 6.4 metres.
An outer circular wall surrounds the walkway.
The walkway is shown shaded in Figure 4 below.

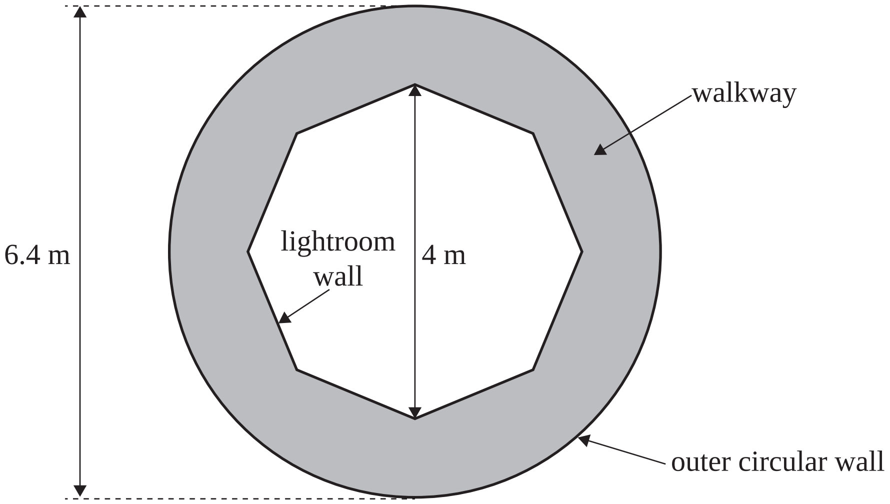


Figure 4

c. Determine the minimum distance between the lightroom wall and the outer circular wall.

1 mark

d. The walkway is the shaded area in Figure 4. Determine its area correct to the nearest square metre.

2 marks

Question 4

The lighthouse tower, shaded on Figure 5 below, is in the shape of a truncated cone.
It has circular cross-sections that decrease uniformly from a radius of 3.5 metres at ground level to a radius of 2 metres at the walkway.
The height of the lighthouse tower is 18 metres.
The angle marked α is the angle that the outer wall of the lighthouse tower makes with the horizontal at ground level.

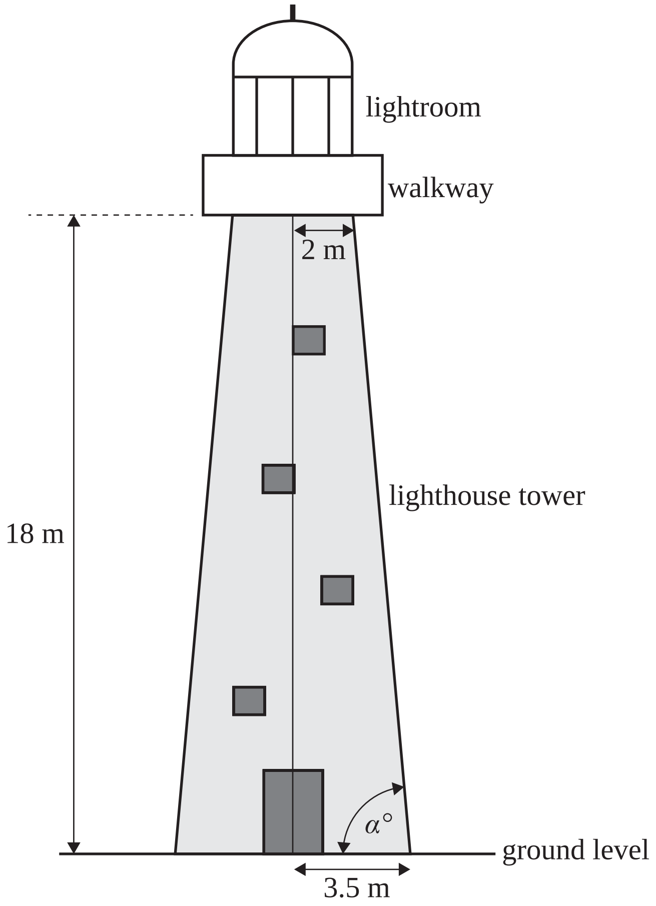


Figure 5

a. Determine the size of the angle α.
Write your answer in degrees correct to one decimal place.

1 mark

The lighthouse tower is part of a cone. The height of this cone is *h* metres and the base radius is 3.5 metres, as shown in Figure 6.

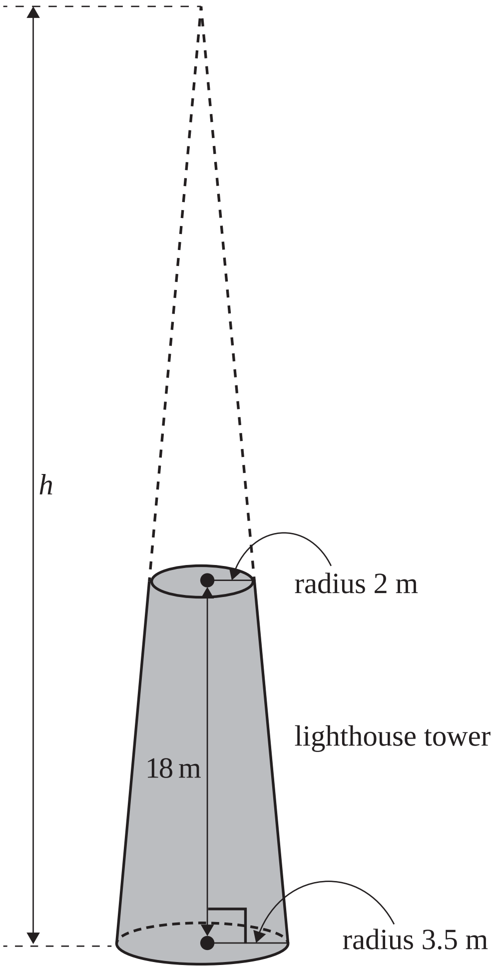


Figure 6

b. **i.** Determine *h*, the height of this cone, in metres.

ii. Determine the volume of the lighthouse tower.
Write your answer to the nearest cubic metre.

2 + 1 = 3 marks

Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

Michael is preparing to hike through a national park.
He decides to make some trail mix to eat on the hike.
The trail mix consists of almonds and raisins.

The table below shows some information about the amount of carbohydrate and protein contained in each gram of almonds and raisins.

	1 g of almonds	**1 g of raisins**
Carbohydrate	0.2 g	0.8 g
Protein	0.2 g	0.04 g

a. If Michael mixed 180 g of almonds and 250 g of raisins to make some trail mix, calculate the weight, in grams, of carbohydrate in the trail mix.

1 mark

Michael wants to make some trail mix that contains 72 g of protein. He already has 320 g of almonds.

b. How many grams of raisins does he need to add?

2 marks

The trail mix Michael takes on his hike must satisfy his dietary requirements.

Let x be the weight, in grams, of almonds Michael puts into the trail mix.
Let y be the weight, in grams, of raisins Michael puts into the trail mix.

Inequalities 1 to 4 represent Michael's dietary requirements for the weight of carbohydrate and protein in the trail mix.

Inequality 1 $x \geq 0$
Inequality 2 $y \geq 0$
Inequality 3 (carbohydrate) $0.2x + 0.8y \geq 192$
Inequality 4 (protein) $0.2x + 0.04y \leq 40$

Michael also requires a minimum of 16 g of **fibre** in the trail mix.
Each gram of almonds contains 0.1 g of fibre.
Each gram of raisins contains 0.04 g of fibre.

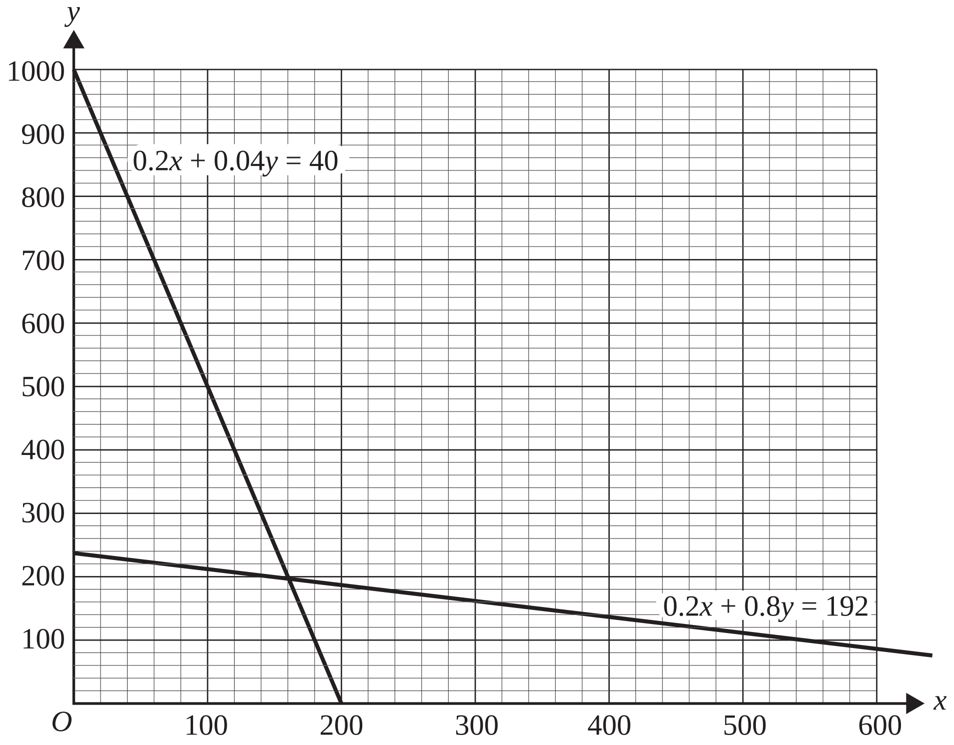
c. Write down an inequality, in terms of x and y, that represents this dietary requirement.

Inequality 5 (fibre) ____________________

1 mark

Module 3: Graphs and relations – Question 1 – continued

The graphs of $0.2x + 0.8y = 192$ and $0.2x + 0.04y = 40$ are shown below.



d. On the graph above

i. draw the straight line that relates to Inequality 5

ii. shade the region that satisfies Inequalities 1 to 5.

1 + 1 = 2 marks

e. What is the maximum weight, in grams, of trail mix that satisfies Michael's dietary requirements?

1 mark

Michael plans to carry at least 500 g of trail mix on his hike.
He would also like this trail mix to contain the greatest possible weight of almonds.
The trail mix must satisfy all of Michael's dietary requirements.

f. What is the weight of the almonds, in grams, in this trail mix?

2 marks

Question 2

Michael began his hike at the national park office and followed a track towards a camping ground, 16 km away. The distance-time graph below shows Michael's distance from the national park office, *d* kilometres, after *t* hours of hiking.

It took Michael seven hours to complete this hike.

a. What was Michael's average speed, in kilometres per hour, during this hike?
Write your answer correct to one decimal place.

1 mark

The equation of Michael's distance-time graph from $t = 3$ to $t = 7$ is

$$d = at + b$$

b. Determine the value of both *a* and *b*.

2 marks

Katie hiked along the same track as Michael, but hiked in the opposite direction.
She began at the camping ground and hiked towards the national park office.

Katie's distance from the national park office, d kilometres, after t hours of hiking, can be determined from the equation

$$d = -3t + 16$$

Katie and Michael both started hiking at the same time.

c. After how many hours did Katie pass Michael?

1 mark

Katie and Michael both carry radio transmitters that allow them to talk to each other while hiking.
The transmitters will not work if Katie and Michael are more than three kilometres apart.

d. For how many hours during the hike were Michael and Katie able to use the radio transmitters to talk to each other?
Write your answer in hours correct to two decimal places.

2 marks

Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

Tony plans to take his family on a holiday.
The total cost of \$3 630 includes a 10% Goods and Services Tax (GST).

a. Determine the amount of GST that is included in the total cost.

1 mark

During the holiday, the family plans to visit some theme parks.

The prices of family tickets for three popular theme parks are shown in the table below.

	Wet World	**Movie Journey**	**Outback Adventure**
Family ticket	\$82	\$220	\$160

b. What is the total cost for the family if it visits all three theme parks?

1 mark

If Tony purchases the **Movie Journey** family ticket online, the cost is discounted to \$202.40

c. Determine the percentage discount.

1 mark

Question 2

Tom and Patty both decided to invest some money from their savings.
Each chose a different investment strategy.

Tom's investment strategy

- Deposit \$5 600 into an account with an interest rate of 7.2% per annum, compounding monthly.
- Immediately after interest is paid into his investment account on the last day of each month, deposit a further \$200 into the account.

a. Determine the total amount in Tom's investment account at the end of the first month.

1 mark

Patty's investment strategy

- Invest \$8 000 at the start of the year at an interest rate of 7.2% per annum, compounding **annually**.

b. The following expression can be used to determine the value of Patty's investment at the end of the first year. Complete the expression by filling in the box.

$$\text{Value of investment} = 8\,000 \times (1 + \boxed{})$$

1 mark

At the end of twelve months, Patty has more money in her investment account than Tom.

c. How much more does she have?

Write your answer to the nearest cent.

2 marks

d. What annual compounding rate of interest would Patty need in order to earn \$1000 interest in one year on her \$8 000 investment?
Write your answer correct to one decimal place.

1 mark

Question 3

Tania purchased a house for $300 000.
She will have to pay stamp duty based on this purchase price.
Stamp duty rates are listed in the table below.

Purchase value	**Rate**
From $25 000 up to $130 000	$350 plus 2.4% of value greater than $25 000
From $130 000 up to $960 000	$2870 plus 6% of value greater than $130 000
Greater than $960 000	5.5% of the entire value

a. Calculate the amount of stamp duty that Tania will have to pay.

1 mark

b. Assuming that her house will increase in value at a rate of 3.17% per annum, what will the value of Tania's house be after 5 years?
Write your answer to the nearest thousand dollars.

1 mark

Tania bought her house at the start of 2011.

c. If the rate of increase in value remains at 3.17% per annum, at the start of which year will the value of Tania's house first exceed $450 000?

2 marks

Question 4

Tania takes out a reducing balance loan of $265 000 to pay for her house.
Her monthly repayments will be $1980.
Interest on the loan will be calculated and paid monthly at the rate of 7.62% per annum.

a. **i.** How many monthly repayments are required to repay the loan?
Write your answer to the nearest month.

ii. Determine the amount that is paid off the principal of this loan in the first year.
Write your answer to the nearest cent.

1 + 1 = 2 marks

Immediately after Tania made her twelfth payment, the interest rate on her loan increased to 8.2% per annum, compounding monthly.
Tania decided to increase her monthly repayment so that the loan would be repaid in a further nineteen years.

b. Determine the new monthly repayment.

Write your answer to the nearest cent.

1 mark

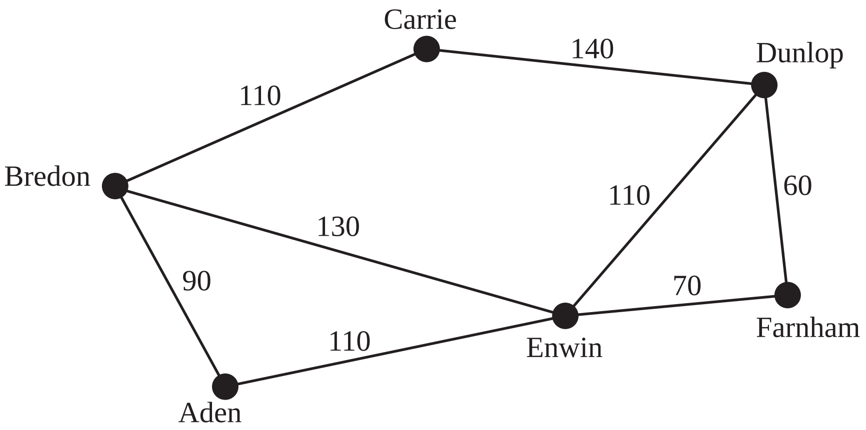
Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1

Aden, Bredon, Carrie, Dunlop, Enwin and Farnham are six towns.
The network shows the road connections and distances between these towns in kilometres.



a. In kilometres, what is the shortest distance between Farnham and Carrie?

1 mark

b. How many different ways are there to travel from Farnham to Carrie without passing through any town more than once?

1 mark

An engineer plans to inspect all of the roads in this network.
He will start at Dunlop and inspect each road only once.

c. At which town will the inspection finish?

1 mark

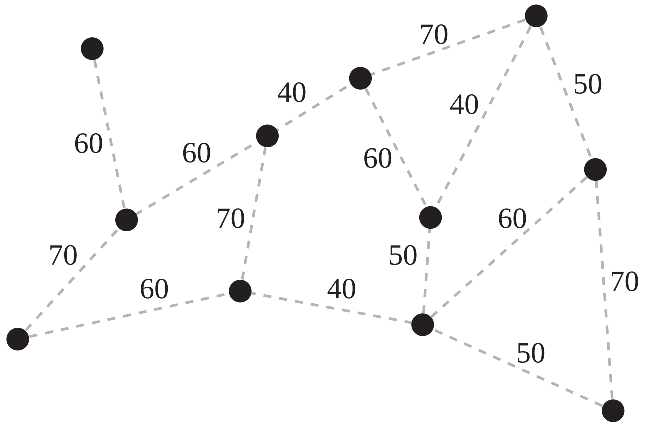
Another engineer decides to start and finish her road inspection at Dunlop.
If an assistant inspects **two** of the roads, this engineer can inspect the remaining six roads and visit each of the other five towns only once.

d. How many kilometres of road will the assistant need to inspect?

1 mark

Question 2

At the Farnham showgrounds, eleven locations require access to water. These locations are represented by vertices on the network diagram shown below. The dashed lines on the network diagram represent possible water pipe connections between adjacent locations. The numbers on the dashed lines show the minimum length of pipe required to connect these locations in metres.



All locations are to be connected using the smallest total length of water pipe possible.

a. On the diagram, show where these water pipes will be placed.

1 mark

b. Calculate the total length, in metres, of water pipe that is required.

1 mark

Question 3

A section of the Farnham showgrounds has flooded due to a broken water pipe. The public will be stopped from entering the flooded area until repairs are made and the area has been cleaned up.

The table below shows the nine activities that need to be completed in order to repair the water pipe. Also shown are some of the durations, Earliest Start Times (EST) and the immediate predecessors for the activities.

Activity	**Activity description**	**Duration (hours)**	**EST**	**Immediate predecessor(s)**
A	Erect barriers to isolate the flooded area	1	0	–
B	Turn off the water to the showgrounds	▨	0	–
C	Pump water from the flooded area	1	2	**A, B**
D	Dig a hole to find the broken water pipe	1	▨	**C**
E	Replace the broken water pipe	2	4	**D**
F	Fill in the hole	1	6	**E**
G	Clean up the entire affected area	4	6	**E**
H	Turn on the water to the showgrounds	1	6	**E**
I	Take down the barriers	1	10	**F, G, H**

a. What is the duration of activity B?

1 mark

b. What is the Earliest Start Time (EST) for activity D?

1 mark

c. Once the water has been turned off (Activity B), which of the activities C to I could be delayed without affecting the shortest time to complete all activities?

1 mark

It is more complicated to replace the broken water pipe (Activity E) than expected. It will now take four hours to complete instead of two hours.

d. Determine the shortest time in which activities A to I can now be completed.

1 mark

Turning on the water to the showgrounds (Activity H) will also take more time than originally expected. It will now take five hours to complete instead of one hour.

e. With the increased duration for Activity H and Activity E, determine the shortest time in which activities A to I can be completed.

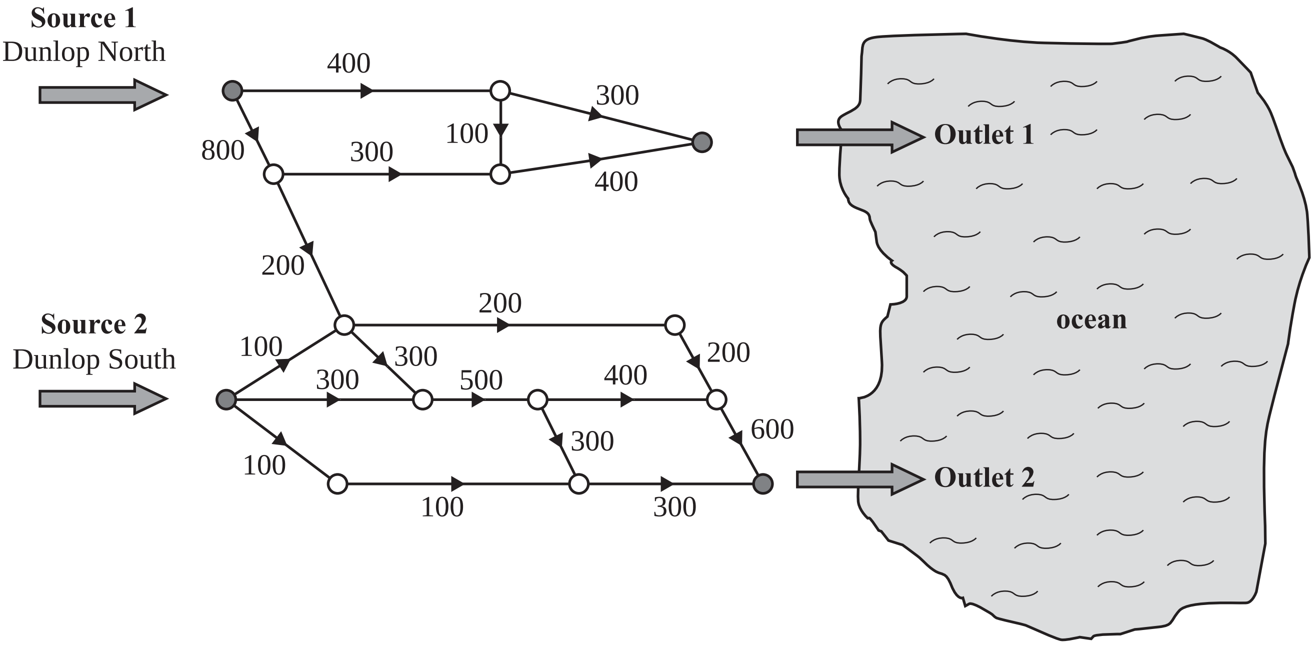
1 mark

Question 4

Stormwater enters a network of pipes at either Dunlop North (Source 1) or Dunlop South (Source 2) and flows into the ocean at either Outlet 1 or Outlet 2.

On the network diagram below, the pipes are represented by straight lines with arrows that indicate the direction of the flow of water. Water cannot flow through a pipe in the opposite direction.

The numbers next to the arrows represent the maximum rate, in kilolitres per minute, at which stormwater can flow through each pipe.



a. Complete the following sentence for this network of pipes by writing either the number 1 or 2 in each box.

Stormwater from Source ☐ **cannot** reach Outlet ☐

1 mark

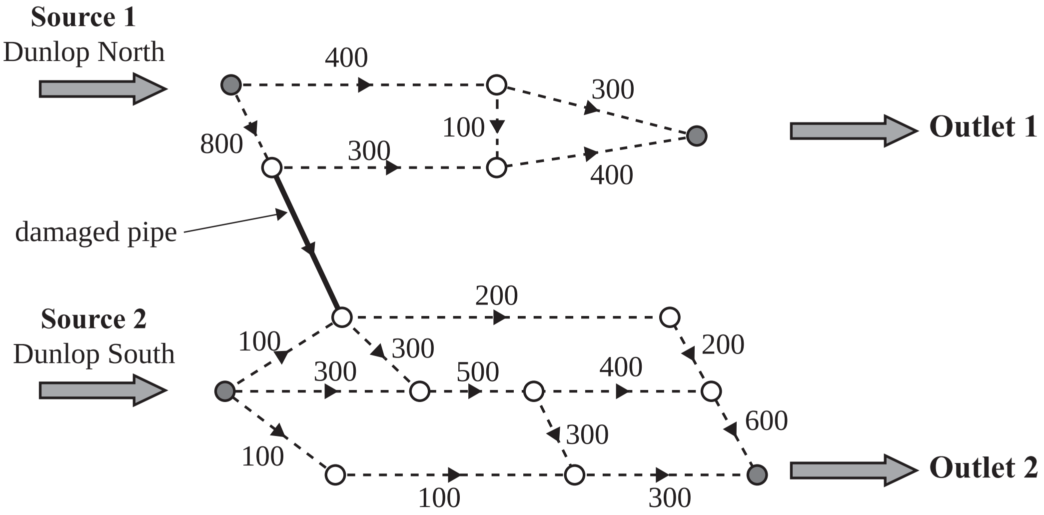
b. Determine the maximum rate, in kilolitres per minute, that water can flow from these pipes into the ocean at

Outlet 1

Outlet 2

2 marks

A length of pipe, shown in **bold** on the network diagram below, has been damaged and will be replaced with a larger pipe.



c. The new pipe must enable the greatest possible rate of flow of stormwater into the ocean from Outlet 2. What minimum rate of flow through the pipe, in kilolitres per minute, will achieve this?

1 mark

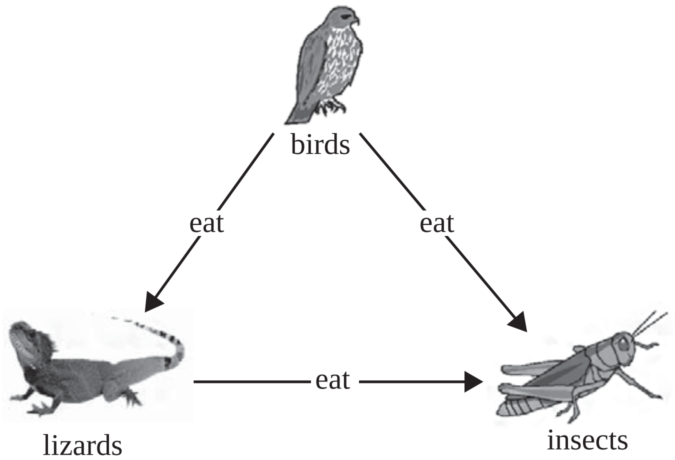
Total 15 marks

END OF MODULE 5
TURN OVER

Module 6: Matrices

Question 1

The diagram below shows the feeding paths for insects (*I*), birds (*B*) and lizards (*L*). The matrix *E* has been constructed to represent the information in this diagram. In matrix *E*, a ‘1’ is read as ‘eat’ and a ‘0’ is read as ‘do not eat’.



$$E = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccc} I & B & L \\ \left[\begin{matrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}\right. & & \left.\right] \end{array}$$

$$E = \begin{array}{c|ccc|c} & I & B & L & \\ \hline & 0 & 1 & 1 & I \\ & 0 & 0 & 0 & B \\ & 0 & 1 & 0 & L \end{array}$$

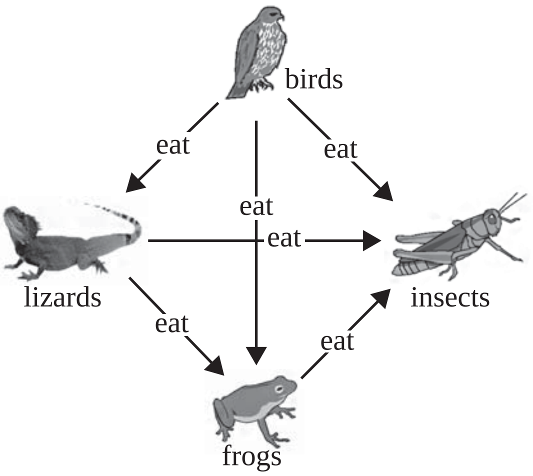
a. Referring to insects, birds or lizards

i. what does the ‘1’ in column *B*, row *L*, of matrix *E* indicate

ii. what does the row of zeros in matrix *E* indicate?

1 + 1 = 2 marks

The diagram below shows the feeding paths for insects (*I*), birds (*B*), lizards (*L*) and frogs (*F*).
The matrix *Z* has been set up to represent the information in this diagram.
Matrix *Z* has not been completed.



$$Z = \begin{array}{cccc|c} I & B & L & F & \\ \hline 0 & 1 & 1 & \_ & I \\ 0 & 0 & 0 & \_ & B \\ 0 & 1 & 0 & \_ & L \\ \_ & \_ & \_ & \_ & F \end{array}$$

b. Complete the matrix *Z* above by writing in the seven missing elements.

1 mark

Question 2

To reduce the number of insects in a wetland, the wetland is sprayed with an insecticide.
The numbers of insects (I), birds (B), lizards (L) and frogs (F) in the wetland that has been sprayed with insecticide are displayed in the matrix N below.

$$\begin{array}{cc} & \begin{array}{cccc} I & B & L & F \end{array} \\ N = & \begin{bmatrix} 100000 & 400 & 1000 & 800 \end{bmatrix} \end{array}$$

Unfortunately, the insecticide that is used to kill the insects can also kill birds, lizards and frogs.
The proportions of insects, birds, lizards and frogs that have been killed by the insecticide are displayed in the matrix D below.

$$\begin{array}{cccc} & \textit{alive before spraying} & & \\ & \begin{array}{cccc} I & B & L & F \end{array} & & \\ D = & \begin{bmatrix} 0.995 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.025 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix} & \begin{array}{c} I \\ B \\ L \\ F \end{array} & \textit{dead after spraying} \end{array}$$

a. Evaluate the matrix product $K = ND$.

$K =$

1 mark

b. Use the information in matrix K to determine the number of birds that have been killed by the insecticide.

1 mark

c. Evaluate the matrix product $M = KF$, where $F = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$M =$

1 mark

d. In the context of the problem, what information does matrix M contain?

1 mark

Question 3

A breeding program is started in the wetlands. It is aimed at establishing a colony of native ducks.

The matrix W_0 displays the number of juvenile female ducks (J) and the number of adult female ducks (A) that are introduced to the wetlands at the start of the breeding program.

$$W_0 = \begin{array}{c} \begin{bmatrix} 32 \\ 64 \end{bmatrix} \end{array} \begin{array}{c} J \\ A \end{array}$$

a. In total, how many female ducks are introduced to the wetlands at the start of the breeding program?

1 mark

The number of juvenile female ducks (J) and the number of adult female ducks (A) in the colony at the end of Year 1 of the breeding program is determined using the matrix equation

$$W_1 = BW_0$$

In this equation, B is the breeding matrix

$$B = \begin{array}{cc} \begin{array}{cc} J & A \end{array} & \\ \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} & \begin{array}{c} J \\ A \end{array} \end{array}$$

b. Determine W_1

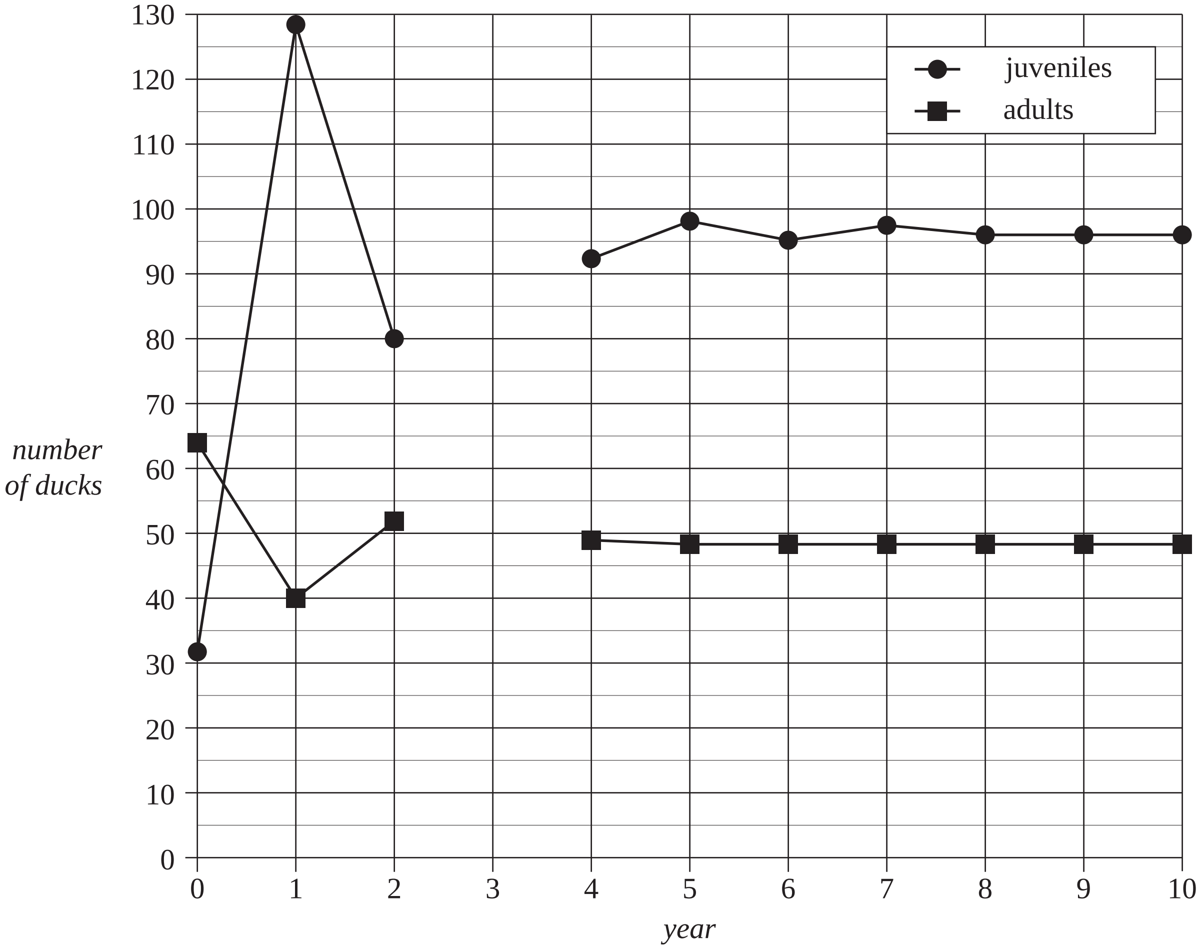
$W_1 =$

1 mark

The number of juvenile female ducks (J) and the number of adult female ducks (A) in the colony at the end of Year n of the breeding program is determined using the matrix equation

$$W_n = BW_{n-1}$$

The graph below is incomplete because the points for the end of Year 3 of the breeding program are missing.



c. **i.** Use matrices to calculate the number of juvenile and the number of adult female ducks expected in the colony at the end of Year 3 of the breeding program.

Plot the corresponding points on the graph.

ii. Use matrices to determine the expected total number of female ducks in the colony in the long term.

Write your answer correct to the nearest whole number.

2 + 1 = 3 marks

The breeding matrix B assumes that, on average, each adult female duck lays and hatches two female eggs for each year of the breeding program.

If each adult female duck lays and hatches only one female egg each year, it is expected that the duck colony in the wetland will not be self-sustaining and will, in the long run, die out.

The matrix equation

$$W_n = PW_{n-1}$$

with a different breeding matrix

$$P = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} J & A \\ \left[\begin{matrix} 0 & 1 \\ 0.25 & 0.5 \end{matrix}\right] & \end{array}\begin{array}{c} \\ J \\ A \end{array}$$

and the initial state matrix

$$W_0 = \begin{bmatrix} 32 \\ 64 \end{bmatrix}\begin{array}{c} J \\ A \end{array}$$

models this situation.

d. During which year of the breeding program will the number of female ducks in the colony halve?

1 mark

Changing the number of juvenile and adult female ducks at the start of the breeding program will also change the expected size of the colony.

e. Assuming the same breeding matrix, P, determine the number of juvenile ducks and the number of adult ducks that should be introduced into the program at the beginning so that, at the end of Year 2, there are 100 juvenile female ducks and 50 adult female ducks.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras’ theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler’s formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET