Unit 3 and 4 Further Mathematics: Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Found a mistake?

Check the Engage Education website for updated solutions, and then email practiceexams@ee.org.au.

Section A

Core: Data analysis

Question 1
The correct answer is B.

The mean is

$$\frac{180 + 172 + 156 + 169 + 189 + 162 + 165}{7}$$

$$= 170.4285 \ldots \approx 170.4$$

Question 2
The correct answer is E.

The median is the centre most value. For the data given this is 169.

Question 3
The correct answer is C.

The number of students he helps and the study score that each one receives at the end of the year are discrete variables as it is not possible to have a fraction of a student or a fraction of a study score. A and D are incorrect because the variables are both numerical.

Question 4
The correct answer is A.

The residual value = actual value – predicted value.
At $x = 7$, the actual value of $y = 9$ and the predicted value is 8.9
Therefore the residual value $= 9 - 8.9 = 0.1$

Question 5
The correct answer is A.

Z-score $= \frac{concentration\ level\ -\ mean}{standard\ deviation}$

To obtain a z-score of 2 solve:

$$2 = \frac{concentration\ level\ -\ 68}{4.5}$$

$$9 = concentration\ level\ -\ 68$$

$$\therefore concentration\ level\ =\ 77\%$$

Question 6
The correct answer is C.

The slope, b, of the regression line $y = bx + a$ where y = concentration from *Experiment 1* and x = concentration from *Experiment 2* is given by:

$$b = \frac{r\,S_y}{S_x}$$

$$b = \frac{0.74 \times 7.2}{4.5}$$

$b = 1.18$ (to two decimal places)

Question 7
The correct answer is C

$r^2 = 0.89$
$r = \pm\sqrt{0.89} = \pm 0.94$

As the relationship is negative (prices increases, number of purchases decreases) then we take the negative sign and $r = -0.94$

Question 8
The correct answer is C.

As Christina's test score at the top of the class is added:
- the mean will increase. This rules out A.
- number of students increases and therefore the centre value (the median) will no longer be the same as each student scored differently. This rules out B.
- the cut off value of the top 25% will increase (the upper quartile). This rules out D.
- the spread of scores will be more varied and the standard deviation will increase. This rules out E.

Therefore the correct answer is C.

Note: Use a diagram if unsure.

Question 9
The correct answer is D.

If we draw the normal distribution we find that all bottles that are discarded lie above one standard deviation above the mean. This equates to 16% of the total bottle production numbers.

$0.16 \times 100000 = 16000$ bottles

Question 10
The correct answer is B.

A 1/y transformation is the only one appropriate for a model of that shape.

Question 11
The correct answer is D.

Both the median and the interquartile range are unaffected by outliers

Question 12
The correct answer is E.

The seasonal indices must add up to 4 as there are 4 seasons.

Therefore the index for winter is $4 - 1.11 - 0.78 - 1.62 = 0.49$

Question 13
The correct answer is A.

To deseasonalise the summer sales we must divide 23 by 1.11. This gives a deseasonalised value of 20.72 bouquets.

In order to match these sales in autumn he must sell 20.72×0.78 16.16 bouquets.

As he cannot sell as portion of a bouquet, the minimum number of bouquets he will need to sell in autumn to increase his summer sales is 17.

Section B

Module 1: Number patterns

Question 1
The correct answer is B.

From the sequence, $a = 7$. The common ratio can be found by dividing subsequent terms by each other to get $r = -4$.

To find the sixth term, t_6

$$t_n = ar^{n-1}$$

$$t_6 = 7 \times (-4)^{(6-1)}$$

$$t_6 = 7 \times (-4)^5$$

$$t_6 = 7 \times -1024$$

$$t_6 = -7168$$

Question 2
The correct answer is D.

From the sequence, $a = -16$ and $d = 5$

Question 3
The correct answer is D.

This scenario follows a geometric sequence with $a = 300$ and $r = 1.04$

$$t_n = ar^{n-1}$$

$$t_n = 300 \times (1.04)^{n-1}$$

Solve for n when $t_n = 1000$

$$n = 31.697$$

So therefore during the 32nd week the amount of chlorine will first exceed 1000 ml.

Question 4
The correct answer is A.

$$t_2 = 4t_1 - 14$$

$$t_2 = 4 \times 12 - 14 = 34$$

$t_3 = 4 \times 34 - 14 = 122$

$t_4 = 4 \times 122 - 14 = 474$

$t_5 = 4 \times 474 - 14 = 1882$

Question 5

The correct answer is D.

A Fibonacci-type sequence is one where the next term is the sum of the two preceding it.

Therefore, we work backwards to find two possible starting terms for our given series.

$30 = 19 + t_{n-1}$
$t_{n-1} = 30 - 19 = 11$

$19 = 11 + t_{n-1}$
$t_{n-1} = 19 - 11 = 8$

$11 = 8 + t_{n-1}$
$t_{n-1} = 11 - 8 = 3$

$8 = 3 + t_{n-1}$
$t_{n-1} = 8 - 3 = 5$

From this we see that two possible starting terms are $f_1 = 5, f_2 = 3$. This is option D.

Question 6

The correct answer is D.

This scenario follows a geometric sequence with $a = 375$ and $r = 0.2$

$t_n = ar^{n-1}$

$t_n = 375 \times (0.2)^{n-1}$

The next four terms are:

$t_3 = 375 \times (0.2)^2 = 15$
$t_4 = 375 \times (0.2)^3 = 3$
$t_5 = 375 \times (0.2)^4 = 0.6$
$t_6 = 375 \times (0.2)^5 = 0.12$

The sum of these four terms is $15 + 3 + 0.6 + 0.12 = 18.72$

Question 7

The correct answer is E.

Let t_n = the account balance at the end of the nth month. Where $t_0 = 50$ (the initial balance)

The important step is to convert the per annum interest rate into one per month:
7.2% per annum = 0.6% per month

$t_1 = t_0 + 25 + 0.006(t_0 + 25)$

$$t_1 = (t_0 + 25)(1 + 0.006)$$

$$t_1 = 1.006(t_0 + 25)$$

In general: $t_{n+1} = 1.006t_n + 25.15$

Question 8
The correct answer is C.

Note: Although the difference equation in D produces the same sequence that option is incorrect as it is not a second-order difference equation like the question asked for.

Question 9
The correct answer is B.

There is a common difference between the number of extra attendees each week and this is confirmed by the linear relationship in the graph. Hence, it is an arithmetic sequence.

Module 2: Geometry and Trigonometry
Question 1
The correct answer is B.

To find the area of this triangle we first use the Sine rule to find another angle in the triangle.

$$\frac{\sin 57}{23} = \frac{\sin C}{18}$$

$$C = \sin^{-1}\frac{18 \times \sin 57}{23} = 41.02°$$

Knowing that the angles of a triangle add up to 180° we can find the last angle $ABC = 180 - 57 - 41.02 = 81.98°$

Now that we have the angle between our two given lengths we can apply the formula $A = \frac{1}{2}ab\sin C$

$$A = \frac{1}{2} \times 23 \times 18 \times \sin 81.98$$

$$A = 204.95 \; cm^2$$

This is closest to 205 cm²

Question 2
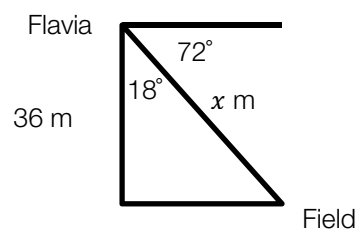The correct answer is A.

The vertical height that Flavia must climb is $240 \times 15 \text{ cm} = 3600 \text{ cm} = 36 \text{ m}$

Using trigonometry:

$$\cos 18 = \frac{36}{x}$$

$$x = \frac{36}{\cos 18} = 37.85$$

The distance the ball travels to Flavia is 37.85 m

Question 3
The correct answer is E.

As the angles are complementary, we can calculate the internal angle to be $180 - 132 = 48°$
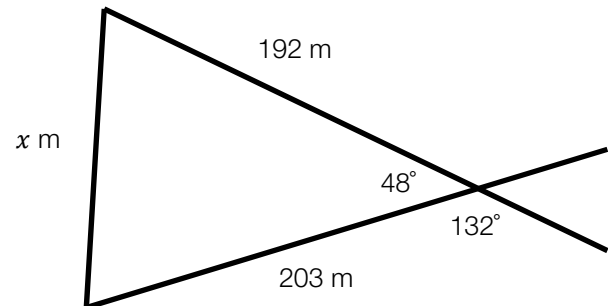
Then we use the cosine rule to find the missing length:

$a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 192^2 + 203^2 - 2 \times 203 \times 192 \times \cos 48°$

$x^2 = 36864 + 41209 - 52160.07$

$x^2 = 25912.93$

$x = 160.97$

Question 4
The correct answer is D.

Using the scale provided, and measuring the distance from A to B, the horizontal distance is 40 m.

The vertical distance between the two points is $250 - 100 = 150$m.
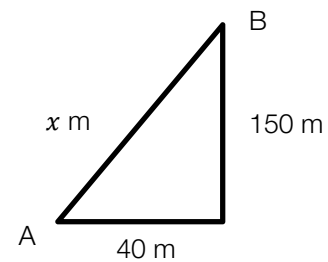
We can find the length of the pipe x, by using Pythagoras' formula:

$c^2 = a^2 + b^2$

$x^2 = 40^2 + 150^2$

$x = 155.24$

This is closest to 155 m

Question 5
The correct answer is E.

$\text{Average slope} = \dfrac{\text{rise}}{\text{run}}$

$\text{Average slope} = \dfrac{150}{40}$

$\text{Average slope} = 3.75$

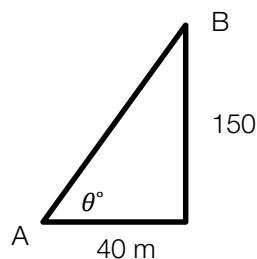### Question 6
The correct answer is

The angle θ is the angle of elevation and can be found with trigonometry.

$\tan \theta = \dfrac{150}{40}$

$\tan \theta = 3.75$

$\theta = \tan^{-1} 3.75$

$\theta = 75.07°$

This is closest to 75˚.

Question 7
The correct answer is D.

40 cm : 25 km
40 cm : 25,000 m
40 cm : 2,500,000 cm
1 cm : 62,500 cm
Therefore the scale ratio is 1: 62,500

Question 8
The correct answer is

Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$

Therefore for this triangle, $s = \frac{15+18+13}{2} = 23$

$$\mathbf{A = \sqrt{23(23-15)(23-18)(23-13)}}$$
$$A = \sqrt{23 \times (8) \times (5) \times (10)}$$

This line is seen in option C.

Question 9
The correct answer is C.

Using co-interior angles we find the 175˚ angle between North and the path between G and H.

Then, using the fact that the angles around a point must sum to 360˚, we obtain the 140˚ pictured.

Lastly, we again use co-interior angles to find the 40˚ between North and the path between H and I.

Now with this information portrayed in the diagram on the right we are left to find the bearing of Point H from Point I which is simply: $360 - 40 = 320˚$
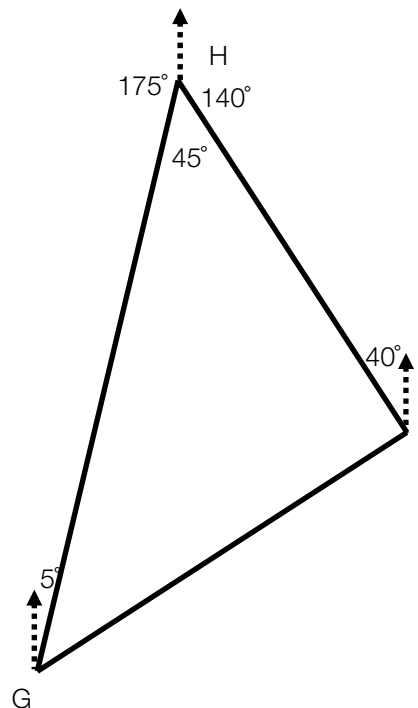
Module 3: Graphs and relations

Question 1
The correct answer is C.

$$m = \frac{1}{7}$$
$$y + 1 = \frac{1}{7}(x - 5)$$
$$x - 7y = 12$$

Question 2
The correct answer is D.

$$\frac{1 - 0}{3 - 0} = \frac{1}{3}$$

Question 3
The correct answer is B.

$$\frac{80}{1.25} = 64$$

Question 4
The correct answer is A.

Sub in points (0,10):
$10 \leq 10$
$10 - 9 > 0$
Therefore, not in region.

Question 5
The correct answer is C.

$7x + 2y = 8.20$
$3x + 5y = 8.90$
$x = .8$ and $y = 1.3$
Therefore, $x + 2y = 3.4$

Question 6
The correct answer is B.

After 2 hours, $C = 30 + 60$, not 120.

Question 7
The correct answer is E.

1 laptop costs $60 for three days.
$60 \times 2 = 120$

Question 8
The correct answer is B.

$50 + (60 \times 2) = 180$
$$\frac{180}{15} = 12$$

Question 9
The correct answer is D.

Rearrange, $y = -2x - 1$
$m = -2$, not 12.

Module 4: Business-related mathematics

Question 1
The correct answer is D.

The interest earned on the investment over four years is:

$$I = \frac{Prt}{100} = \frac{6000 \times 5 \times 4}{100} = \$1200$$

The value of the investment after four years is the interest added to the principal = $7200

Question 2
The correct answer is B.

The value of the investment after 5 years is $2000 \times (1.03)^5 = \$2318.55$

Question 3
The correct answer is B.

$r_f = \frac{I}{P} \times \frac{m}{n} \times 100$ where I is the interest on the hire-purchase agreement, P is the principal minus deposit, m is the number of repayments per year and n is the number of repayments all together

$$r_f = \frac{260}{700} \times \frac{12}{24} \times 100$$

$$r_f = 18.57\%$$

Question 4
The correct answer is A.

$r_e = r_f \times \frac{2n}{n+1}$ where n is the number of repayments

$$r_e = 18.57 \times \frac{2 \times 24}{24 + 1}$$

$$r_e = 18.57 \times \frac{48}{25}$$

$$r_e = 35.65\%$$

Question 5
The correct answer is C.

This is somewhat of a trick question. Dave will still end up paying the same amount for his phone on the new hire-purchase plan. The important thing to realise here is that a hire-purchase agreement works differently to a reducing-balance loan when altering monthly payments and the length of the loan.

Question 6
The correct answer is D.

Convert the percentage increases and decreases into decimal form and then multiply them together:

$1.02 * 1.12 * 0.93 * 1.06 * 0.96 = 1.081$. This is closest to an 8% increase

Question 7
The correct answer is C.

The reducing balance depreciation method utilises the following formula:

$$\text{Book Value} = PR^n \text{ where } R = 1 - \frac{r}{100}$$

After 8 years: Book Value = $3000 \times (0.833)^8 = \$695.47$

After 9 years: Book Value = $3000 \times (0.833)^9 = \$579.33$

After 10 years: Book Value = $3000 \times (0.833)^{10} = \$482.58$

The computer is above the scrap value ($500) after 9 years but below the scrap value after 10 years. Therefore, the computer can be written off during the 10th year.

Question 8
The correct answer is B.

Using the perpetuity formula:

$$Q = \frac{Pr}{100}$$

$$Q = \frac{246000 \times 6.1}{100} = \$15006$$

Therefore Anna receives $15006 annually from the perpetuity as the interest rate is per annum.

The amount she receives monthly is:

$$\frac{\$15006}{12} = \$1250.50$$

Question 9
The correct answer is E.

Using TVM solver:

N = 15 * 12 = 180
I% = 8.9
PV = 180000
PMT = ?
FV = 0
P/Y = 12
C/Y =12

Solving for PMT will produce a value of -1814.99

This means that the monthly repayment is $1814.99

Module 5: Networks and decision mathematics

Question 1

The correct answer is D.

Need to ascertain how many vertices have an even number of edges coming from it – the number of edges coming from it is known as that vertex's *degree*,

Question 2

The correct answer is B.

Need to substitute the number 8 into Euler's law:
That is, $vertices - edges + faces = 2$
$\therefore$ use $v - 8 + f = 2$
$10 = v + f$
Then use trial and error using the values from the answers – there will only be one correct answer.

Question 3
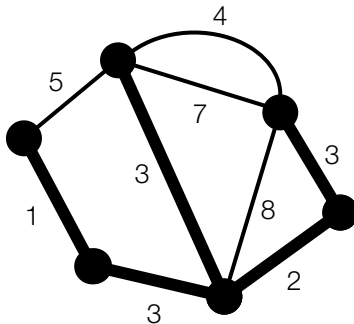
The correct answer is D.

Need to realise that a Hamiltonian Path passes through each vertex only once, as opposed to a circuit, which returns to the first vertex.

Also need to note that whilst answer A is a Hamiltonian Path, it does not begin at F, as specified by the question.

Question 4

The correct answer is A.

The minimal spanning tree is:

The value of 1 is the only option in the answers included in the spanning tree.

Question 5

The correct answer is A.

If students find the critical path, they will see that the minimum completion time is 27 weeks rather than 29 weeks, and that the other options are correct.
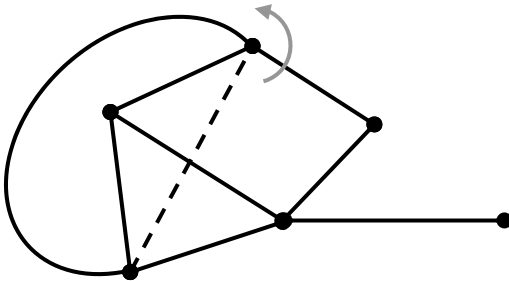
Question 6

The correct answer is C.

If students find the earliest and latest completion times for each activity, the critical path will become apparent (it includes each of the activities whose earliest and latest start times are the same).

Question 7
The correct answer is E.

Students need to make the graph 'planar' (by ensuring that two lines do not intersect) before they count the faces. An example of how they might do this:

They must also remember that the outside area surrounding the graph is a face too.

Question 8
The correct answer is B.

The final tally of two-step dominances was: A – 1, B – 3, C – 1, D – 2.

Question 9
The correct answer is D.

Students could use the formula $\frac{n(n-1)}{2}$, where n is the number of vertices in the graph for both 2011 and 2012, and find the difference:

2011: $\frac{4(4-1)}{2} = 6$, 2012: $\frac{6(6-1)}{2} = 15$, so the difference is 9.

Module 6: Matrices
Question 1
The correct answer is C.

(1 row – horizontal) x (2 columns – vertical)

Question 2
The correct answer is D.

Question 3
The correct answer is C.

Determinant $= ad - bc$ (using the formula provided on the formula sheet)
$(4 \times 3) - (8 \times -3) = 36$

Question 4
The correct answer is C.

Question 5
The correct answer is A.

$S_3 T = S_4$

Question 6
The correct answer is B.

Question 7
The correct answer is C.

AB is not defined:
A (2 x 2) matrix cannot be multiplied by a (1 x 2) matrix

Question 8
The correct answer is A.

Process of elimination
Sub in multiple choice answers one at a time
When 4 is used, produces same result as given

Question 9
The correct answer is B.

All transition matrices follow the same pattern as this:
$$\begin{bmatrix} A|A & A|B \\ B|A & B|B \end{bmatrix}$$
Therefore Let Crumby's = C and let Baker's Joy = B
$$\begin{bmatrix} C|C & C|B \\ B|C & B|B \end{bmatrix}$$