The Mathematical Association of Victoria

FURTHER MATHEMATICS 2012

Trial Written Examination 1-SOLUTIONS

SECTION A: Core--Data analysis
Answers:

1.	D	**2.**	D	**3.**	C	**4.**	D	**5.**	A
6.	B	**7.**	D	**8.**	B	**9.**	A	**10.**	A
11.	D	**12.**	C	**13.**	A				

SECTION B: MODULES
Module 1: Number Patterns

1.	B	**2.**	E	**3.**	A	**4.**	C	**5.**	C
6.	A	**7.**	C	**8.**	E	**9.**	C		

Module 2: Geometry and trigonometry

1.	C	**2.**	C	**3.**	D	**4.**	A	**5.**	B
6.	A	**7.**	C	**8.**	B	**9.**	B		

Module 3: Graphs and relations

1.	C	**2.**	E	**3.**	B	**4.**	A	**5.**	B
6.	D	**7.**	C	**8.**	D	**9.**	B		

Module 4: Business-related mathematics

1.	D	**2.**	E	**3.**	B	**4.**	C	**5.**	D
6.	B	**7.**	B	**8.**	B	**9.**	C		

Module 5: Networks and decision mathematics

1.	D	**2.**	D	**3.**	B	**4.**	B	**5.**	C
6.	C	**7.**	B	**8.**	B	**9.**	E		

Module 6: Matrices

1.	D	**2.**	D	**3.**	C	**4.**	B	**5.**	E
6.	E	**7.**	A	**8.**	A	**9.**	C		

Worked solutions--Core: Data analysis

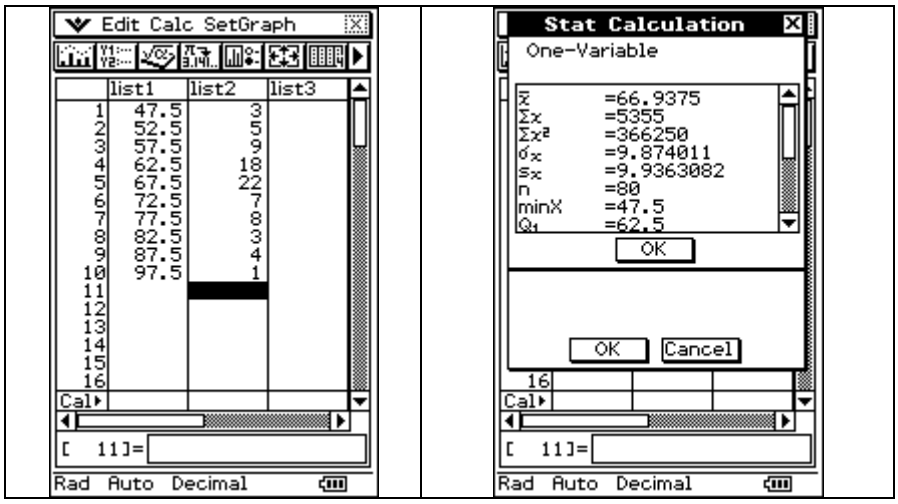
Question 1

Eight people have a heart rate over 80, so $\frac{8}{80} \times 100 = 10\%$ are at risk.

Answer D

Question 2

Entering the middle heart rate value of each class interval and the corresponding frequency



will give a mean value of 66.9

Answer D

Question 3

Parallel boxplots are used to display bivariate data where one set is categorical and the other is numerical.

Options A, B and E are incorrect – these refer to two categories.

Options D is incorrect – this refers to two numerical data.

Answer C

Question 4

Option A is incorrect because $\bar{x} - 3s = 64 - 3 \times 12 = 28$ means that 0.15% of the population of students will score less than 28.

Option B is incorrect because $\bar{x} - s = 64 - 12 = 52$ means that 16% of the population of students will score less than 52.

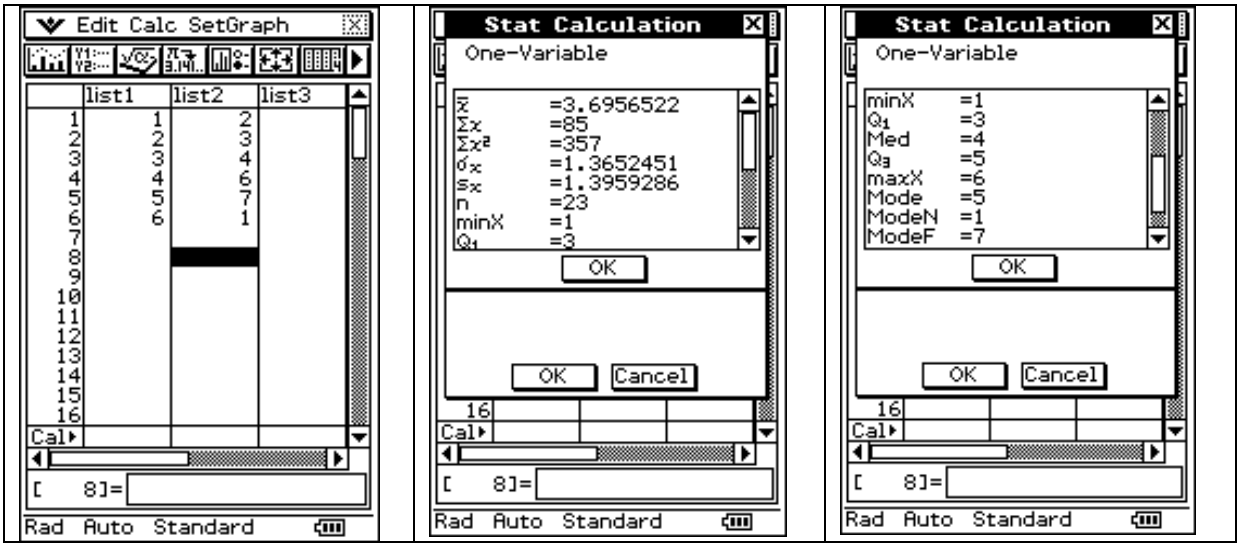
Option C is incorrect because $\bar{x} + 2s = 64 + 2 \times 12 = 88$ means that 2.5% of the population of students will score more than 88.

Option D is **correct** because $\bar{x} - 2s = 40$ and $\bar{x} + s = 78$ means that half of 95% plus half of 68% of the population of students scored between 40 and 76. That is 47.5 +34 = 81.5% (more than 80%) lie between 40 and 76.

Option E is incorrect because 99.7% of scores lie in the interval between $\bar{x} - 3s$ and $\bar{x} + 3s$, which in this case is between 28 and 100.

Answer D

Question 5



Mean is 3.7, Median is 4 and mode is 5 so the order from lowest to highest is mean, median, mode.

Answer A

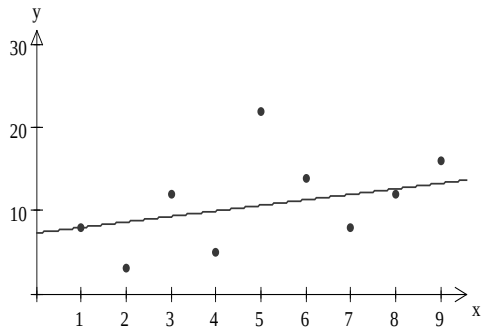
Question 6

The gradient of the line is $m = \dfrac{10}{4} = 2.5$, so the interpretation of the gradient on the graph is:

For every hour worked $2.50 is earned.

Answer B

Question 7



The three median line shown on the graph passes above the left and right median points and has a positive y-intercept which is less than 10. It appears to pass through one data point on the scatterplot, not one – third of the points. The median point will always pass through the average point of the three median points, that is through the point

$$\left(\frac{x_L + x_M + x_R}{3}, \frac{y_L + y_M + y_R}{3}\right)$$

Answer D

Question 8

The IQR is 35.7 – 24.3 = 11.4 so option A is correct

Upper Boundary = $35.7 + 1.5 \times 11.4 = 52.8$ so option B is **incorrect**

75% if the data lie above the lower Quartile of 24.3 so option C is correct

The median lies anywhere between the two quartiles 24.3 and 35.7 so option D is correct

Lower Boundary = $24.3 - 1.5 \times 11.4 = 7.2$ so option E is correct

Answer B

Question 9

Option A is false because $r^2 = 0.757$ so $r = -\sqrt{0.757} = -0.87$ (see graph for direction - negative)

Option B is true because there is a negative gradient

Option C is true because $m = -0.8 = \frac{-8}{10}$, so for every increase in 10 minutes of talking, text time decreases by 8 minutes

Option D is true because $y = -0.8 \times 100 + 166.4 = 86.4$ so residual = 60 – 86.4= –26.4

Option E is correct interpretation of coefficient of determination.

Answer A

Question 10

To linearise the data, stretch the *x* scale by applying an x^2 transformation or stretch the y scale by applying a y^2 transformation

Answer A

Question 11

Day	Sales	Median (4 point)	Median centred
Monday	23		
Tuesday	32		
Wednesday	18		
		Median (32, 18, 34, 40) is 33	
Thursday	34		$\frac{33+37}{2}=35$
		Median (18, 34, 40, 48) is 37	
Friday	40		
Saturday	48		
Sunday	33		

Answer D

Question 12

The sum of the seasonal indices is equal to the number of seasons. $x+0.83+y+1.06=4$

Hence the sum of the seasonal indices for summer and winter is $4-1.89=2.11$

Option A $0.83 + 1.06 = 1.89$

Option B $1.06 + 0.83 = 1.89$

Option C $0.83 + 1.28 = 2.11$

Option D $1.06 + 1.28 = 2.34$

Option E $1.28 + 1.06 = 1.34$

Answer C

Question 13

Given that $a=\bar{y}-b\bar{x}$ and $b=r\frac{s_y}{s_x}$ will give the y-intercept and gradient of the least squares regression line $y=a+bx$ and $a=148,\ \bar{x}=21,\ s_x=4,\ b=3.5$ and $r=0.7685$

Substituting in $a=\bar{y}-b\bar{x}$ gives $148=\bar{y}-3.5\times 21$ so $\bar{y}=148+74.5=221.5$

Substituting in $b=r\frac{s_y}{s_x}$ gives $3.5=0.7685\times\frac{s_y}{4}$ so $s_y=\frac{3.5\times 4}{0.7685}=18.22$

Answer A

Module 1: Number patterns

Question 1

$t_1 = -20, \quad t_2 = -20 \times -1.5 = 30, \quad t_3 = -20 \times (-1.5)^2 = -45$

so the difference between 30 and -45 is 75

Answer B

Question 2

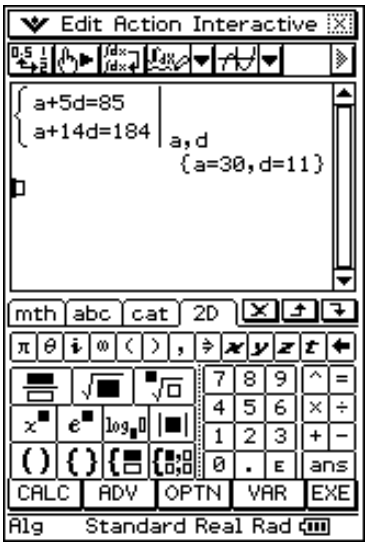
The worm population is increasing by the same amount therefore the sequence is arithmetic

$t_6 = 85$

so $a + 5d = 85$ (equation 1)

$t_{15} = 184$

so $a + 14d = 184$ (equation 2)



equation 2 – equation 1

$9d = 99$

$d = 11$

sub in equation 1

$a + 5 \times 11 = 85$

so $a = 30$

Answer E

Question 3

Substitute $n = 2$ in $t_{n+1} = 2t_n - 1$

$t_3 = 2t_2 - 1$ since $t_3 = 9$

$9 = 2 \times t_2 - 1$

so $t_2 = 5$

$t_2 = 2k - 1$

$5 = 2k - 1$

$k = 3$

Answer A

Question 4

If $a = 3$ and $b = 0$ then $B_{n+1} = 3B_n$, where $B_1 = 1$ generates the sequence 1, 3, 9, 27, . . . This is a geometric sequence (not arithmetic).

If $a = 3$ and $b = 1$ then $B_{n+1} = 3B_n + 1$, where $B_1 = 1$ generates the sequence 1, 4, 13, 40, ... This sequence is neither arithmetic nor geometric.

If $a = 1$ and $b = 3$ then $B_{n+1} = B_n + 3$, where $B_1 = 1$ generates the sequence 1, 4, 7, 10, . . .This sequence is arithmetic

Answer C

Question 5

The arithmetic sequence is given by $a,\ a+d,\ a+2d,\ a+3d,\ a+4d$

$S_1 = a = 3$

$S_2 = 2a + d = 11$

$S_3 = 3a + 3d = 24$ etc

$$\begin{aligned} d &= S_2 - 2S_1 \\ &= 11 - 2 \times 3 \\ &= 5 \end{aligned}$$

so the original sequence is 3, 8, 13, 18, 23 where $d = 5$

Answer C

Question 6

From the graph it is observed that the sum of an even number of terms ($S_2, S_4,\ S_6$) is less than the sum of an odd number of terms ($S_1, S_2,\ S_3$) which must mean that every second term of the original sequence is a negative number. The sequence must be geometric with the first term a positive number and the common ratio a negative number. So options A and D are possible answers.

Since the difference between each sum is getting smaller then the original sequence must be converging to zero so $-1 < r < 0$

Answer A

Question 7

For the sequence $2, 5, 8, \ldots$ $t_n > 100$ $a + (n-1)d > 100$ $2 + (n-1)(3) > 100$ using solve on CAS gives $n > 33\frac{2}{3}$ when n = 34	For the sequence $-20, -13, -6, \ldots$ $t_n > 100$ $a + (n-1)d > 100$ $-20 + (n-1)(7) > 100$ using solve on CAS gives $n > 18.14$ ie when n = 19	For the sequence $0.2, 0.3, 0.45, \ldots$ $t_n > 100$ $ar^{n-1} > 100$ $0.2(1.5)^{n-1} > 100$ using solve on CAS gives $n > 16.327$ ie when n = 17
For the sequence $99, 99.05, 99.1, \ldots$ $t_n > 100$ $a + (n-1)d > 100$ $99 + (n-1)(0.05) > 100$ using solve on CAS gives $n > 20$ ie when n = 21	For the sequence $0.5, -0.6, 0.72, \ldots$ $t_n > 100$ $ar^{n-1} > 100$ $0.5(-1.2)^{n-1} > 100$ generate sequence and scroll when n = 31	

Answer C

Question 8

For a Fibonacci sequence $f_6 + f_7 = f_8$ $b + f_7 = a$ $f_7 = a - b$ option A is true	Also $f_7 + f_8 = f_9$ $a - b + a = f_9$ $2a - b = f_9$ option B is true	Also $f_5 + f_6 = f_7$ $f_5 + b = a - b$ $f_5 = a - 2b$ option C is true
$f_6 + f_7 = f_8$ $f_6 + f_7 = a$ option D is true	$f_4 + f_5 = f_6$ $f_6 + f_7 = b$ so option E is not true	

Answer E

Question 9

Diameter is *increasing* by 0.8 of previous increase where the first increase is $a = 6$

The series generated is $6 + 6 \times (0.8) + 6 \times (0.8)^2 + 6 \times (0.8)^3 + ...$

So maximum increase is given by $S_\infty = \frac{a}{1-r} = \frac{6}{1-0.8} = 30$

Maximum diameter is 15 + 30 = 45 cm

Answer C

Module 2: Geometry and trigonometry

Question 1

$2x^o + 58^o = 180^o$

$x = 61^o$

Answer C

Question 2

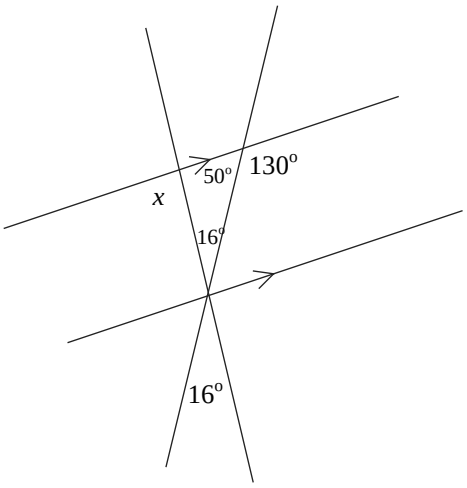
The interior angles of a regular polygon with n sides is given by $180 - \frac{360}{n}$

solving $180 - \frac{360}{n} = 140$ gives n = 9

Answer C

Question 3

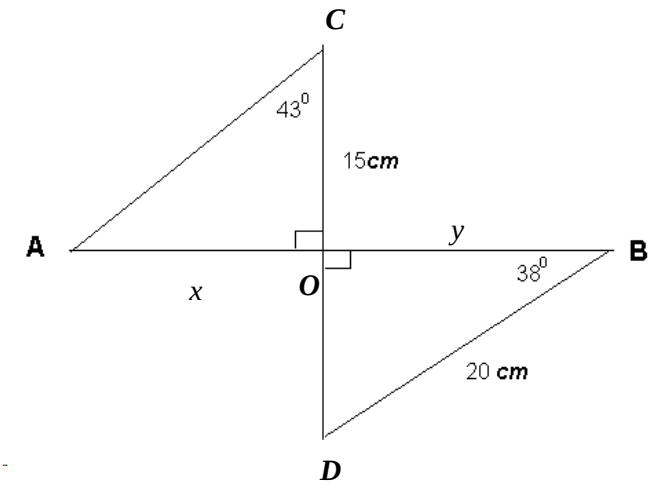
Inside the triangle:
Using the opposite angle of 16^o
and the supplementary angle of 130^o
(50^o) inside the triangle then
$x = 50^o + 16^o = 66^o$



Answer D

Question 4

Using triangle AOC



Let x be the distance AO

$$\tan(43^o) = \frac{x}{15}$$

$$x = 15\tan(43^\circ)$$

Using triangle BOD

Let y be the distance OB

$$\cos(38^o) = \frac{y}{20}$$

$$y = 20\cos(38^\circ)$$

$$AB = x + y = 15\tan(43^\circ) + 20\cos(38^\circ)$$

Answer A

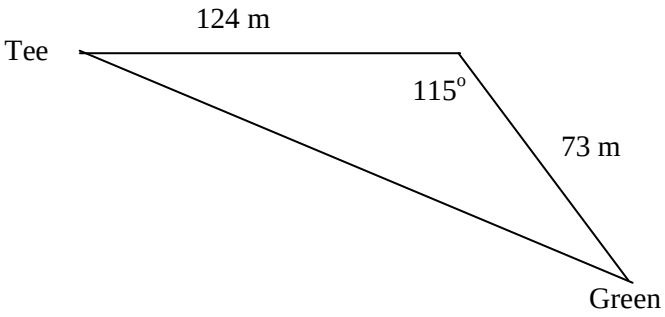
Question 5

Let *x* be the distance from the Tee to the Green

Using the cosine rule

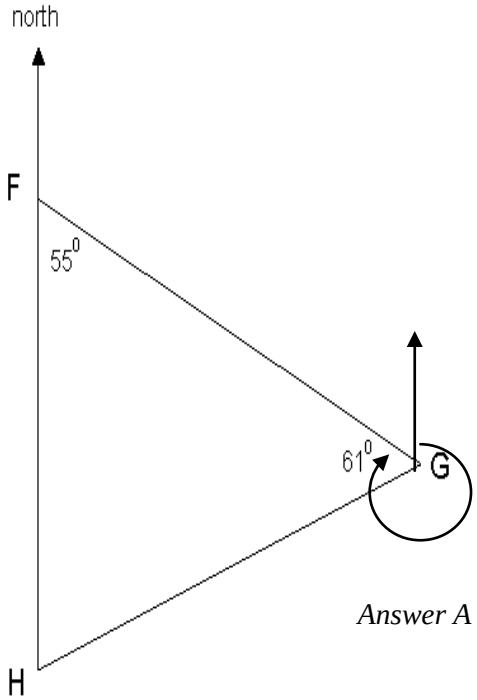
$$x^2 = 124^2 + 73^2 - 2(124)(73)\cos 115^\circ$$

$$x = \sqrt{124^2 + 73^2 - 2(124)(73)\cos 115^\circ}$$



Answer B

Question 6



The bearing of F from G is more than 270^o

The bearing of G from F is less than 180^o

The bearing of G from H is less than 90^o

The bearing of H from G is more than 180^o but less than 270^o

The bearing of H from F is 180^o

Answer A

Question 7

Volume ratio = 27:125

$= 3^3 : 5^3$

Length ratio = 3 : 5

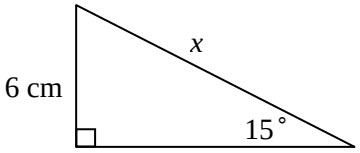
Area ratio = $3^2:5^2$

$$\begin{matrix} \times 10 \\ \\ \times 10 \end{matrix} \begin{matrix} \curvearrowright \\ = 9 : 25 \\ 90 : x \end{matrix} \begin{matrix} \curvearrowleft \\ \\ \\ \times 10 \end{matrix}$$

If area of smaller cone is 90 cm^2 then area of larger is 250 cm^2

Answer C

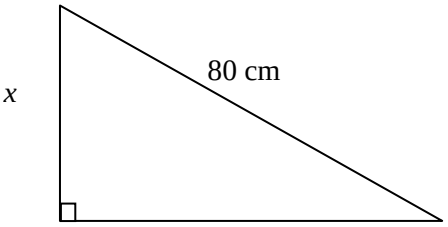
Question 8



Let x be the depth of the desk

$$\sin(15^\circ) = \frac{6}{x}$$

$$x = \frac{6}{\sin(15^\circ)} = 23.18$$



using Pythagoras' Theorem

$$length^2 + x^2 = 80^2$$

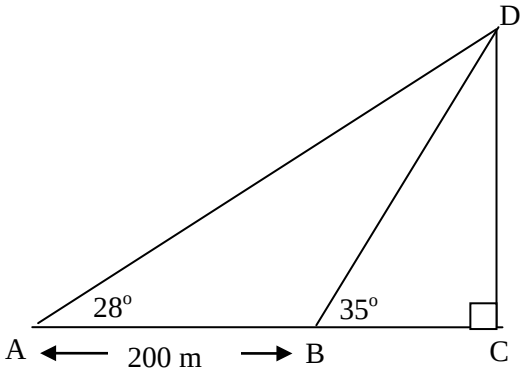
$$length = \sqrt{80^2 - 23.18^2} = 76.6$$

length

Answer B

Question 9

Method 1



Find BD using triangle ABD

Angle ABD = $180^o - 35^o = 145^o$

Angle ADB = $180^o - 145^o - 28^o = 7^o$

$$\frac{200}{\sin(7^\circ)} = \frac{BD}{\sin(28^\circ)}$$

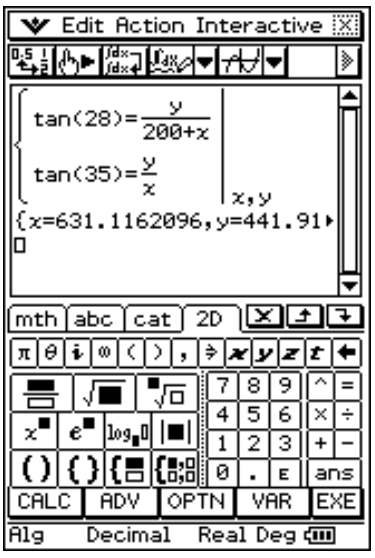
$$BD = \frac{200\sin(28^\circ)}{\sin(7^\circ)} = 770.45$$

Find CD using triangle BCD

$$\sin(35^\circ) = \frac{CD}{770.45}$$

$$CD = 770.45\sin(35^\circ) = 441.9\,\text{m}$$

Method 2



Using triangle ACD

Let BC = x and let the height = CD = y

$$\tan(28^\circ) = \frac{y}{200 + x}$$ **equation 1**

Using triangle BCD

$$\tan(35^\circ) = \frac{y}{x}$$ **equation 2**

solving on the calculator gives height = 441.9 m

Answer B

Module 3: Graphs and relations

Question 1

45 litres of petrol was used over 8 days. Average Rate $= \frac{45}{8} = 5.6$ litres per day

Answer C

Question 2

$3x - 4y - 2 = 0$ can be written as $4y = 3x - 2$

so $y = \frac{3x}{4} - \frac{2}{4}$

The line has a positive gradient of $\frac{3}{4}$ and a y-intercept of $-\frac{1}{2}$. So options A , B and C are not true

When $x = 3, \quad y = \frac{3 \times 3}{4} - \frac{2}{4} = \frac{7}{4}$ so the line does not pass through (3, 2)

When $y = 0, \quad x = \frac{2}{3}$ so the x intercept is at $x = \frac{2}{3}$

For the line $6x - y - 4 = 0$ (option E)

when $y = 0, \quad 6x - 4 = 0 \ \therefore \ x = \frac{2}{3}$ so its *x* intercept is also at $x = \frac{2}{3}$

Answer E

Question 3

Let *m* = cost of a muesli bar; *f* = fruit strap

$8m + 6f = 14.12$ I

$12m + 5f = 17.90$ II

1.5 × I $12m + 9f = 21.18$ III

or solve on calculator

III – II $4f = 3.28$

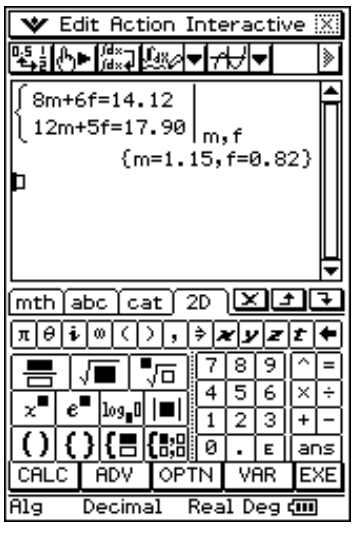
$f = 0.82$ substitute into II to find *m*.

$12m + 5 \times 0.82 = 17.90$

$12m + 4.10 = 17.90$

$12m = 22.00$

$m = \$1.15$



For three muesli bars and two fruit straps Zoe will pay 3(1.15) + 2(0.82) = \$5.09

and will receive \$4.90 change from \$10.00

Answer B

Question 4

The graph shown has the equation $y = \frac{k}{x^2}$ where $k =$ gradient of the line

$k = \frac{6}{2} = 3$ so the equation is $y = \frac{3}{x^2}$

Answer A

Question 5

To break even Profit = 0. This can be read from the graph i.e. when n = 150

Answer B

Question 6

From the graph shown the gradient can be found by the points (0, −1200) and (150, 0)

$m = \frac{1200}{150} = 8$ y-intercept is -1200

Profit $= 8n - 1200$

The selling price is \$35 per T-Shirt so

Revenue $= 35n$

Profit = Revenue – Cost

Rearranging this gives

Cost = Revenue – Profit

$Cost = 35n - (8n - 1200)$

$Cost = 27n + 1200$

Answer D

Question 7

John's travel card was used over a 2.5 hour period so the charge is \$5.
The 10 year old child used their travel card over a 2.5 hour period so the charge is also \$5.
The 7 year old child used their travel card for exactly 2.5 hours so $2.5 \times 0.5 = \$1.25$.
Sarah's travel card was used over a 6.5 hour period so \$8 was charged.

Total charge = \$5 + \$5 + 1.25 + 8 = \$19.25

Answer C

Question 8

Taxi company A has a fixed charge of \$3 (y-intercept) and a rate of \$4 per kilometre (gradient) so $C = 4x + 3$ eliminate option A and E

Taxi company B has two charges based on the distance travelled.

- \$5 per kilometre is charged for the first 6 km so $C = 5x, \quad 0 \le x \le 6$ end point is (6, 30) and
- \$3 per kilometre is charged after the first 6 km

e.g If x = 10 then the first 6 km is charged \$5 each and the remaining 4 km is charged \$3 each.

$C = 5 \times 6 + 3 \times 4 = 42$ (10, 42)

If x = 15 then the first 6 km is charged \$5 each and the remaining 9 km is charged \$3 each

$C = 5 \times 6 + 3 \times 9 = 57$ (15, 57)

Using the two points (10, 42) and (15, 57) gives the linear equation $C = 3x + 12$

Generally, if x is greater than 6 km then the first 6 km is charged \$5 each and the remaining $(x - 6)$ km is charged \$3 so

$$\begin{aligned} C &= 5 \times 6 + (x - 6) \times 3 \\ &= 30 + 3x - 18 \\ &= 3x + 12 \end{aligned}$$

Answer D

Question 9

The equation passing through (0, 1) and (–1, 0) is $y = x + 1$ and

the equation passing through (0, 4) and (8, 0) is $y = -\frac{1}{2}x + 4$

The lines $y = x + 1$ and $y = -\frac{1}{2}x + 4$ intersect at the point B (2, 3)

The extreme points of the feasible region are:

A (0, 1) where the objective function $M = k \times 0 + 1 = 1$

B (0, 4) where the objective function $M = k \times 0 + 4 = 4$

C (2, 3) where the objective function $M = k \times 2 + 3 = 2k + 3$

For M to be a maximum at B

$$\begin{aligned} 2k + 3 &> 4 \\ 2k &> 1 \\ k &> \frac{1}{2} \end{aligned}$$

Answer B

Module 4: Business-related mathematics

Question 1

Minimum monthly balance was brought forward from August into September \$326.36

Monthly interest on minimum balance is $\frac{3.4}{1200} \times \$326.26 = \$0.92$

Answer D

Question 2

The interest paid is $\$8250 - \$6000 = \$2250$

Let T be the time of the investment in years.

$$I = \frac{PRT}{100}$$

$$2250 = \frac{6000 \times 7.5 \times T}{100}$$

$$T = \frac{2250 \times 100}{6000 \times 7.5}$$

$T = 5$ years

$5 \times 12 = 60$ months

Answer E

Question 3

Vickie's investment is determined using

$A = PR^n$ where $R = \left(1 + \frac{10}{100}\right) = 1.1$ and $n = 3 \times 1 = 3$

$A = 1000 \times 1.1^3$

$I = 1000 \times 1.1^3 - 1{,}000 = 331$

Rob's investment is determined using

$$I = \frac{PRT}{100}$$

$$I = \frac{1000 \times 10 \times 3}{100} = 300$$

Vickie's interest exceeds Rob's by $\$331 - \$300 = \$31$

Answer B

Question 4

10% of \$50 = \$5 so the charge per session is \$55

The only value listed that is exactly divisible by \$55 is \$1100

Answer C

Question 5

Photocopier has depreciated by $\frac{5\,000\,000 \times 12}{100} = 600\,000$ cents

Depreciated by \$6 000

Value = 9 400 – 6 000 = \$3 400

Answer D

Question 6

The shirt is discounted at $\frac{8}{100} \times 150 = \$12$ and the jacket is discounted at $\frac{6}{100} \times 250 = \$15$

The total discount is \$12 + \$15 = \$27 for a total marked price of \$150 + \$250 = \$400

The percentage discount for the two items combined = $\frac{27}{400} \times 100 = 6.75\%$

Answer B

Question 7

Guilia's investment is determined using

$A = PR^n$ where $R = \left(1 + \frac{7.2}{1200}\right) = 1.006$ and $n = 2 \times 12 = 24$

$A = 2000 \times 1.006^{24}$

Answer B

Question 8

A depreciation of 25% means that the television will retain 75% of its value

June 1 2011 \$3299

June 1 2012 \$3299 × 0.75 = \$2474.25

June 1 2013 \$3299 × 0.75^2 = \$1855.69

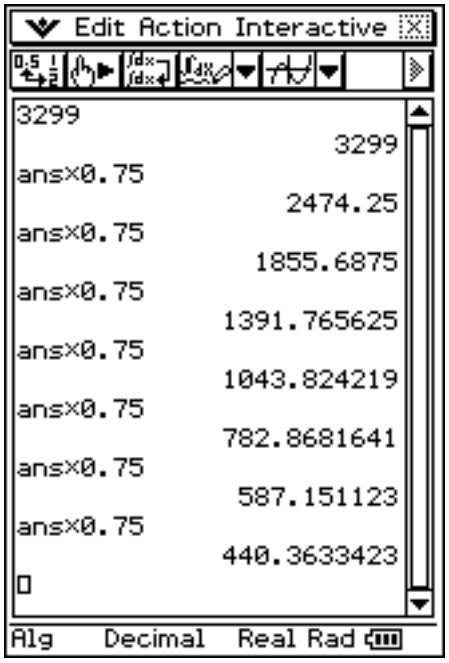
June 1 2014 \$3299 × 0.75^3 = \$1391.77

June 1 2015 \$3299 × 0.75^4 = \$1043.82

June 1 2016 \$3299 × 0.75^5 = \$782.87

June 1 2017 \$3299 × 0.75^6 = \$587.15

June 1 2018 \$3299 × 0.75^7 = \$440.36



Answer B

Question 9

$$R = 1 + \frac{6}{1200} = 1.005$$

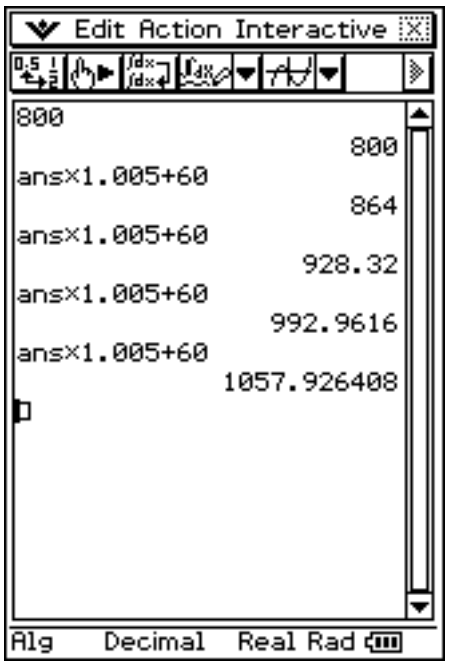
The amount of the investment at the end of:

April is \$800 × 1.005 + 60 = \$864

May is \$864 × 1.005 + 60 = \$928.32

June is \$928.32 × 1.005 + 60 = \$992.96

July is \$928.32 × 1.005 + 60 = \$1057.93



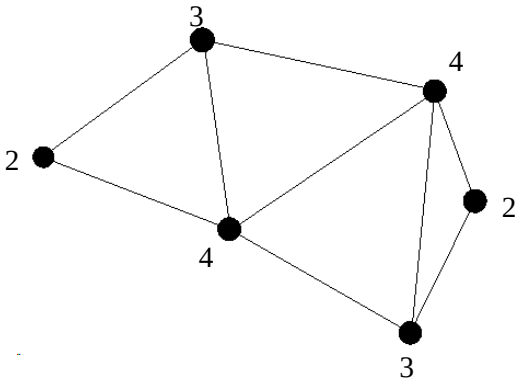
Answer C

Module 5: Networks and decision mathematics

Question 1

There are two vertices of degree 2
Two vertices of degree 4 and
Two vertices of degree 3

So four are even and two are odd

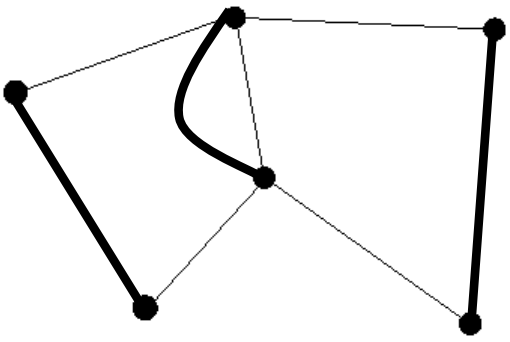


Answer D

Question 2

The degree of all vertices must be even on a connected graph for it to be an Eulerian circuit.

Adding the three paths as shown will create a graph with four vertices of degree two and two vertices of degree four hence satisfying the requirements of an Eulerian circuit.



Answer D

Question 3

The dominance matrix for the tournament is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$ This gives the dominance vector $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$ so the final will be between Teams 1 and 4

Answer B

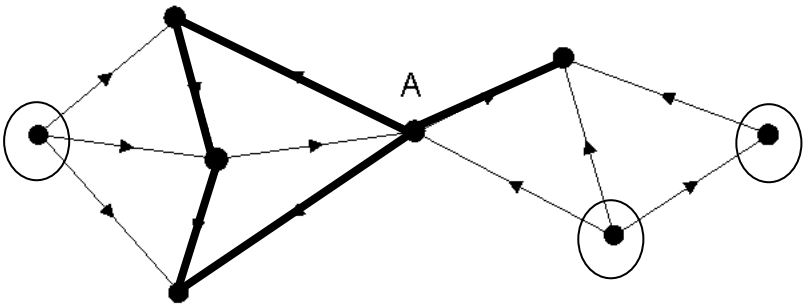
Question 4

An illustration of the relationship between two categorical variables: *player* and *position* on basketball court needs to be displayed. The only graph that is used for this purpose is a bipartite graph.

Answer B

Question 5

Three vertices (circled) cannot be reached from vertex A



Answer C

Question 6

For any connected planar graph v + f = e + 2. All graphs except C satisfy this relationship

Graph	Edges	Faces	Vertices	v + f – e = 2
A	5	4	3	4 + 3 – 5 = 2
B	7	5	4	5 + 4 – 7 = 2
C	**10**	**10**	**5**	**10 + 5 – 10 = 5**
D	11	7	6	6 + 7 – 11 = 2
E	13	8	7	7 + 8 – 13 = 2

Answer C

Question 7

A complete graph with five vertices has 4 + 3 + 2 + 1 = 10 edges to be a complete graph.

If the graph is already connected with the minimum number of edges then it already has 4 edges.

It will require an extra 6 edges to make it complete

Answer B

Question 8

The dominance matrix for the water polo competition is

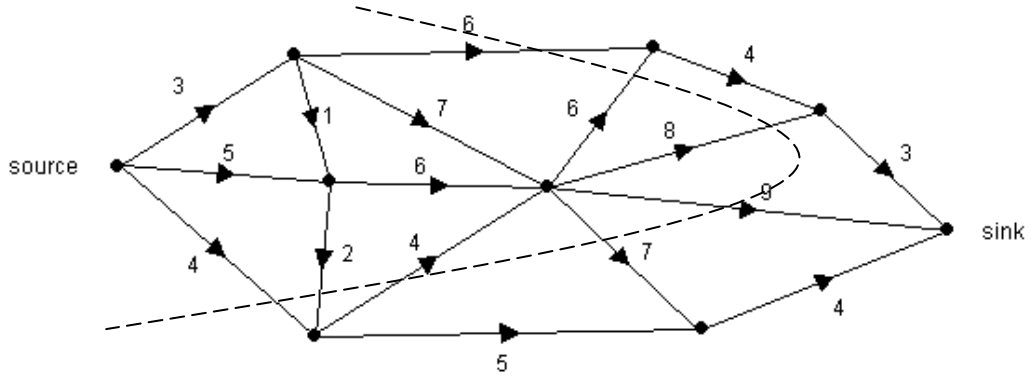
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

The two step dominance matrix is $D^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

This gives the two step dominance vector $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$

Answer B

Question 9



A maximum of seven edges.

Answer E

Module 6: Matrices

Question 1

Determinant

$= ad - bc$

$= 2 \times 2 - 3 \times 3$

$= 4 - 9$

$= -5$

Answer D

Question 2

If $CX = D$

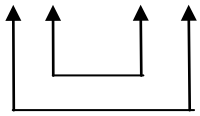
To solve for X then pre-multiply by the inverse of C

$X = C^{-1}D$

Answer D

Question 3

$$A \times B = AB$$
$$3 \times 4 \times m \times n = 3 \times 1$$



For the Matrix AB to exist the number of columns in matrix A must equal the number of rows in matrix B , therefore $m = 4$

The resulting matrix has the same number of columns as B, therefore $n = 1$

So B is a 4×1 matrix

Answer C

Question 4

The inverse of the matrix $\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ is

$$= \frac{1}{2\times 4 - 6\times 1}\begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

The only element not in the inverse matrix is 3

Answer B

Question 5

A^{-1} will exist if the determinant does not equal zero

$ad - bc \neq 0$

$xy - 24 \neq 0$

$xy \neq 24$

The product of x and y for all options listed, except E, give 24

Answer E

Question 6

$$\begin{aligned} t_1 - 2t_2 &= 0 \\ t_1 - 3a &= 87 \\ t_2 + 2a &= 29 \end{aligned}$$ can be represented as $$\begin{aligned} t_1 - 2t_2 + 0a &= 0 \\ t_1 + 0t_2 - 3a &= 87 \\ 0t_1 + t_2 + 2a &= 29 \end{aligned}$$

so the matrix can be set up as $$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

Answer E

Question 7

The transition matrix $\begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$

The steady state matrix is $\begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}^{50} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.33333 \\ 0.66666 \end{bmatrix}$ Mario's / Sienna's

Answer A

Question 8

The initial state does not have an affect on the long term outcome. The transition matrix dictates the final outcome so Mario's percentage of customers will be unaltered.

Answer A

Question 9

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.8 & 0.3 \\ 0.2 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = \begin{bmatrix} 210 \\ 250 \\ 140 \end{bmatrix}$$

Labour lost 90 votes, Liberal gained 50 votes and the other party gained 40 votes.

The total change in votes was 90 + 50 + 40 = 180 votes.

Answer C