The Mathematical Association of Victoria

FURTHER MATHEMATICS 2012

Trial Written Examination 2 - SOLUTIONS

SECTION A: Core--Data analysis

Question 1

a. 35% A1

b. Yes. As age increased the percentage who slept more than 10 hours decreased from 65% for 10 year olds to about 53% for 11 year olds to 45% for 12 year olds to 35% for 13 year olds to about 18% for 14 year olds to about 14 % for 15 year olds A1

or similar statement for another category

Question 2

a.i. mean BMI = $\frac{500}{16} = 31.25$ A1

a.ii. percentage overweight = $\frac{5}{16} = 31.25\%$ A1

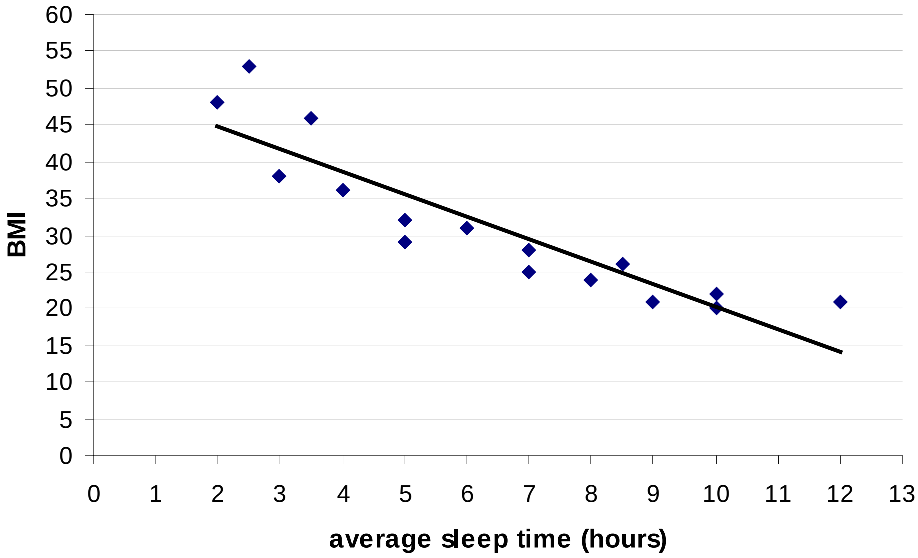
b.i Coefficient of determination = $r^2 = (-0.90)^2 = 0.81$ A1

bii. 81% of the variation in BMI can be explained by the variation in hours of average sleep A1

c. The slope of -3.08 tells us that for each increase of one hour in average sleep, BMI decreases by 3.08. Therefore increasing average sleep by two hours means BMI decreases by $2 \times 3.08 = 6.16$ A1

d. Two points correctly plotted from the equation, for example, $(0,51)$ and $(10,20.2)$ M1

Line correctly drawn similar to that below M1



e. From the equation when average hours = 9, BMI =23.28 M1

Residual = actual value – predicted value

$=28-23.28=4.72$ A1

f. The residual plot shows a systematic pattern (not random) therefore the linear model has failed to capture the essential features of the relationship. A1

Question 3

a. $\frac{1}{BMI}=0.015+0.003\times average\,sleep\,time$ A2

b. Solve for average sleep time when BMI =30

$\frac{1}{30}=0.015+0.003\times average\,sleep\,time$

Average sleep time = 6.1 hours A1

SECTION B: MODULES

Module 1: Number patterns

Question 1

a. Arithmetic sequence: a = 144, d = -2, $t_n = a + (n-1)d$

$t_{10} = 144 + 9(-2) = 126$ A1

b. Arithmetic series: $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{10} = \frac{10}{2}\left[288 + 9\,(-2)\right] = 1350$ A1

c. $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

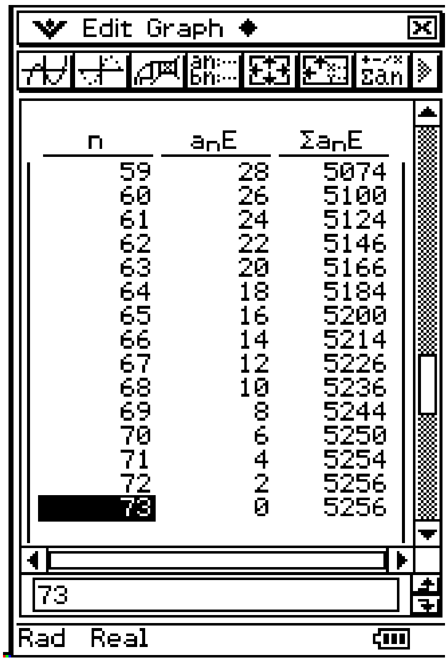
$S_n = \frac{n}{2}[288 + (n-1)(-2)]$ M1

$S_n = \frac{n}{2}[290 - 2n]$

$S_n = 145n - n^2$

$b = 145\ , c = -1$ A1

d. From a table of values the 73rd recharge will be the first that adds no charge time to the battery. The total number of hours the battery will work is the sum of 73 terms which equals **5256 hours** A1



n	anE	ΣanE
59	28	5074
60	26	5100
61	24	5124
62	22	5146
63	20	5166
64	18	5184
65	16	5200
66	14	5214
67	12	5226
68	10	5236
69	8	5244
70	6	5250
71	4	5254
72	2	5256
73	0	5256

Or let $t_n = 0$

$0 = 144 + (n-1)(-2)$

$2n = 146 \Rightarrow n = 73$

Substitute in equation from **c.** to give 5256 hours

Question 2

a. Geometric sequence $a = 144$, $r = 0.96$

$t_n = ar^{n-1}$

$t_2 = 144 \times 0.96 = 138.24$ hours A1

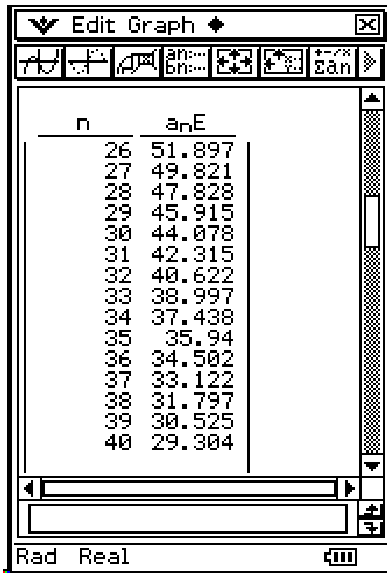
b. $k = 0.96$ A1

c. Infinite Geometric series: $S_\infty = \dfrac{a}{r-1}$

$S_\infty = \dfrac{144}{1-0.96} = 3600$ A1

d. The 40th recharge A1

Using a table of values



Question 3

a. $C_2 = 1.05 \times 144 - 8.2 = 143$ hours and
$C_3 = 1.05 \times 143 - 8.2 = 141.95$ hours A1

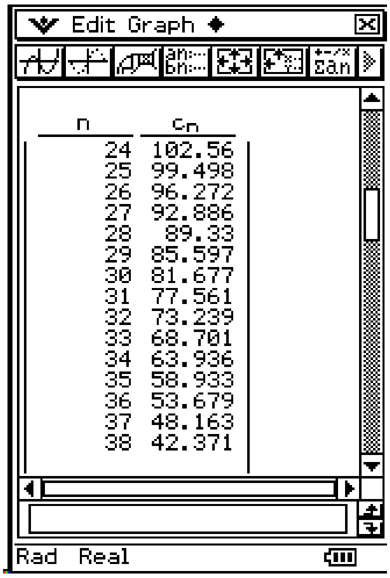
b. First three terms of the sequence generated are 144, 143, 141.95

To be arithmetic $t_2 - t_1 = t_3 - t_2$
$t_2 - t_1 = -1$ and $t_3 - t_2 = -1.05$ therefore not arithmetic M1

To be geometric $\frac{t_2}{t_1} = \frac{t_3}{t_2}$
$\frac{t_2}{t_1} = 0.9931$ and $\frac{t_3}{t_2} = 0.9927$ therefore not geometric M1

c. 37 recharges A1

Using a table of values



n	cn
24	102.56
25	99.498
26	96.272
27	92.886
28	89.33
29	85.597
30	81.677
31	77.561
32	73.239
33	68.701
34	63.936
35	58.933
36	53.679
37	48.163
38	42.371

d. Solve for $T_{n+1} = T_n = 48$

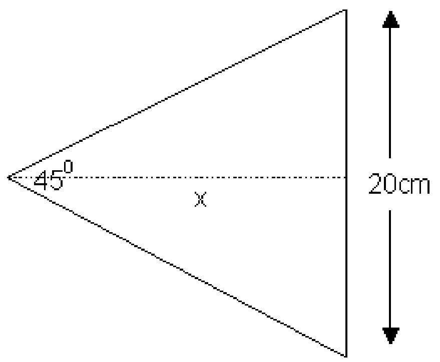
$48 = 1.05 \times 48 - d$ M1

$d = 2.4$ A1

Module 2 Geometry and Trigonometry

Question 1

a. The octagon can be divided into 8 triangles as illustrated



Using half of this triangle

$\tan 22.5° = \frac{10}{x}$ M1

Length of side of square = $2x = 2 \times \frac{10}{\tan(22.5°)} = 48.28\,\text{cm}$ A1

b. $180° - 22.5° - 90° = 67.5°$

$2 \times 67.5° = 135°$ M1

c.i. Area of each triangle $= \frac{1}{2}bh = \frac{1}{2} \times 20 \times 24.14 = 241.4$ M1

Area octagon = $8 \times 241.4 = 1931.2\ \text{cm}^2$ A1

c.ii. Area of each rectangle = $20 \times 40 = 800$

Total area = $8 \times 800 + 1931.2 = 8331\text{cm}^2$ A1

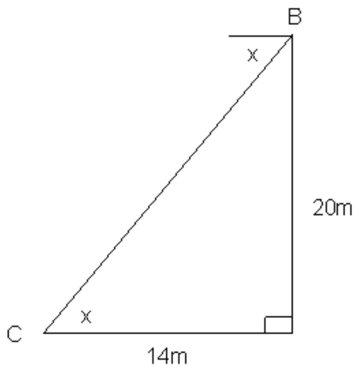
d.i. Length ratio = 3:4

$\frac{3}{4} \times 20 = 15\,\text{cm}$ A1

d.ii. Volume ratio = $3^3 : 4^3 = 27 : 64 \Rightarrow \frac{27}{64}$ A1

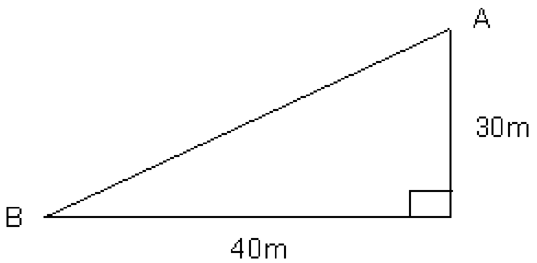
Question 2

a. Actual horizontal distance = $1.4\,\text{cm} \times 1000 = 1400\,\text{cm} = 14\,\text{m}$
Vertical distance =20 m M1



Angle of depression = x where $\tan(x) = \frac{20}{14} = 55°$ A1

b. Horizontal distance = 40 m, vertical distance = 30 m



Direct distance = $\sqrt{30^2 + 40^2} = 50$m A1

Question 3

a. angle PMQ = $226° -$ bearing of M
= $226° - 90° = 136°$ M1

b.i. Using the cosine rule

distance PQ = $\sqrt{640^2 + 430^2 - 2 \times 640 \times 430 \times \cos 136°}$
= 995.2 m A1

b.ii. $\frac{\sin MPQ}{430} = \frac{\sin 136°}{995.2}$ M1

angle MPQ = $17.47°$

Bearing of Q from P = $46° + 17.47° = 63.47°$ therefore $063°$ A1

Module 3: Graphs and relations

Question 1

a. $5 \times 1000 = \$5000$ A1

b. $3 \times 500 + 6 \times 500 = \$4500$ A1

c. Amount received

$= 3 \times 500 + 6(n - 500)$

$= 6n - 1500$ M1

$\Rightarrow k = 6 \;, \; c = -1500$

A1

d. Point A $(500, 1500)$ A1

At Point B $5n = 6n - 1500 \Rightarrow n = 1500$

When $n = 1500$ amount received = \$7500

$(1500, 7500)$ A1

e. Sienna is better off with Option 1 when selling less than 1500 bracelets and better off with Option 2 when selling more than 1500 bracelets A1

Question 2

a. Consider $n = 2$ and $n = 3$ A1

b. Substitute in a point e.g. $(4, 9600)$

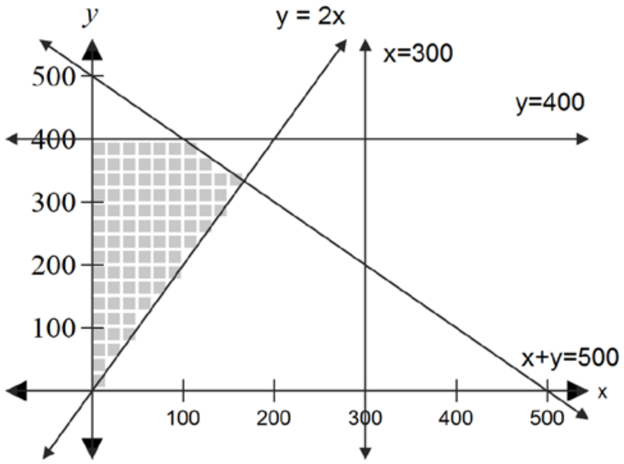
$9600 = k \times 64 \Rightarrow k = 150$ A1

Question 3

a. The total number of necklaces and rings sold cannot exceed 500 A1

b. $y \geq 2x$ A1

c.



Shaded region is the feasible region.

A1

d. Consider any profit expression similar to $P = 2x + y$

Determine the extreme point which is the intersection of $x + y = 500$ and $y = 2x$ M1

This point is $(166\frac{2}{3}, 333\frac{1}{3})$ however considering integer solutions within the feasible region gives 166 necklaces and 334 rings A1

e. Consider any profit expression similar to $P = x + y$

Maximum profit occurs along the line $x + y = 500$ between $(100, 400)$ and $(166\frac{2}{3}, 333\frac{1}{3})$

Again considering integer solutions minimum is 334 rings and maximum 400 rings A1

Module 4 Business related mathematics

Question 1

a. Total cost $= 25000 + 1200 + 300 = \$26500$ A1

b. $\frac{300}{1500} \times 100 = 20\%$ A1

Question 2

a. Finance Solver

$N = 130$

$I(\%) = 8.2$

$PV = 25\,000$

Pmt = ?

FV = 0

$PpY = 26$

$CpY = 26$

Fortnightly repayment is \$234.72 A1

b. Total Interest = 234.72 × 130 – 25 000

Total Interest = \$5513.60 A1

c. Finance Solver

$N = 130$

$I(\%) = 6.7$

$PV = 50000$

$Pmt = ?$

$FV = 0$

$PpY = 26$

$CpY = 26$

Fortnightly repayment is \$453.12 M1

When PV =75000 the Fortnightly repayment is \$679.68

Saving = $130 \times 234.72 - 130 \times (679.68 - 453.12) = \$1060.80$ A1

Question 3

a. $\frac{1320}{11} = \$120$ A1

b. $250 + 6 \times 190 = \$1390$ **A1**

c. Interest paid in 6 months is $6 \times 190 - (1320 - 250) = \$70$ M1

This is equivalent to \$140 interest per annum

$\frac{140}{1070} \times 100 = 13.1\%$ A1

d. The effective interest rate is calculated on the average amount of principal owed over the term of the loan. It is higher than the flat rate as it takes into account **the reductions in the principal owed**

A1

e. Depreciation per annum = $\frac{1320 - 330}{3} = \$330$ M1

Annual depreciation rate = $\frac{330}{1320} \times 100$ =25% A1

f. Depreciated value = Purchase price $\times \left(1 - \frac{depreciation\,rate}{100}\right)^{number\ of\ years}$

$330 = 1320\left(1 - \frac{d}{100}\right)^3$ M1

$1 - \frac{d}{100} = \sqrt[3]{0.25}$

$\frac{d}{100} = 0.37004$

depreciation rate =37% A1

Module 5: Networks and decision mathematics

Question 1

a. 5 A1

The five paths are AEIJ, ACFGIJ, ACFHJ, BDFGIJ and BDFHJ

b. The critical path is the longest path from start to finish
Critical path is BDFGIJ A1

Minimum time for project completion is the length of the critical path = 17 weeks A1

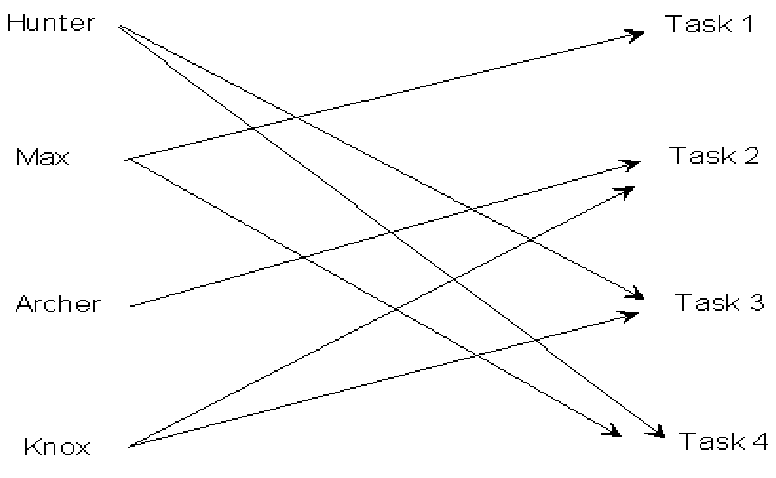
c. Activity H is non-critical with a float (slack) time of 1 week therefore this will have **no effect** on the minimum completion time A1

d. Activity E A1
It has an earliest start time of 2 weeks and a latest start time of 4 weeks

Question 2

a. Minimum overall time will occur with Knox doing either Task 2 or Task 3 A1

b.



Recognition of the significance of the zero elements M1

All arrows correct A1

c. Hunter Task 4, Max Task 1, Archer Task 2, Knox Task 3 A1

d. $12+9+10+20=51$ minutes A1

Question 3

a. Archer 1 (victory over Max) and Knox 2 (victories over Hunter and Archer) A1

b. Knox defeated Hunter who has dominance over one other (Archer) and Knox defeated Archer who has dominance over one other (Max) A1

c. First Max, Second Knox, Third Archer, Fourth Hunter A1

d. Maximum =8 A1

This will occur when the competition is as even as possible i.e. with 2 players having 2 wins and 2 players having 1 win (as in the original scenario)

Minimum =4 A1

This will occur when the competition is as 'one-sided' as possible i.e. when 1 player has 3 wins (giving 3 two-step dominances), 1 player having 2 wins (giving 1 two-step dominance), 1 player having 1 win (no two-step dominances) and 1 player having 0 wins (no two-step dominances)

Module 6: Matrices

Question 1

a. $C = \begin{bmatrix} 8 & 6 & 4 \end{bmatrix}$ A1

b. 3×1 A1

c. $\begin{bmatrix} 8 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 30 \end{bmatrix} = \begin{bmatrix} 156 \end{bmatrix}$ therefore \$156 A1

Question 2

a. $\begin{bmatrix} 5 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} 33.50 \\ 32.70 \end{bmatrix}$ A1

b. As the determinant is non zero (=18) A1

c.

$$\begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{9} \\ -\frac{1}{6} & \frac{5}{18} \end{bmatrix} \begin{bmatrix} 33.50 \\ 32.70 \end{bmatrix}$$ A1

d. Solving from **c.** gives $h = \$3.90$ and $s = \$3.50$
Charge is $3 \times 3.90 + 2 \times 3.50 = \$18.70$ A1

Question 3

a. Red and Yellow A1

b. The Green monster cannot destroy any of the monsters A1

c. The diagonal of zeros A1

d. 90% of players who play Devil's disguise on any visit also play it on their next visit A1

e. $T \times S_1 = \begin{bmatrix} 363 \\ 327 \end{bmatrix}$

363 play Monster's Ink and 327 play Devil's Disguise A1

f. Using a large value of n the steady state can be found e.g. $T^{100} \times S_1 = \begin{bmatrix} 230 \\ 460 \end{bmatrix}$

Proportion playing Monster's Ink is $\frac{230}{690} = \frac{1}{3}$

A1

Question 4

Columns must add to 1 therefore $x = 0.7$ and $y + z = 1$ M1

For 50% at each venue in the long run, the proportions switching each way must be equal therefore $y = 0.3$ and $z = 0.7$ A1