2012 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

There were 28 937 students who sat the Further Mathematics examination 2 in 2012. The selection of modules by students is shown in the table below.

MODULE	% 2012
1 – Number patterns	28%
2 – Geometry and trigonometry	70%
3 – Graphs and relations	41%
4 – Business-related mathematics	35%
5 – Networks and decision mathematics	44%
6 – Matrices	72%

Most students completed the Core section and three modules. In general, students were able to answer the first question in each module very well but were challenged by the later, more complex questions.

Some students wrote answers without showing any working.

Students should be aware that, even if their answer is incorrect, they may be awarded some marks for demonstrating a correct method in a question worth more than one mark.

They may also have access to consequential marks for some questions. Consequential errors can arise where an answer from a previous question must be used determine the answer to a subsequent question.

Consequential marks will only be awarded where
- the question is structured so that this is applicable
- a calculation is written that shows the application of the previous answer
- the answer found applies to the written calculation
- the answer is reasonable within the context of the question.

Without written calculations, students who only provide answers do not have access to rounding error or further engagement error consideration and allowances.

A rounding error occurs where the student has clearly rounded a number incorrectly. For example, if a student, when rounding off to one decimal place as required, had written 3.3496… = 3.5, then this is considered a rounding error for which a maximum of one mark for the entire paper is deducted. If this student had already been penalised for making a rounding error earlier in the paper, then full marks would be given for writing 3.3496… = 3.5 in this instance. However, if the student had only written an answer of 3.5 with no other working before it, then it is not clear if a rounding error has been made. This is then an absolute error, which will be penalised without limit.

A further engagement error occurs where a student has written down a correct answer but has then continued with the question to eventually end up with an incorrect answer. For example, a question asks for an answer correct to the nearest cent and $123.47 is the expected answer. Any student whose final answer was $123.47 would get full marks. A student who then worked on to obtain $123.45 will lose one mark as a further engagement error.

There are other types of further engagement errors that sometimes occur in Further Mathematics exams but the above illustrates the most common type.

Students will lose a maximum of one mark for each type of further engagement error for the entire paper.

So, a student who wrote 12.49 = 12.5 metres (expected answer is 12.49 metres) on one page and then later wrote $123.47 = $123.45 on another page would lose only one mark for the first further engagement error and get full marks for the second. But, if this same student then subsequently wrote, in the matrices module for example, $\begin{bmatrix} 2 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 13 \end{bmatrix}$ where the first matrix is correct, then this would be a different type of further engagement error and would again incur the loss of a mark.

In Module 3 – Business-related Mathematics, a small number of students use the annuities formula instead of the TVM calculator function. As pointed out repeatedly in recent assessment reports, the use of the annuities formula is not required in the current study design. It is neither an efficient nor an effective way of solving annuities-related problems.

Failure to read the question carefully cost many students marks. For example, Question 3a. in the Core section asked for ‘The wind direction with the lowest recorded wind speed… ’ However, many students answered this question as if it were ‘The lowest recorded wind speed …’

Some students seemed to have a limited understanding of terms such as ‘residual value’, ‘right-triangular prism’ and ‘perpetuity’. The development of a glossary of key terms throughout the year is suggested.

There were fewer examples of unreasonable numerical answers this year than has been seen in the past. Students are encouraged to review each answer, relate it to the context of the question and ask themselves whether it is reasonable. For example, 1322.0 metres is not a reasonable answer for the length of fencing around the top of the excavation shown in Question 1c. in Module 2 – Geometry and Trigonometry.

Core

Question 1a–b.

Marks	0	1	2	3	Average
%	6	17	35	41	**2.1**

1ai.
20 °C

1aii.
23.3 %

1b.
97.5%

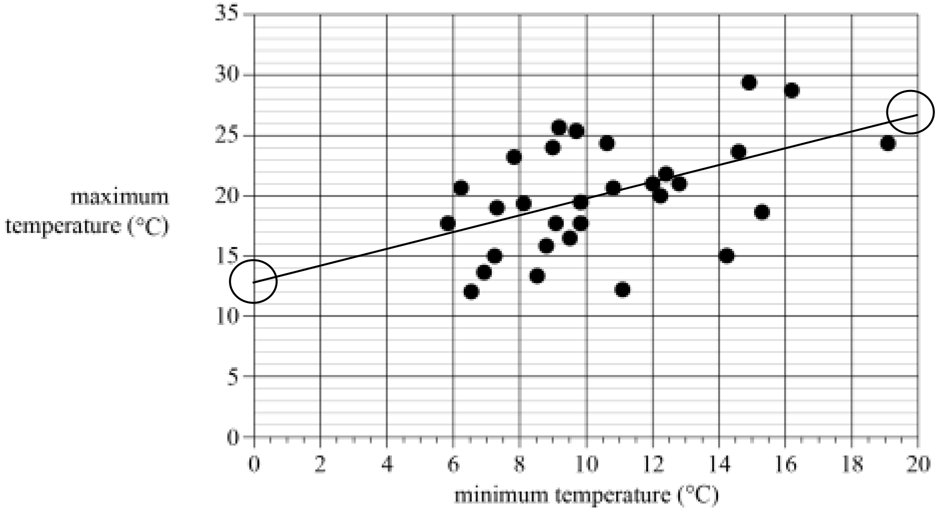
1– 0.025 = 0.975

Some students were unable to apply the 68-95-99.7% rule.

Question 2a–f.

Marks	0	1	2	3	4	5	6	7	Average
%	30	14	9	8	8	9	10	13	**2.8**

2a.
A line should have been drawn on the graph through (0, 13) and (20, 26.4), circled below.



Some students did not understand that this question required a line to be drawn on the scatterplot. Many students who attempted to draw the line, did so with insufficient accuracy, particularly with regard to the need to pass through or close to the point (20, 26.4)

2b.
On average, it is predicted that, when the minimum temperature is 0 °C, the maximum temperature will be 13 °C.

2c.
Moderate, positive

2d.
On average, it is predicted that the maximum temperature increases by 0.67 °C for each 1 °C increase in the minimum temperature.

Some students gave the dependent and independent variable names in reverse order.

2e.
40%

2f.
–8 °C

Predicted value = 13 + 0.67 × 11.1 = 20.437…
Residual value = 12.2 – 20.437 … = –8.237…

The negative sign on the residual was required.

This question was often left unanswered. Many students incorrectly calculated the residual as 12.2 − 11.1 = 1.1 ≈ 1.

Question 3a–4b.

Marks	0	1	2	3	4	5	Average
%	15	35	9	15	16	9	**2.1**

3a.
The wind direction with the lowest recorded wind speed was south-east.
The wind direction with the largest range of recorded wind speeds was north-east.

3b.
2, 2, 2, 3, 4, 4, 4, 4

Few students answered this question correctly.

Often, incorrect answers included 3.5 as a value, while others included numbers less than 2 or greater than 4.

The eight wind speeds were recorded correct to the nearest whole number and, since the median = 3.5, the fourth and fifth values either side of this median must be 3 and 4. Further, the lack of whiskers on the relevant boxplot means that the minimum value and the first quartile (Q_1) coincide and thus are both 2. Similarly, the maximum value and Q_3 are both 4.

This means the data set must look like {2, _, _, 3, 4, _, _, 4}.

Since $Q_1 = 2$ and Q_1 represents the midpoint of the second and third data points in this group of eight, both these data points must be 2.

Similarly, since $Q_3 = 4$, the sixth and seventh data points must both be 4.

Hence, the data values are 2, 2, 2, 3, 4, 4, 4, 4.

The available mark was awarded for these eight numbers written in any order.

4a.
3.4, 6.6

Many students did not answer this question. Some students who did answer this question did not seem to know how to deal with the squared transformation of the dependent variable.

4b.
13 km/h

$(ws3pm)^2 = 3.4 + 6.6 \times 24 = 161.8$
$\therefore\ ws3pm = \sqrt{161.8} = 12.720\ \ldots$

Many students who performed the correct transformation and got the correct coefficients for the equation in Question 4a. gave an incorrect rounded answer of 162. They needed to find the square root to obtain the value of *ws3pm* rather than leave it as $(ws3pm)^2$.

Module 1 – Number patterns

Question 1

1a.–1dii.

Marks	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**Average**
%	2	4	9	21	37	6	22	**3.9**

1a.
162 – 168 = –6 or 156 – 162 = –6

1b.
138

$t_6 = 168 + (6–1) \times (–6) = 138$

1c.
12

1di.
2106

$$S_{18} = \frac{18}{2}\left(2\times 168 + (18-1)\times(-6)\right) = 2106$$

1dii.
330

Many students did not attempt this question and others made little progress toward a solution.

The sequence can go on for $\frac{168}{6} = 28$ months.

$$S_{28} = \frac{28}{2}\left(2\times 168 + 27\times(-6)\right) = 2436$$

Therefore, the number of blocks left after 18 months in which 2106 blocks had been sold = 2436 – 2106 = 330.

Question 2a–2e.

Marks	0	1	2	3	4	5	6	Average
%	13	6	10	16	18	18	19	3.5

2a.
36

$16 \times (1.5)^{3-1} = 36$

2b.
Year 6

$16 \times 1.5^{n-1} = 100$
$\therefore\ n = 5.5197..$, which is in the sixth year.

2c.
211

Solve $S_5 = \dfrac{16(1.5^5 - 1)}{0.5} = 211$

2d.
Year 9

$\dfrac{16(1.5^n - 1)}{0.5} = 1000 \;\; \therefore n = 8.57...$, which is in the ninth year.

or

determine when the terms of the sequence first exceed 1000 as in $S_8 = 788$ and $S_9 = 1198$

2e.
$a = 1.5$, $b = 0$, $c = 16$

Many students were unable to determine the value of b.

Question 3a.–c.

Marks	0	1	2	3	Average
%	40	16	26	18	1.2

3a.
1026

$P_1 = 50$
$P_2 = 548$
$P_3 = 1026.08$

3b.
Year 11

n	***P***
9	3518.6…
10	3877.9…
11	4222.8…

Many students did not attempt this question.

3c.
12 500

Find when $P_{n+1} = P_n$
$P = 0.96P + 500$
$\therefore 0.04P = 500$
$\therefore\ P = 12\ 500$

Many students did not attempt this question, while others obtained an answer of 12 499.999… and then inappropriately rounded this to 12 499.

Module 2 – Geometry and trigonometry

Question 1a.–c.

Marks	0	1	2	3	Average
%	5	22	37	37	2.1

1a.
4250 m^2

1b.
1000 m^3

$$V = \frac{1}{2} \times 25 \times 4 \times 20 = 1000$$

A surprising number of students used $V = LWH$, failing to see this was a triangular prism rather than a rectangular prism.

1c.
90.6 m

$$Length = 2 \times 20 + 2 \times \sqrt{25^2 + 4^2} = 90.635...$$

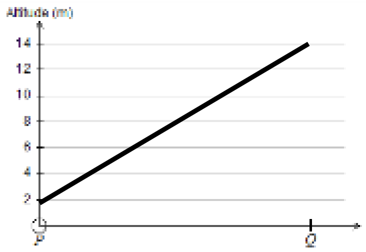
This question was not well done by many students, with 50.6 being a common incorrect answer that did not include the two 20 m sections of fence FE and AD. This is likely to be the result of incomplete reading of the question.

Question 2a.–2cii.

Marks	0	1	2	3	4	Average
%	13	29	23	15	20	2

2a.
12 m

2b.



The cross-section is continuous and must start at (0, 2) and end at (Q, 14). It need not have been ruled, since an incline on natural ground is not perfectly smooth or straight. However, substantial deviations from a straight line were not accepted.

Common errors included starting at (0, 0) or ending short of where point Q was on the horizontal axis or where 14 was on the vertical axis.

2ci.
132°

This question was quite poorly answered by many students. Working with bearings seems to be a problem for students.

2cii.
222°

This question was often not answered or very poorly answered by many students.

Question 3a–b.

Marks	0	1	2	Average
%	24	50	26	1

3a.

$$\frac{ST}{\sin(47^\circ)} = \frac{50}{\sin(61^\circ)}$$

$$\therefore ST = \frac{50\ \sin(47^\circ)}{\sin(61^\circ)} \approx 41.81$$

This ‘show that’ question required the end result of an appropriate calculation to be 41.81.

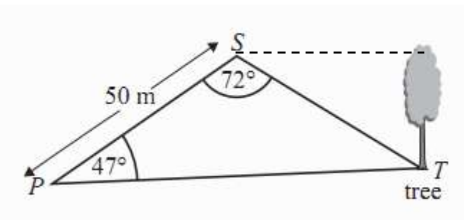
3b.
16.9 m

Height = 41.81 × tan (22°) = 16.892…..

This question was very poorly answered by many students, who appeared to be unable to interpret the diagram and written description.

The most common incorrect answer came from students who connected point S to the top of the tree, as shown in the diagram below, then incorrectly assumed a right triangle had been formed with ST as the hypotenuse and then used

$$\sin\left(22^\circ\right) = \frac{x}{41.81} = 15.7$$



Question 4a.–d.

Marks	0	1	2	3	4	5	6	Average
%	22	11	18	13	15	6	14	2.7

4a.
8.66 m

$10 \times \cos(30°) = 8.660\ldots$

4b.
21.7 m^3

$\frac{1}{2} \times 10 \times 8.66 \times \sin(30^\circ) = 21.65...$

Some applied Heron's rule instead of $Area = \frac{1}{2} bc \sin(A)$.

4c.
68.6 m^2

$\left(\frac{14}{10}\right)^2 \times 35 = 68.6$

While most students correctly found the scale factor $\frac{14}{10}$, many did not apply it properly by failing to square it when calculating the scaled-up area.

4d.
43°

$BC = \sqrt{10^2 + 14^2 - 2(10)(14)\cos(30^\circ)} = 7.315...$

$\therefore \frac{\sin BCO}{10} = \frac{\sin 30}{BC = 7.315...}$

$\therefore \angle BCO = 43.118\ldots = \angle CDO$

Many students did not attempt this question.

The key to the quickest solution starts with realising that angle *CDO* is the same as angle *BCO*.

Module 3 – Graphs and relations

Rounding of technology results without reference to the context of a question resulted in many students losing marks in this module. Some students gave annotations such as 'always round up if objects or people are involved'. Such general statements do not take into account the context of the question and would be correct if applied to Question 2a. but incorrect if applied to Question 3d. In each case, the rounding depended on whether the whole number result had to be a minimum, as in Question 2a. or a maximum as in Question 3d.

Question 1a–c.

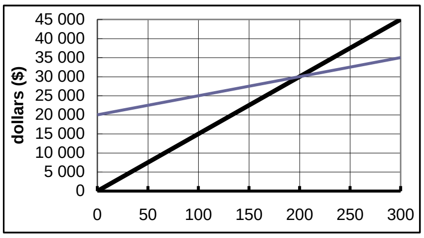
Marks	0	1	2	3	4	5	Average
%	10	9	9	9	15	49	3.6

1ai.
50

1aii.
$C = 50n + 20\ 000$

A number of students used the variable x rather than n.

1bi.



1bii.
360

$54\ 000 = 150\ n$

1c.
200 phones

$50n + 20\ 000 = 150n$
$\therefore n = 200$

Question 2a–b.

Marks	0	1	2	Average
%	49	24	27	0.8

2a.
447 laptops

$320n + 125\ 000 = 600n$

$\therefore$ break even at 446.428 laptops and so must sell one more than 446 for a profit

A common error involved not relating the answer to the context and rounding 446.428… down to 446, which is then on the loss side of the break-even point, rather than the profit side as required.

2b.
$682.50

New Cost equation is $C = 370n + 125\ 000$

For 400 phones, cost = $273 000

$$\therefore \text{ per phone} = \frac{273\ 000}{400} = 682.5$$

Question 3a–b.

Marks	0	1	2	Average
%	18	29	52	1.4

3a.
The time available for repairing phones and laptops is not more than 1750 minutes each day.

The answer required a reference to a maximum time of 1750 minutes being available for repairs of both phones and laptops. Some incomplete responses said that ‘less than 1750 minutes for repairs …’ was available but these should have read ‘less than **or equal to** 1750 minutes for repairs …’

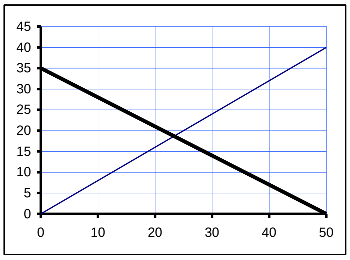
Many responses incorrectly referred to 1750 ‘phones and laptops’, while others did not indicate what the 1750 referred to at all.

3b.
8

Question 3c.–3fii.

Marks	0	1	2	3	4	5	6	Average
%	29	24	17	11	11	2	7	**1.9**

3c.



3d.
18

Intersection of lines at (23.3, 18.7)

The maximum number of laptops must be fewer than 18.7.

The most common incorrect answer was 35, which resulted from not addressing Constraint 4.

Many students found $x \approx 18.7$ and rounded up to give 19 laptops. However, it can be seen that this value is above the intersection point and thus outside the feasible region.

3e.
37 phones

$35x + 50 \times (y = 9) = 1750$

$\therefore\ x = 37.14\ldots$

From the graph, the line $y = 9$ crosses both $y = \frac{4}{5}x$ and $35x + 50y \le 1750$, with the greatest value for x occurring at the second line crossing through $35x + 50y \le 1750$.

A common incorrect answer was 11 phones found by using only the rule or graph of $y = \frac{4}{5}x$.

3fi.
24 phones and 18 laptops

One mark was awarded for each of the two numbers correctly labelled.

By substitution, we can determine that the maximum profit occurs at the intersection of the two lines or at a point below this intersection.

Alternatively, the ‘sliding line’ method could be used.

P = $60x + 100y$ (gradient = –0.6)

$35x + 50y = 1750$ (gradient = –0.7) which is steeper than –0.6

Therefore, slide the profit line up to get maximum value of y at 18 (see answer to Question 3d.).

Then substitute $y = 18$ into Constraint 3.

$\therefore 35x + 50 \times (y = 18) = 1750$

$\therefore\ x = 24.28\ldots$

When $y = 18$, $x = 23$ or $x = 24$, as both fit inside the feasible region. Take the larger value for x.

Then, maximum profit is at (24, 18).

The most common incorrect answer was 23 laptops and 19 phones.

3fii.
$3240

$P = 60 \times 24 + 100 \times 18 = 3240$

Very few students obtained this answer.

Module 4 – Business-related mathematics

Question 1a.–c.

Marks	0	1	2	3	4	Average
%	5	3	14	35	43	3.1

1a.
$1254

$8360 \times 0.15 = 1254$

1bi.
$7106

$8360 - 1254 = 7106$

1bii.
$694

Repaid = $12 \times 650 = 7800$

Interest = $7800 - 7106 = 694$

1c.
$7600

$1.1x = 8360$

$\therefore x = 7600$

As in previous years, many students had difficulty with this reverse GST calculation.

A very common incorrect answer was \$7524, found from 90% of \$8360.

Question 2a.–d.

Marks	0	1	2	3	4	Average
%	22	15	19	23	20	**2.1**

2a.
\$5852

$8360 - 0.22 \times 3800 \times 3 = 5852$

A common error was to give the total depreciation over three years rather than the depreciated value after three years.

2b.
$8360 \times 0.1 = 0.22 \times 3800 = \$836$

This 'show that' question required two calculations, but many students did not follow the instructions and showed only one calculation. Others determined the depreciated *value* for a specific year instead of the annual depreciation *amount* in any one year as required.

2c.
10 years

$0.22 \times 3800 \times n = 8360$
$\therefore n = 10$

2d.
\$1850

$8360 \times 0.86^{10} = 1850.08\ldots$

Question 3a.–c.

Marks	0	1	2	3	4	Average
%	45	20	18	12	5	**1.1**

3a.
\$807.23

N = 60
I % = 7.8
PV = 40 000
PMT = **–807.232 51…**
FV = 0
P/Y = 12
C/Y = 12

3b.
47 months

N = **46.474 250…**
I % = 7.8
PV = 40 000
PMT = –1000
FV = 0
P/Y = 12

C/Y = 12

3ci.

$$A = 40\ 000 \times \left(1 + \frac{7.8}{1200}\right)^{12}$$

3cii.
\$281.02

New balance = \$43 234

$$I = 43\ 234 \times \frac{7.8}{1200} = 281.021...$$

Many students did not answer this question.

Question 4a–c.

Marks	0	1	2	3	Average
%	54	22	14	9	**0.8**

4a.
6.3%

$$\frac{1260 \times 4}{80\ 000} \times \frac{100}{1} = 6.3$$

This question was poorly answered. A common incorrect answer was $\frac{1260}{80\ 000} \times \frac{100}{1} = 1.575\%$.

4b.
\$80 000

Most students did not show an understanding of a 'perpetuity', in which payments are calculated so that its original value (capital) is always preserved.

The most common incorrect answer was 80 000 – 20 × 1260 = \$54 800.

4c.
\$35 208

N = 40
I % = 9.4
PV = **–35 208.002 54…**
PMT = 1260
FV = 7000
P/Y = 4
C/Y = 4

Module 5 – Networks and decision mathematics

Question 1ai.–bii.

Marks	0	1	2	3	4	5	Average
%	3	10	23	31	24	10	**3**

1ai.
160 m

This question was not answered well by many students.

Incorrect answers included 70 + 60 + 80 = 210 and 50 + 40 + 60 + 80 = 230.

1aii.
2

A large number of incorrect responses ranged between 1 and 7 inclusive.

1aiii
1250 m

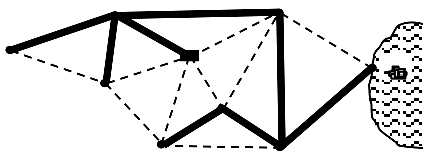
1180 + 70 = 1250

An Euler circuit would be an ideal solution but this is not possible due to the presence of two odd vertices: one at the house and one at the end of the edge marked 70, leading from the house. However, an 1180-metre long Euler path commencing at the house is possible, provided it ended at the other odd vertex. To return to the house, we must then add 70 metres for the length of the shortest path between these two odd vertices.

This question was very poorly answered, with a common incorrect answer of 1180.

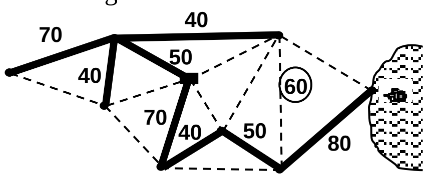
Some students wrote out all, or most of, the edge lengths and showed their (sometimes incorrect) total, despite this being given in the question.

1bi.



This question was not well done by many students.

The following is a common incorrect answer.



This answer may have been found by starting at the pump and then selecting the shortest edge from only the very last vertex connected, rather than any of the already-connected vertices.
After choosing the third edge (from the pump) marked 40, the edge marked 60 should have been selected next rather than just choosing the smallest edge that immediately followed the 40.

The apparent inability of many students to apply the algorithm for determining a minimal spanning tree is a weakness that needs to be addressed.

1bii
Minimal spanning tree

Many students provided incorrect answers such as maximum flow, Hamiltonian path, minimum cut, shortest path and others.

Question 2a.–e.

Marks	0	1	2	3	4	5	Average
%	14	26	16	13	17	14	**2.4**

2a.
12

2b.
Activity *F* has only activity *B* as a predecessor, while activities *G* and *H* have both *B* and *C* as predecessors. As there cannot be two activities called *B*, a dummy activity (with zero time) is drawn as a form of extension of *B* to the start of *G* and *H* to indicate that *B* is a predecessor for these two activities as well.

Many unacceptable answers seemed to be direct excerpts from notes about dummy activities in general, rather than clear explanations of the purpose of the specific dummy activity in the given context. Most of these did not include an explanation or demonstrate any clear understanding of what was being written.

Many students said that '… we cannot have parallel activities …', without an explanation of what this meant in the context of the question.

Similarly, many students stated that the dummy was used 'in order to satisfy the two conventions', without explaining what these conventions were and how they related to this context.

2c.
15

Few students were able to answer this question correctly. Many ignored the dummy activity and obtained the incorrect answer of 13.

2d.
A B H I L M

Common incorrect answers included *ABFJ* and *ACGM*.

2e.
25 days

A consequential mark was available for a correct calculation that showed the addition of the times, in days, for each of the activities in an incorrect critical path given for Question 2e. Instead, most students wrote a single number here without showing the calculation and were ineligible for the consequential mark.

Question 3a.–b.

Marks	0	1	2	Average
%	13	42	45	**1.4**

3a.
17

3b.
Four lines are needed before an allocation of four tasks to four people may be attempted and there are only three at the moment.

This question was answered quite poorly by many students.

Question 3c.–d.

Marks	0	1	2	3	Average
%	23	11	9	56	**2**

3c.

Task \ Worker	**Julia**	**Ken**	**Lana**	**Max**
W	0	0	4	0
X	2	2	0	10
Y	1	3	0	0
Z	7	0	3	5

3d.

Worker	Task
Julia	*W*
Ken	*Z*
Lana	*X*
Max	*Y*

Module 6 – Matrices

Question 1a.–1d.

Marks	0	1	2	3	4	Average
%	2	5	15	30	48	**3.2**

1a.
On this airline, you can fly directly from Berga to Anvil and Dantel.

1b.
Anvil – Berga – Dantel – Cantor

1c.

$$G = KF = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 1 \end{bmatrix}$$

1d.
The matrix *G* lists, for each city, the total number of direct flight connections from that city to another city in the network. It refers specifically to the number of direct flights out of each of the four cities to another city in the network, not the inward flights or just 'connections'.

The matrix $K = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ is an example of a 'summing matrix' that finds the sum of each column of the 4×4 matrix labelled *F*.

Question 2ai.–2b.

Marks	0	1	2	3	Average
%	9	18	37	36	**2**

2ai.

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 4 \\ 1 & 3 & 2 \end{bmatrix}$$

2aii.
133926

2b.
$A = B^{-1} C$

Other equivalent forms of this answer were accepted.

Question 3a.–d.

Marks	0	1	2	3	4	5	6	7	8	Average
%	21	15	13	12	11	9	8	6	5	3

3a.
275

3b.
Once a worker leaves the site, they never return (or equivalent statement).

Most students were unable to explain the meaning of the 1.00 figure in the matrix within the context of the question. Students were expected to respond with a statement such as ‘Once a worker leaves the site, they never return’.

The most common unacceptable answer was ‘100% of those who leave this year will leave the next year’, while others said that ‘100% of the people who do job *L* this year and will again do job *L* next year’.

3ci.

$$\begin{bmatrix} 70 \\ 170 \\ 65 \\ 45 \end{bmatrix}$$

$$S_{2012} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 70 \\ 170 \\ 65 \\ 45 \end{bmatrix}$$

3cii
143

$$S_{2013} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix}^2 \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 49 \\ 143 \\ 75.5 \\ 82.5 \end{bmatrix}$$

The most common incorrect answer was 170, found from S_{2012} rather than from S_{2013} as required.

3ciii.
2021

$$S_{2019} = \begin{bmatrix} 5.8 \\ 44.6 \\ 81.7 \\ 218 \end{bmatrix} \quad S_{2020} = \begin{bmatrix} 4 \\ 36.2 \\ 78 \\ 231.8 \end{bmatrix} \quad S_{2021} = \begin{bmatrix} 2.8 \\ 29.4 \\ 73.8 \\ 244 \end{bmatrix}$$

An answer of 'the 10th year' was not accepted, since actual years were used and required.

3civ.

No staff will remain at the site in the long-term.

$$S_{\infty} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix}^{100} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 350 \end{bmatrix}$$

The most common incorrect answer was 350.

3d.
182

$$S_{2012} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} + \begin{bmatrix} 30 \\ 20 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 190 \\ 75 \\ 45 \end{bmatrix}$$

$$\therefore S_{2013} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{bmatrix} 100 \\ 190 \\ 75 \\ 45 \end{bmatrix} + \begin{bmatrix} 30 \\ 20 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 182 \\ 96.5 \\ 91.5 \end{bmatrix}$$

From the equation given, the state matrix for each year is calculated by using the state matrix from the year before.

Many students instead found $S_{2013} = T^2 S_{2011} + A$ rather than $S_{2012} = T S_{2011} + A$ and therefore $S_{2013} = T S_{2012} + A$.

Other students inappropriately used the matrix $S_{2012} = \begin{bmatrix} 70 \\ 170 \\ 65 \\ 45 \end{bmatrix}$ from Question 3ci. to find the matrix here for S_{2013}. This matrix for S_{2012} cannot be applied here since it was determined from a different relation that did not involve matrix A.
The most common incorrect answer was 163.