The Mathematical Association of Victoria

FURTHER MATHEMATICS 2013

Trial Written Examination 1--SOLUTIONS

SECTION A: CORE--Data analysis
Answers:

1.	B	**2.**	D	**3.**	D	**4.**	B	**5.**	D
6.	A	**7.**	A	**8.**	B	**9.**	E	**10.**	D
11.	E	**12.**	D	**13.**	C				

SECTION B: MODULES
Module 1: Number Patterns

1.	A	**2.**	C	**3.**	E	**4.**	D	**5.**	B
6.	D	**7.**	C	**8.**	A	**9.**	B		

Module 2: Geometry and trigonometry

1.	C	**2.**	C	**3.**	A	**4.**	E	**5.**	E
6.	C	**7.**	B	**8.**	C	**9.**	B		

Module 3: Graphs and relations

1.	E	**2.**	E	**3.**	B	**4.**	B	**5.**	A
6.	A	**7.**	C	**8.**	E	**9.**	D		

Module 4: Business-related mathematics

1.	B	**2.**	D	**3.**	A	**4.**	B	**5.**	C
6.	B	**7.**	A	**8.**	B	**9.**	B		

Module 5: Networks and decision mathematics

1.	D	**2.**	D	**3.**	B	**4.**	E	**5.**	A
6.	A	**7.**	E	**8.**	D	**9.**	C		

Module 6: Matrices

1.	A	**2.**	A	**3.**	B	**4.**	A	**5.**	D
6.	B	**7.**	B	**8.**	A	**9.**	E		

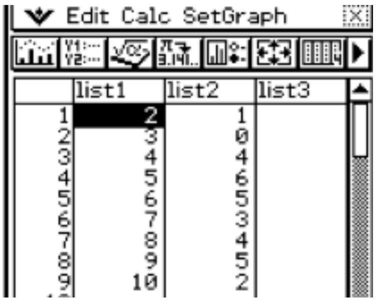
Worked solutions--Core: Data analysis

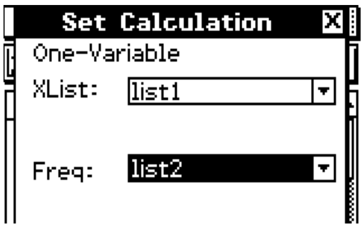
Question 1

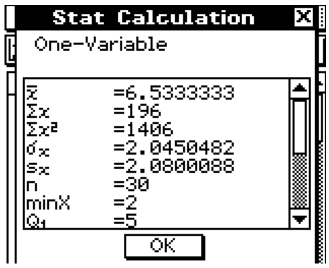
$$\frac{11}{30} \times 100 = 36.6 \approx 37\%$$

Answer B

Question 2







Answer D

Question 3

Outlier must lie above the upper boundary

$$\begin{aligned}\text{Upper Boundary} &= Q_3 + 1.5 \times IQR \\ &= 11 + 1.5 \times (11 - 5) \\ &= 20\end{aligned}$$

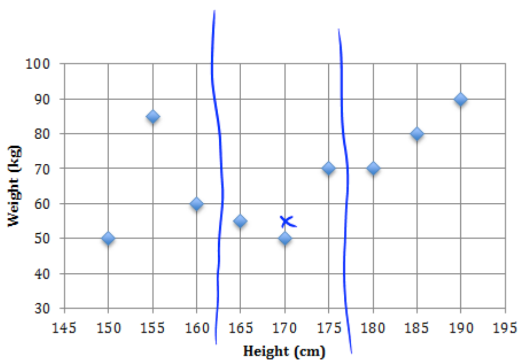
Answer D

Question 4

Two categorical variables are being displayed.

Answer B

Question 5



Answer D

Question 6

The gradient is given by $m = \frac{rise}{run} = \frac{\text{change in weight}}{\text{change in height}} = \frac{2}{3}$ kg/cm

This means that as the height increases by 3 cm the weight increases by 2 kg

so if the height increases by 9 cm then the weight increases by 6 kg

Note:
Option C is false since it is extrapolation
Option D is false since a height of 185 cm gives a weight $= 75\frac{1}{3}$ kg which is an under prediction
Option E is false because the left median point of (155, 60) will not be altered

Answer A

Question 7

Since the relationship is negative $r = -\sqrt{0.81} = -0.9$

$$m = \frac{r \times S_y}{S_x} = \frac{-0.9 \times 3}{0.5} = -5.4$$

Answer A

Question 8

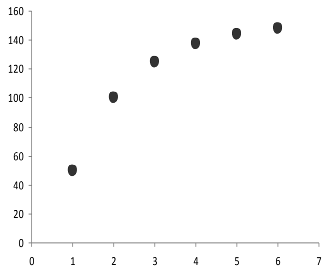
$r = 0.7$ The value of r measures the strength of the linear relationship.

This value suggests that there is a *positive* linear relationship between the weekly hours of study and the expenditure on energy drinks. So as the hours of study increase then so does the weekly expenditure.

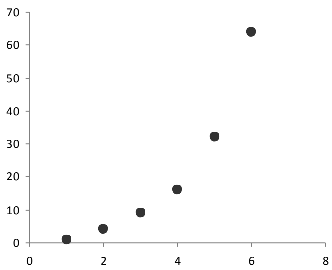
Note:
Option D: "49% of the variation of *hours studied* is explained by the variation of expenditure on energy drinks" is incorrect because hours studied is the *independent vari*able on this occasion.

Answer B

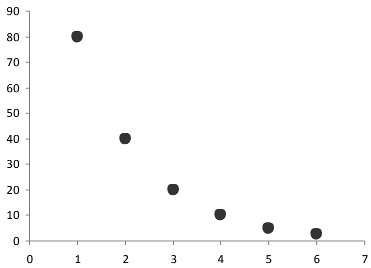
Question 9



GRAPH A



GRAPH B

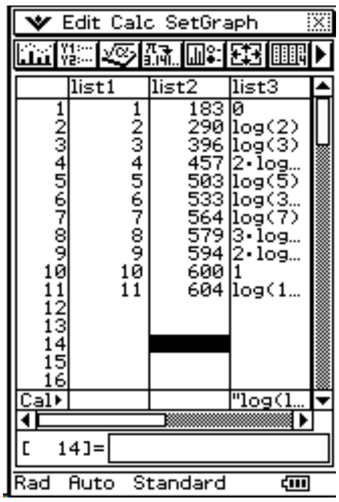


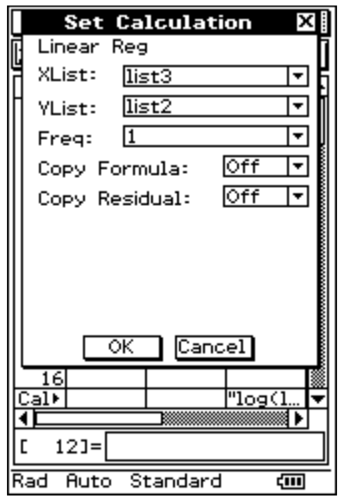
GRAPH C

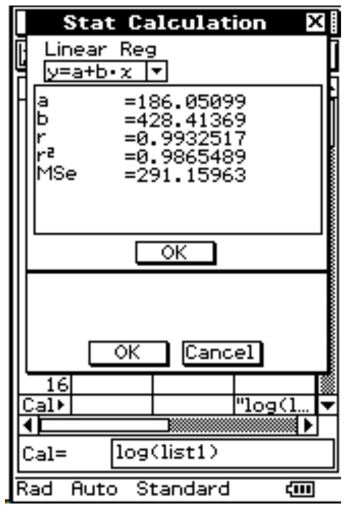
Graph A can be potentially linearised by squaring the dependent variable
Graph B can be potentially linearised by finding the reciprocal or logarithm of the dependent variable
Graph C can be potentially linearised by finding the reciprocal or logarithm of the dependent variable

Answer E

Question 10







Answer D

Question 11

Residual = Actual ice-creams sold – Predicted ice-creams sold

- 25 = 240 – Predicted ice-creams sold

Predicted ice-creams sold = 265

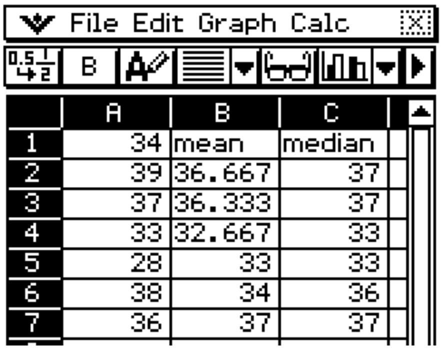
Substituting 265 ice-creams in the equation gives

265 = 0.2 × Max Daily Temperature^2 + 10.2

Max Daily Temp = 35.69

Answer E

Question 12



File Edit Graph Calc

	A	B	C
1	34	mean	median
2	39	36.667	37
3	37	36.333	37
4	33	32.667	33
5	28	33	33
6	38	34	36
7	36	37	37

Thursday and Saturday have the same 3 point mean and 3 point median values of 33 and 37 respectively

Answer D

Question 13

The sum of the Autumn and Winter seasonal sales is 4 – 1.20 – 1.05 = 1.75

So any combination of seasonal indices are possible as long as they add up to 1.75

Option A is false since either one of Autumn or Winter could potentially have an S.I. greater than 1.20

Option B is false since one season can have a S.I. below 0.75 then other will be greater then one

Option C is true because each of Autumn and Winter could have an S.I. $= \frac{1}{2} \times 1.75$

Option D is false since the S.I. of Summer = 1.20 is greater than that of spring = 1.05

Option E is false since one of the two seasons has a S.I. greater than one then the other must be less than one

Answer C

Module 1: Number patterns

Question 1

$d = t_2 - t_1 = -4 - -7 = -4 + 7$

Answer A

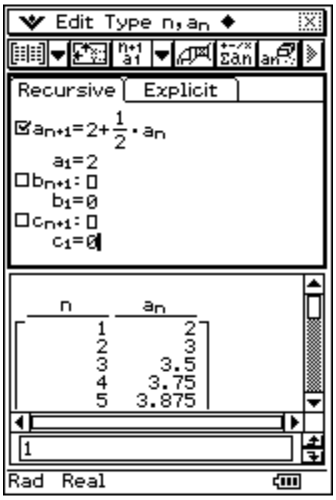
Question 2

For 7, 0.7, 0.07, …	$t_2 - t_1 \neq t_3 - t_2$ $0.7 - 7 \neq 0.07 - 0.7$ $-6.3 \neq -6.93$ is not arithmetic	$\frac{t_2}{t_1} = \frac{t_3}{t_2}$ $\frac{0.7}{7} = \frac{0.07}{0.7} = 0.1$ is geometric
For -15.2, -13.8, -12.4,…	$t_2 - t_1 = t_3 - t_2$ $-13.8 - -15.2$ $= -12.4 - -13.8$ $= 1.4$ is arithmetic	$\frac{t_2}{t_1} \neq \frac{t_3}{t_2}$ $\frac{-13.8}{-15.2} \neq \frac{-12.4}{-13.8}$ $\frac{69}{76} \neq \frac{62}{69}$ is not geometric
For $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$	$t_2 - t_1 \neq t_3 - t_2$ $\frac{1}{6} - \frac{1}{4} \neq \frac{1}{8} - \frac{1}{6}$ $-\frac{1}{12} \neq -\frac{1}{24}$ is not arithmetic	$\frac{t_2}{t_1} \neq \frac{t_3}{t_2}$ $\frac{1}{6} \div \frac{1}{4} \neq \frac{1}{8} \div \frac{1}{6}$ $\frac{2}{3} \neq \frac{3}{4}$ is not geometric
For 4, 9, 16, ….	$t_2 - t_1 \neq t_3 - t_2$ $9 - 4 \neq 16 - 9$ $5 \neq 7$ is not arithmetic	$\frac{t_2}{t_1} \neq \frac{t_3}{t_2}$ $\frac{9}{4} \neq \frac{16}{9}$ is not geometric

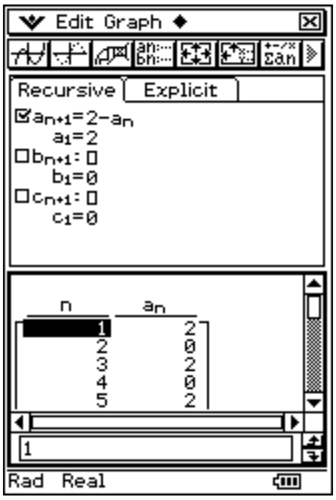
The first sequence is geometric, the second is arithmetic and the last two are neither

Answer C

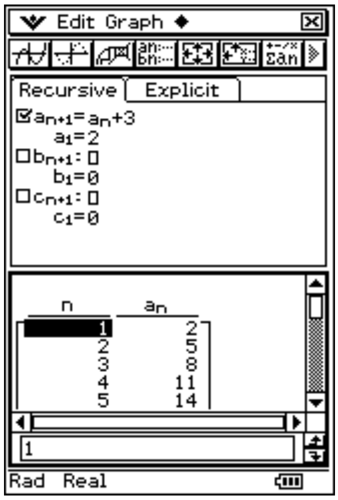
Question 3



$\frac{3}{2} \neq \frac{3.5}{3}$ not geometric

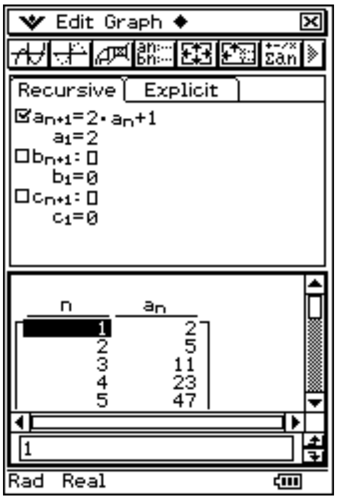


$\frac{0}{2} \neq \frac{2}{0}$ not geometric

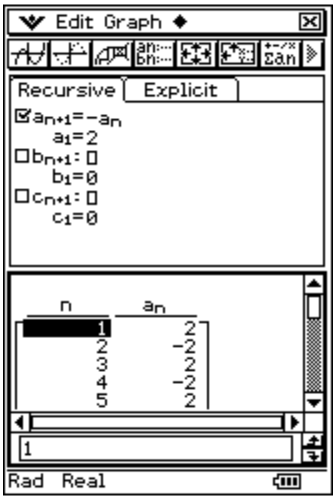


$\frac{5}{2} \neq \frac{8}{5}$ not geometric

(this is an arithmetic sequence)



$\frac{5}{2} \neq \frac{11}{5}$ not geometric



$\frac{-2}{2} = \frac{2}{-2} = -1$

has a common ratio of -1 so it is geometric

Answer E

Question 4

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{20}{2}(14 + 71) = 850$$

Answer D

Question 5

Equation 1

$t_5 = 29$

$a + 4d = 29$

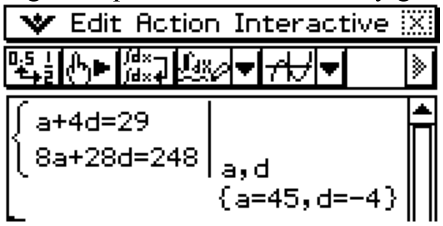
Equation 2

$S_8 = 248$

$\frac{8}{2}[2a + 7d] = 248$

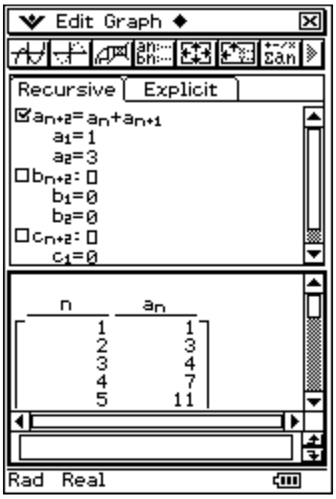
$8a + 28d = 248$

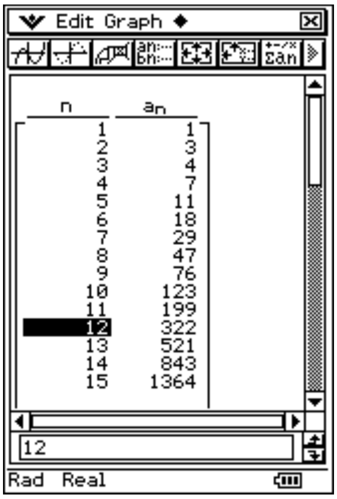
Solving the equations simultaneously gives



Answer B

Question 6





Answer D

Question 7

$T_2 = 3T_1 + b, \quad T_1 = 4$

$T_2 = 12 + b$

$T_3 = 3T_2 + b\ , \quad T_3 = 48$

$48 = 3(12 + b) + b$

$48 = 36 + 3b + b$

$48 = 36 + 4b$

$b = 3$

Answer C

Question 8

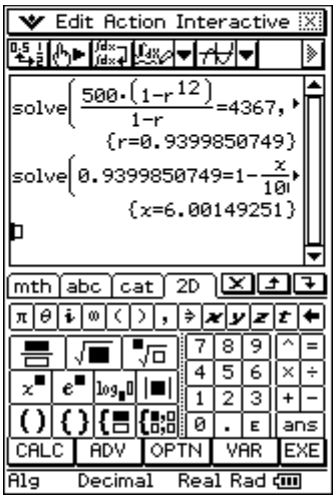
Reduction in x % means that this is a geometric sequence where $r = 1 - \frac{x}{100}$

Equation

$S_{12} = 4367, \quad a = 500$

$S_n = \frac{a(1-r^{12})}{1-r}$

$4367 = \frac{500(1-r^{12})}{1-r}$



$r = 0.939985...$

$r = 1 - \frac{x}{100}$

$x = 6$

Answer A

Question 9

Area is *increasing* by 0.2 of previous increase where the first increase is $a = 12$

The series generated is $12 + 12 \times (0.2) + 12 \times (0.2)^2 + 12 \times (0.2)^3 + ...$

So maximum increase is given by $S_\infty = \frac{a}{1-r} = \frac{12}{1-0.2} = 15$

Maximum area is 15 + 25 = 40 cm^2

Answer B

Module 2: Geometry and trigonometry

Question 1

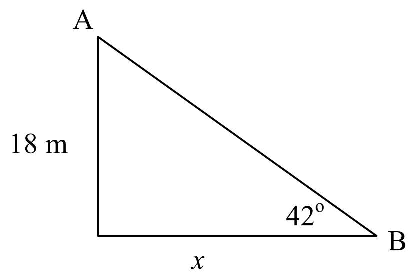
$$\frac{360^o}{8} = 45^o$$

Answer C

Question 2

$$\tan 42^o = \frac{18}{x}$$

$$x = \frac{18}{\tan 42^o} = 19.99$$



Answer C

Question 3

$$\sqrt{8\times1\times3\times4} = \sqrt{8(8-a)(8-b)(8-c)}$$

$$8-a=1,\quad 8-b=3,\quad 8-c=4$$
$$a=7,\quad b=5,\quad c=4$$

so the triangle has side lengths 7, 5 and 4

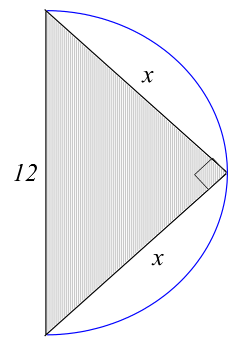
Answer A

Question 4

$$\cos x = \frac{3^2+4^2-6^2}{2\times3\times4}$$
$$= -\frac{11}{24}$$

Answer E

Question 5



Using Pythagoras

$x^2 + x^2 = 12^2$

$2x^2 = 144$

$x = \sqrt{72}$

Perimeter = $12 + 2 \times \sqrt{72} = 28.97$

Answer E

Question 6

Area of semi circle = $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times 6^2 = 18\pi$

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \sqrt{72} \times \sqrt{72} = 36$

Area of unshaded region = $18\pi - 36$

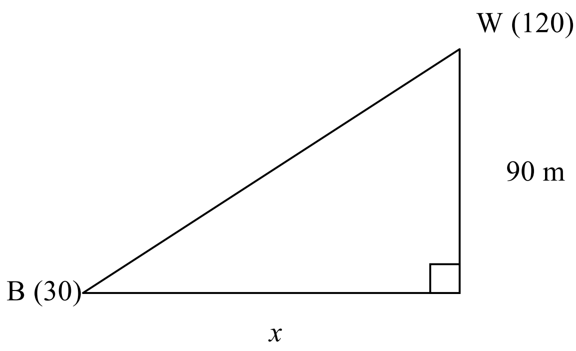
Answer C

Question 7

Let x = horizontal distance

$m = \frac{rise}{run} = \frac{90}{x} = \frac{2}{3}$

$x = 135\ m$

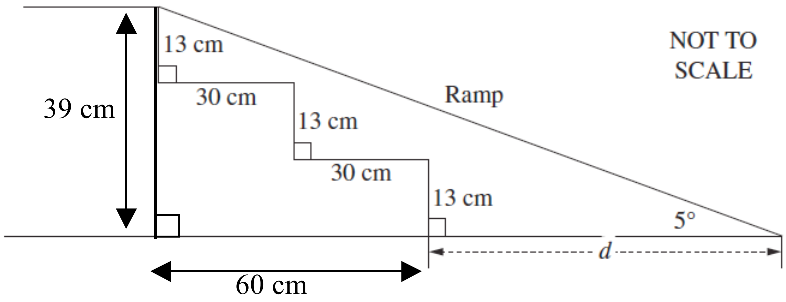


using Pythagoras the direct distance

$BW = \sqrt{90^2 + 135^2}$

Answer B

Question 8



Using the right angle triangle where

height = 39 cm and a base length = $d + 60$ cm

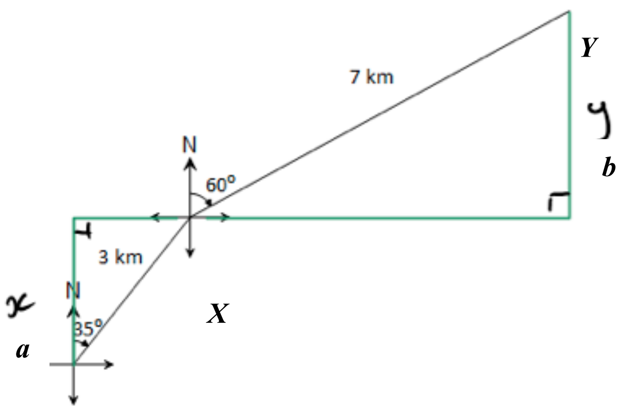
$$\tan 5^o = \frac{39}{d+60}$$

$$d = \frac{39}{\tan 5^o} - 60$$

$$= 385.772..cm$$

Answer C

Question 9



$$a + b = 3\cos 35^o + 7\sin 30^o = 3\cos 35^o + 7\cos 60^o = 3\sin 55^o + 7\sin 30^o$$

Answer B

Module 3: Graphs and relations

Question 1

Using the points (0, -2.5) and (3, 3.5)

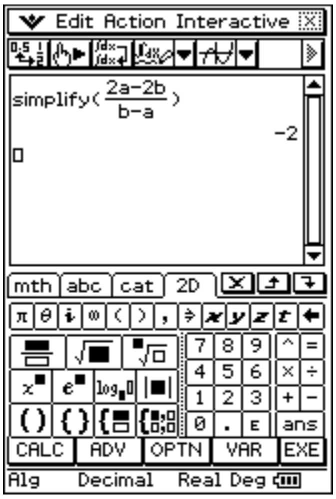
$$m = \frac{3.5 - -2.5}{3 - 0} = \frac{6}{3} = 2$$

The y intercept is at – 2.5 so the equation is $y = 2x - 2.5$

Doubling the equation gives $2y = 4x - 5$ and rearranging gives $5 = 4x - 2y$

Answer E

Question 2



$$m = \frac{rise}{run} = \frac{2a - 2b}{b - a}$$

Answer E

Question 3

Rate of change represents the gradient between the two years

From the graph the rate of change between :

1987 to 1988 is approximately 0

1988 to 1990 is approximately $\frac{rise}{run} = \frac{-3200}{2} = -1600$

1990 to 1992 is approximately $\frac{rise}{run} = \frac{-800}{2} = -400$

1992 to 1996 is approximately $\frac{rise}{run} = \frac{-3600}{4} = -900$

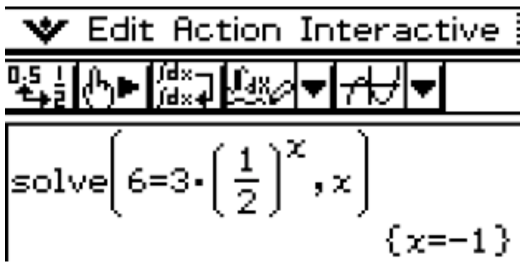
1996 to 2000 is approximately $\frac{rise}{run} = \frac{-100}{4} = -25$

Answer B

Question 4

Substituting the point $(\frac{1}{2}, 6)$ in the equation

$y = 3x^n$ gives $6 = 3 \times \left(\frac{1}{2}\right)^n$ and solve



Answer B

Question 5

The revenue equation is $5x + 4.50y = 355$

When doubled it becomes $10x + 9y = 710$

This is found in option A and C

The total number of sandwiches sold is given by the equation $x + y = 75$

This is found in options A B and D

Answer A

Question 6

Method 1

When hired for 10 days the cost is $10 \times \$100 = \$1000$ Point (10, 1000)

\$80 per day thereafter means that the gradient, $m = 80$

substitute point and $m = 80$ into $y = mx + c$

$1000 = 80 \times 10 + c$

$c = 200$

Equation is $y = 80x + 200$

Method 2

Choose two days that are at least 10 e.g. (10, 1000) and when hired for 20 days then

Cost = $10 \times \$100 + 10 \times \$80 = \$1800$ so other point is (20, 1800)

$m = \dfrac{1800 - 1000}{20 - 10} = 80$ substituting (10, 1000) in $y = 80x + c$ gives c = 200

Equation is $y = 80x + 200$

Answer A

Question 7

The passenger train travels for 300 km at 80 km/hr and stops at Springfield station for 45 min = 0.75 hours. The total time of the journey $= \dfrac{300}{80} + 0.75 = 4.5$ hours

Answer C

Question 8

If Springfield is 120 km from Melbourne then the passenger train travels 300 – 120 = 180 km to reach Springfield.

Given that the passenger train travels at 80 km/hr it will take $\frac{180}{80} = 2.25$ hours to reach Springfield and then wait for 45 minutes = 0.75 hours. So the passenger train is at Springfield when $2.25 \le t \le 3$ hours.

The Freight train must travel no faster than 120 km in 2.25 hours i.e. speed = $\frac{120}{2.25} = 53\frac{1}{3}$ km/h and no slower than 120 km in 3 hours ie speed = $\frac{120}{3} = 40$ km/h

So the speed of the freight train, x ,must be between 40 and $53\frac{1}{3}$ km/h to cross the passenger train at Springfield.

Answer E

Question 9

All values of x and y are positive so they follow the inequalities $x \ge 0$ and $y \ge 0$.

$y \le 6 - x$ means that $x + y \le 6$. That is, the sum of the coordinates must be less that or equal to 6. All points follow this inequality.

$y \ge \frac{1}{2}x$ means that the y value must be greater than half the x value.
All but (4, 1) follow this inequality.

Point	$x + y \le 6$	$y \ge \frac{1}{2}x$	
(1,3)	1 + 3 = 4	$3 \ge 0.5$	True
(1,5)	1+ 5 = 6	$5 \ge 0.5$	True
(3,2)	3 + 2 = 5	$2 \ge 1.5$	True
(4,1)	4 + 1 = 5	$1 \ge 2$	False
(3,3)	3+ 3= 6	$3 \ge 1.5$	True

Answer D

Module 4: Business-related mathematics

Question 1

8% of 150 = $12

6% of 250 = $15

Total discount = $27

Percentage discount $= \dfrac{27}{400} \times 100 = 6.75\%$

Answer B

Question 2

The value of the computer after three years $= 4000 \times 0.85^3 = 2456.50$

The amount depreciated $= 4000 - 2456.50 = \$1543.50$

Answer D

Question 3

$$I = \frac{PRT}{100}$$

$95 = \dfrac{P \times 5.5 \times \dfrac{112}{365}}{100}$ solves to give $\$5629.06$

Answer A

Question 4

Finance Solver

$\text{N} = 300$

$\text{I}(\%) = 6.5$

$\text{PV} = 250000$

$\text{Pmt} = ?$

$\text{FV} = 0$

$\text{PpY} = 12$

$\text{CpY} = 12$

Solve for Payment to give $\$1688.02$

Answer B

Question 5

\$1500 interest per quarter therefore \$6000 interest per annum.

$$I = \frac{PRT}{100}$$

$$6000 = \frac{120000 \times R \times 1}{100}$$

solving gives $R = 5\%$

Answer C

Question 6

Charge before GST is added is 85 + 140×4

10% GST therefore this charge is multiplied by 1.1

Total amount charged is therefore $(85 + 140 \times 4) \times 1.1$

Answer B

Question 7

$R = 1 + \frac{r}{100}$ where r is the interest rate per month $= \frac{6.6}{12} = 0.55$

$$R = 1 + \frac{0.55}{100} = 1.0055$$

Answer A

Question 8

Depreciation of 12 cents in dollars = $\frac{12}{100}$

Depreciation per sheet = $\frac{12}{100 \times 1000}$

To find the total depreciation multiply by number of sheets = $\frac{3000000 \times 12}{100 \times 1000}$

Initial value is 860 + depreciation = $860 + \frac{3000000 \times 12}{100 \times 1000}$

Answer B

Question 9

Choose an amount to be invested e.g. $1000

Account 1 value after x months is $1000+\dfrac{1000\times 7.5\times x}{\dfrac{12}{100}}$

Account 2 value after x months is 1000×1.005833^{x}

Edit T-Fact Graph

x	y1	y2
20	1125	1123.4
21	1131.3	1129.9
22	1137.5	1136.5
23	1143.8	1143.1
24	1150	1149.8
25	1156.3	1156.5
26	1162.5	1163.3
27	1168.8	1170.0
28	1175	1176.9
29	1181.3	1183.7
30	1187.5	1190.6

Account 2 first exceeds Account 1 in month 25

Answer B

Module 5: Networks and decision mathematics

Question 1

In a simple graph the sum of the degrees is equal to twice the number of edges.

Therefore, a simple graph with 8 edges will have 16 as the sum of the degrees of the vertices.

Answer D

Question 2

For any connected planar graph v + f = e + 2.

Since $v = f$, $2f = e + 2$ therefore $e = 2f - 2$

So the number of edges is equal to twice the number of regions (faces) minus two.

Answer D

Question 3

G is unreachable from any other vertex, but G can reach every other vertex.

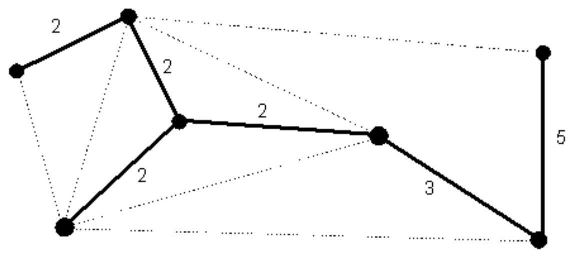
Answer B

Question 4

Answer E as it has 2 paths from A to C rather than from B to C

Answer E

Question 5



Minimum spanning tree shown in bold.

Length $= 2 + 2 + 2 + 2 + 3 + 5 = 16$

Answer A

Question 6

Optimal allocation is found by taking the shortest time for each allocated task

Mowing – Nigel

Trimming – Michael

Planting – Oscar

Mulching – Peter

The task with the longest allocation time is Mowing with 35 minutes

Answer A

Question 7

The minimum length of hose required to connect the tap to each sprinkler will be a minimum spanning tree.

Answer E

Question 8

The earliest start time for activity I is the longest path from the start to I

This will be path A-C-G that is $4+3+4=11$ days

Answer D

Question 9

The earliest start time for activity D is 4

The latest start time is $17-10=7$ (this is the length of the critical path minus length of path D-F-I)

The slack time is the latest start time $-$ earliest start time which equals $7-4=3$

Answer C

Module 6: Matrices

Question 1

For the Matrix AB to exist the number of columns in matrix A must equal the number of rows in matrix B. These values are not equal as demonstrated below

$$\begin{matrix} A & \times & B \\ 2\times 1 & \times & 2\times 2 \end{matrix}$$

Answer A

Question 2

Only square matrices may be raised to a power. A is a 1×1 square matrix

Answer A

Question 3

The inverse will not exist if the determinant is equal to zero

$4y-2x=0$ when $x=4$ and $y=2$

Answer B

Question 4

7 is in position row 1 and column 3 so is obtained by multiplying row 1 x column 3

Answer A

Question 5

$$\det\left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right)=0-6=-6$$

$$\det\left(\begin{bmatrix} 7 & 9 \\ 3 & 0 \end{bmatrix}\right)=28-27=1$$

$$\det\left(\begin{bmatrix} -1 & -3 \\ 3 & 4 \end{bmatrix}\right)=-4+9=5$$

$$\det\left(\begin{bmatrix} 5 & 7 \\ 3 & 0 \end{bmatrix}\right)=0-21=-21$$

$$\det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)=4-6=-2$$

Smallest value is -21 *Answer D*

Question 6

The transition matrix is

$$T = \begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

After 2 transition periods the state matrix is $S_3 = T^2 S_1 = \begin{bmatrix} 0.37 \\ 0.63 \end{bmatrix}$

Answer B

Question 7

Steady state will be $\dfrac{0.4}{0.4+0.7} = \dfrac{0.4}{1.1}$

Answer B

Question 8

The solution is found by evaluating $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

This uses the inverse of the original 2×2 matrix

Therefore the equations in matrix form are $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

So the original equations are $2x + 3y = 3$ and $3x + 4y = -1$

Answer A

Question 9

The number of holiday-makers who dance on the second night is
$0.6 \times 100 + 0.5 \times 100 + 0.2 \times 100 = 130$

$\dfrac{130}{300} \times 100 = 43.33\%$ therefore E is not true

Answer E