Neap:

Trial Examination 2014

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – Core: Data analysis

Question 1 D

We require approximately 35% of 5176. This will give a slight over-estimate.

$$\frac{35}{100} \times 5176 = 1812$$

The closest value is 1806.

Question 2 E

If the mean weight of an egg is 60 grams, then the total must be $12 \times 60 = 720$ grams. If we add the weight of the box we achieve a result of 740 grams.

Question 3 A

The median for both events is 14. Students may think that the New Era event has more variation but the interquartile range shows otherwise. The IQR is 5 for both.

Question 4 D

There are two requirements. The first is that they must show the most frequent ages. This is not evident at all from a box plot. A stem and leaf plot will allow us to determine such ages but it may not be evident immediately from the plot itself. A histogram is best as it allows us to see the frequency of each age.
The second requirement is that the interquartile range of ages be evident. Only the box plot allows this. Thus we need both box plot and histogram.

Question 5 C

This is clearly time series data and thus a time series graph is required. Only the third graph is thus appropriate. The second is clearly unsuitable as it fails to consider the sequential nature of the data at all. The first graph also fails to connect the data sequentially and thus fails for the same reason.

Question 6 B

This is a normal distribution and confidence limits question. The question asks about the number of fruit exceeding 1000 grams. This represents 2 standard deviations above the mean. It is known that 95% of all fruit will weigh within 2 standard deviations of the mean. Thus there will be 2.5% on each extreme.

$$\frac{2.5}{100} \times 600 = 15$$

Question 7 A

This question determines whether students confuse correlation with gradient. The B-trendline fits the data well and thus correlation is high. The C-trendline is less successful and thus clearly it will have a lower correlation. Those students who confuse gradient with correlation will be inclined to choose options **B** or **D** as they accommodate that error.

Question 8 A

This is a straight calculator question. The temperature data can be placed in list L1 and the rainfall data in L2 (or vice versa). Ensure diagnostics are turned on and then perform a *LinReg* calculation on the stat menu.

Question 9 **C**

The most appropriate least squares regression line is the one that fits the transformed data. As can be seen from the two scatterplots, the transformed data is much more correlated than the original plot and thus the trendline for this data should be used.
If a line is fitted to the transformed data it would have gradient close to 1 and the y-intercept would also be approximately 1. Thus $y = 1\log(x) + 1$.

Question 10 **C**

The seasonality is clear from the graph. The thickness peaks every winter and is least every summer. The shape is almost identical every year. What is also apparent, however, is that there is a decreasing trend also.

Question 11 **A**

We have 10 data points. 5-point moving mean would lose 2 at the start and 2 at the end. That leaves 6 points which fails to satisfy Janet's requirements. 4-point moving mean is a 2-step process but it also loses 4 data points and thus fails the requirements. The choice is thus between 3-point moving mean (losing 2 data points) and deseasonalising.

There is no reason to think that daily data is seasonal. The question states that the variations are apparently random and thus not seasonal.

Question 12 **A**

The June seasonal index is missing from the table but we know that the sum of all indices must be 4. Thus the June index is $4 - 1.05 - 1.10 - 0.95 = 0.90$.

Accordingly, the actual price for June quarter 2010 was $\$306\ 000 \times 0.90 = \$275\ 400$.

Question 13 **E**

First we calculate the deseasonalised figure.

$$\frac{330\ 000}{0.95} = 347\ 368.42$$

Now we check the prediction from the regression equation. It is quarter number 8.

$$\text{deseasonalised price} = 1560 \times 8 + 320\ 000 = 332\ 480$$

The residual is equal to $\text{actual} - \text{predicted}$ and thus the residual is 14 888.

SECTION B

Module 1: Number patterns

Question 1 **A**

$a = 3$

$r = -2$

$t_8 = 3(-2)^7 = -384$

Question 2 **B**

Geometric sequence

$a = 10$

$r = 0.7$

$t_5 = ar^4$

$t_5 = 10(0.7^4) = 2.40$

Question 3 **E**

Arithmetic sequence

$a = 100$

$d = 50$

$S_{18} = \frac{18}{2}(2 \times 100 + 17 \times 50) = 9450$

Question 4 **E**

Geometric sequence

$a = 10\ 000$

$r = \frac{6000}{10\ 000} = 0.6$

$S_\infty = \frac{a}{1-r} = \frac{10\ 000}{1-0.6} = 25\ 000$

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Question 5 B

It would not be possible for an arithmetic sequence to alternate between positive and negative terms. The difference between the first 2 terms would be negative whereas the difference between terms 2 and 3 would be a positive. These differences must be the same in order for the sequence to be arithmetic and this is clearly not the case. Thus the correct response cannot be **A**.

The sequence could be geometric. We have no means to confirm that it is geometric but we do not need to prove that anyway. The question states “could be” not “must be”.

If the sequence is geometric then it must have a negative common ratio due to its alternating terms. Option **E** is thus also invalid.

We are told that term 3 is less than term 1. Since $t_3 = ar^2$ it follows that $r^2 < 1$. This matches the criteria for the existence of an infinite sum. Options **C** and **B** are the only that remain possible. Now we choose between these by performing some calculations on the infinite sum.

$$S_\infty = \frac{a}{1-r}$$

Since $r < 0$, $1 - r > 1$. The infinite sum is thus term 1 divided by a number exceeding 1. The infinite sum must thus be less than the first term.

Question 6 B

This is a geometric sequence with first term 4 and common ratio 3. Once we recognise this to be true it is easy to apply the appropriate formula.

$$t_n = ar^{n-1} = 4(3)^{n-1}$$

Question 7 A

The sequence can be seen to have only negative terms. That rules out geometric with negative common ratio as such a sequence would alternate between positive and negative.

If the sequence was arithmetic, there would be a common difference between successive terms. That would result in the points forming a line. This is untrue also.

Some quick calculations reveal that the ratio of terms is approximately constant. It is 0.75 throughout. Thus the best option is **A**.

Question 8 C

$$S_{12} = 6000$$

$$\frac{12}{2}(a + 100) = 6000$$

$$6(a + 100) = 6000$$

$$a + 100 = 1000$$

$$a = 900$$

The largest payment is $900.

Question 9 **D**

$P_{14} = \frac{1}{2} P_{13}$

$P_{15} = 2P_{14} - P_{13}$

$= 2\left(\frac{1}{2} P_{13}\right) - P_{13} = 0$

They die out in that year. Population is zero.

Module 2: Geometry and trigonometry

Question 1 **B**

$(XY)^2 + 7^2 = 12^2$

$XY = 9.75$

Question 2 **E**

Must include 1 side from each group squared.

Only option is $A^2 + I^2 + F^2$

Question 3 **C**

$2x - 10 + 10x + 5x = 180$

$10x + 5x = 180 - 2x + 10$

Question 4 **B**

$4^2 + 12^2 + 5^2 = AD^2$

$AD = 13$

Question 5 **A**

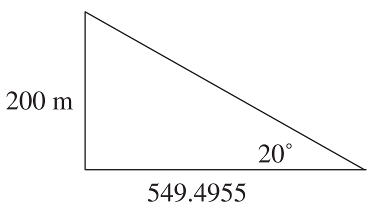
$AB^2 = 5^2 - 4^2$

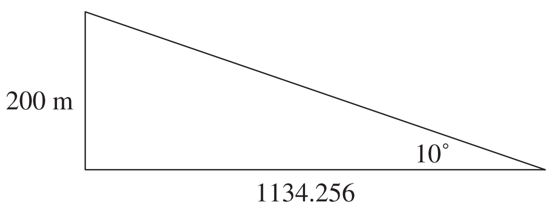
$AB = 3$

Volume = $0.5 \times$ width $\times$ height $\times$ length

$= 0.5 \times 3 \times 4 \times 12$

Question 6 **B**





$x = 1134.256 - 549.4955$

$x = 584.8$ m

Question 7 **D**

$\frac{H}{700} = \frac{2}{3}$

$H = 466.7$ cm

Question 8 **B**

$$\left(\frac{3}{700}\right)^3 \times 80 \times 10^{-6} = 6.297 \text{ m}^3$$

Question 9 **C**

Use Geometry On CAS (Draw Triangle placing values given).

Module 3: Graphs and relations

Question 1 C

Each point can be tried individually.
Point C is the line that does not contain the point required.

$2x - 3y = 2 \times -1 - 3 \times 1 = -5$ not 1.

Question 2 B

The lines shown graphed are $y = 2x - 3$ and $y = 3 - \frac{3}{2}x$

Neither of these appear in that format amongst the options listed. Options **A** and **D** can be rejected immediately, however, as they are in the same format but not correct.
We just need to pick the option that has the correct intercepts.

Question 3 A

x cappuccinos cost $3x$.
y lattes cost $2y$

$\text{total cost} = 3x + 2y$

Question 4 C

Cost for Smyths is $2x$ for the adults and $3y$ for the children. We are told that this cost is a total of 38 and thus

$2x + 3y = 38$

Cost for Nguyens is $4x$ for the adults and $5y$ for the children. Thus $4x + 5y = 70$

Question 5 C

The vertical intercept gives the base cost. That is approximately 24 on the graph.

The gradient gives the rate at which this cost is rising per km. The gradient is $\frac{1}{16}$ and thus we can say that the extra cost per km is $\frac{1}{16} = 0.0625$.

Question 6 C

The gradient of the graph is $\frac{5}{4} = 1.25$

Thus $v = \frac{5}{4} \times \frac{1}{d}$

Multiply both sides by d,

$vd = \frac{5}{4}$

Question 7 **A**

If $c < 4$, the third line will have y-intercept below the lower of the two other lines. This would prevent point A existing as shown in the diagram.
The other potential problem is that this third line may intersect the other two lines above the intersection of these other two lines with each other. Since these other two lines intersect at (2.25,2.5), the third equation gives $c = 4.75$ for the case when all 3 lines pass through the same point. Any value of *c* below 4.75 will avoid this.

Question 8 **D**

Let us label each gridline on the vertical axis as *a*. Parking for 1 hour costs 2.5*a* as it is 2.5 gridlines up from the origin.
Parking for 3 hours costs approximately 4.2*a*.

Thus $4.2a = 2.5a + 3.40$

$$1.7a = 3.4$$

$$a = 2$$

Each gridline is worth $2.
So, parking for 5 hours would cost $10 as it is 5 gridlines up.

Question 9 **D**

At a gradient of 3, a line with run 2 will have rise $3 \times 2 = 6$. This rules out only option **E**.
The other options differ in the *y*-intercept of the graph and also the intervals for each part.
Since the last section has a gradient of –1, it will be 6 units long (run = rise magnitude). The last section must start at 14. Options **A** and **D** are the only to do this.
Finally the correct line must pass through (20,0). Option **D** is thus correct.

Module 4: Business-related mathematics

Question 1 **A**

$$\frac{5000}{250\ 000} = 2\%$$

Question 2 **E**

$$11\ 100 \times 1.1 = \$12\ 210$$

Question 3 **C**

$$18\ 5000 - 1200 \times 6.5 = \$10\ 700$$

Question 4 **B**

$$\text{time} = \frac{SI \times 100}{P \times r}$$

$$\text{time} = \frac{8800 \times 100}{44\ 000 \times 5}$$

$$\text{time} = 4$$

Question 5 **D**

$$22\ 800 \times 1.055^7 = \$33\ 166.68$$

Question 6 **C**

$$P = 8100 - 900 = \$7200$$

$$I = (900 + 30 \times 250) - 8100 = \$300$$

$$r_e = \frac{I}{P} \times \frac{m}{n} \times \frac{2n}{n+1}$$

$$r_e = \frac{300}{7200} \times \frac{12}{30} \times \frac{2 \times 30}{30+1}$$

$$r_e = 0.0322\ldots \approx 3.2\%$$

Question 7 **D**

$$r_f = \frac{I}{P} \times \frac{m}{n}$$

$$0.1 = \frac{7500}{30\ 000} \times \frac{12}{n}$$

$$10 = \frac{30\ 000}{7500} \times \frac{n}{12}$$

$$n = \frac{10 \times 7500 \times 12}{30\ 000}$$

$$n = 30$$

Question 8 **E**

Use the finance solver.

$N = 50$ (any number)

$I\% = 4.5$

$PV = -120\ 000$

$Pmt = 450$

$Fv = 120\ 000$ (enter)

$P_PY = 12$

Question 9 **B**

Use the finance solver to calculate the monthly repayment.

$N = 20 \times 12$

$I = 5.1$

$PV = 125\ 000$

$Pmt = -831.87$

$FV = 0$

$P_PY = 12$

Calculate the amount owing after 12 months.

$N = 12$

$I = 5.1$

$PV = 125\ 000$

$Pmt = -831.87$

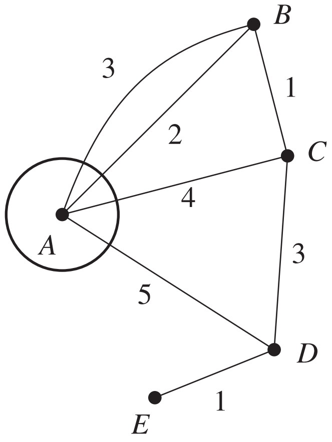
$FV = -121\ 307.03$

$P_PY = 12$

$121\ 307.03 - (125\ 000 - 12 \times 831.87) = \$6289.47$

Module 5: Networks and decision mathematics

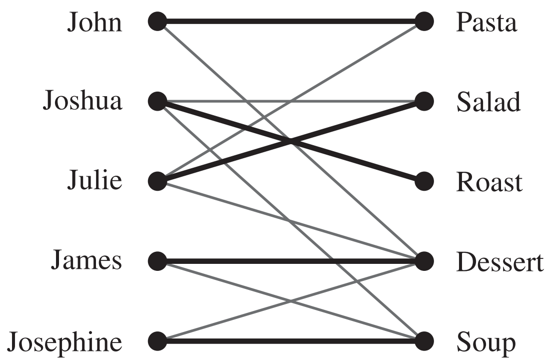
Question 1 **A**



Question 2 **E**

All vertices must be even therefore connect *AF*.

Question 3 **B**



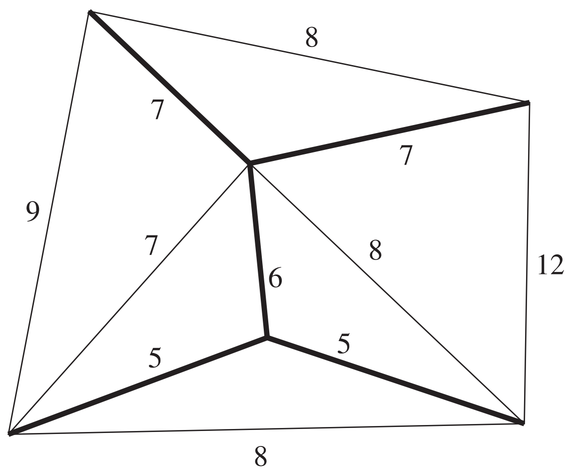
James and Josephine can be swapped, but Julie must bring salad.

Question 4 **C**

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
+
\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}^2
=
\begin{bmatrix} 3 & 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}
\begin{array}{c} 8 \\ 6 \\ 9 \\ 7 \\ 4 \end{array}
$$

9 is largest, so option **C** is correct.

Question 5 **C**



total = 30

Question 6 **C**

$A = 1$ win

$B = 1$ win

$C = 2$ wins

$D = 4$ wins

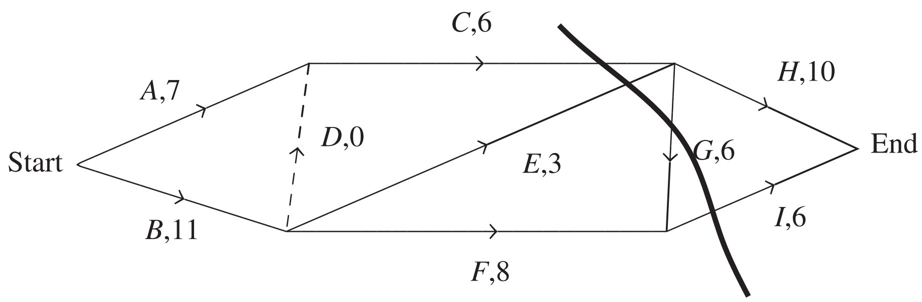
$E = 1$ win

Therefore by placing these in order, we see player C would be 2nd.

Question 7

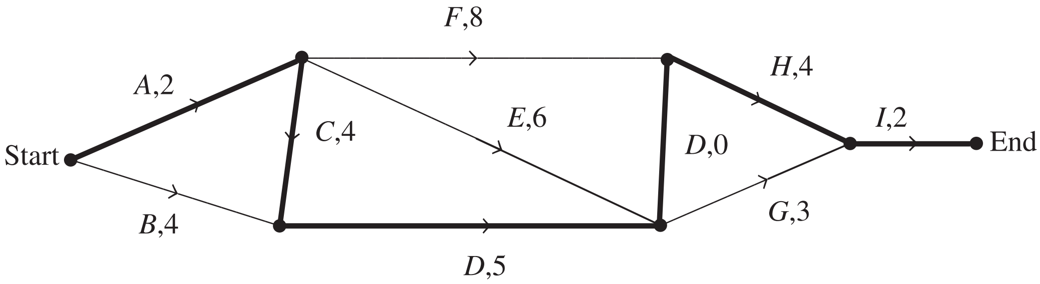
$$v - e + f = 2$$
$$v - 12 + 6 = 2$$
$$v = 8$$

Question 8 **A**



minimum cut = 15

Question 9 **A**



Module 6: Matrices

Question 1 **C**

A	$\times$	B	$=$	C
5×4	$\times$		$=$	5×3

B must be 4×3 as:

- the rows of B must match the columns of A.
- the columns of B must match the columns of the product, C.

Question 2 **B**

We want the different types of coffee in matrix P to multiply by the number of each in matrix R. This will happen if we calculate RP.

Question 3 **B**

The equation provided, in matrix form is $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & -2 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}$

Thus $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & -2 \\ 0 & 1 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}$

Question 4 **D**

There will be an inverse if the matrix is **not** singular.

Find the determinant.

$\Delta = ad - bc = 10 - 3a$

If not singular, $10 - 3a \neq 0$

$a \neq \frac{10}{3}$

Question 5 **E**

Each column of a transition matrix must sum to 1. Thus $a = 0.9$

2012 state matrix will be $\begin{bmatrix} 0.7 & 0.1 & 0.05 \\ 0.2 & 0.8 & 0.05 \\ 0.1 & 0.1 & 0.90 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.4 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.395 \\ 0.425 \\ 0.180 \end{bmatrix}$

Question 6 A

The question tells us that applying the transition matrix has no effect. That means that the initial state was the final state. Thus the question really just wants the final state.

If we find T^{100} and T^{101} they are both $\begin{bmatrix} 0.500 & 0.500 & 0.500 \\ 0.500 & 0.500 & 0.500 \\ 0 & 0 & 0 \end{bmatrix}$ to 3 decimal places. Thus this is the final state.

Question 7 C

Many students will confuse the order in which the operations of matrix multiplication and subtraction should be carried out.

$$S_n - Q = \begin{bmatrix} 88 \\ 90 \\ 45 \end{bmatrix}$$

$$S_{n+1} = P(S_n - Q) = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.4 & 0.6 & 0.4 \\ 0.2 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 88 \\ 90 \\ 45 \end{bmatrix} = \begin{bmatrix} 84.7 \\ 107.2 \\ 31.1 \end{bmatrix}$$

Question 8 B

$S_7 = P(S_6 - Q)$

Thus $P^{-1}S_7 = S_6 - Q$

$S_6 = P^{-1}S_7 + Q$

Question 9 D

We see from matrix that 70% of science students do the same course in the following year (element $A_{1,\,1}$). Thus 30% do change. Likewise, element $A_{3,\,3}$ indicates that 50% of Law students stay in that course for the following year. Thus the statements in options **A** and **B** are both true.

We need to calculate T^2 so that we can check on options **C** and **D**.

$$T^2 = \begin{bmatrix} 0.54 & 0.22 & 0.38 \\ 0.28 & 0.44 & 0.28 \\ 0.18 & 0.34 & 0.34 \end{bmatrix}$$

This confirms that option **C** is a true statement but that option **D** is false.

To find the proportion of students enrolled for science in year 3 we calculate $\begin{bmatrix} 0.54 & 0.22 & 0.38 \\ 0.28 & 0.44 & 0.28 \\ 0.18 & 0.34 & 0.34 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 0.38 \\ 0.33 \\ 0.29 \end{bmatrix}$

Thus option **E** is also a true statement so **D** is the correct response.