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Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

# *Units 3 and 4 Further Maths: Exam 2*

*Practice Exam Question and Answer Booklet*

Duration: 15 minutes reading time, 1 hour 30 minutes writing time

Structure of book:

| Section | Number of questions | Number of questions to be answered | Number of modules | Number of modules to be answered | Number of marks |
|---|---|---|---|---|---|
| Core | 4 | 4 | | | 15 |
| Modules | | | 6 | 3 | 45 |
| | | | | Total | 60 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied:

- This question and answer booklet of 32 pages, with a sheet of miscellaneous formulas.

Instructions:

- Detach the formula sheet from this book during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Write all your answers in the spaces provided in this booklet.

## *Instructions*

This examination consists of a core and six modules. Students should answer all questions in the core then select three modules and answer all questions within the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

## *Core: Data analysis*

### Question 1

The following stem and leaf plot show the age at which 15 women had their first child.

Key: 2|5 = 25 years

| | |
|---|---|
| 1 | 7 , 8 |
| 2 | 0 , 0 , 2 , 2 , 5 , 6 , 7 |
| 3 | 0 , 0 , 0 , 1 , 2 |
| 4 | |
| 5 | 1 |

a. Calculate the mean, median and mode of the above data, showing all working out.

3 marks

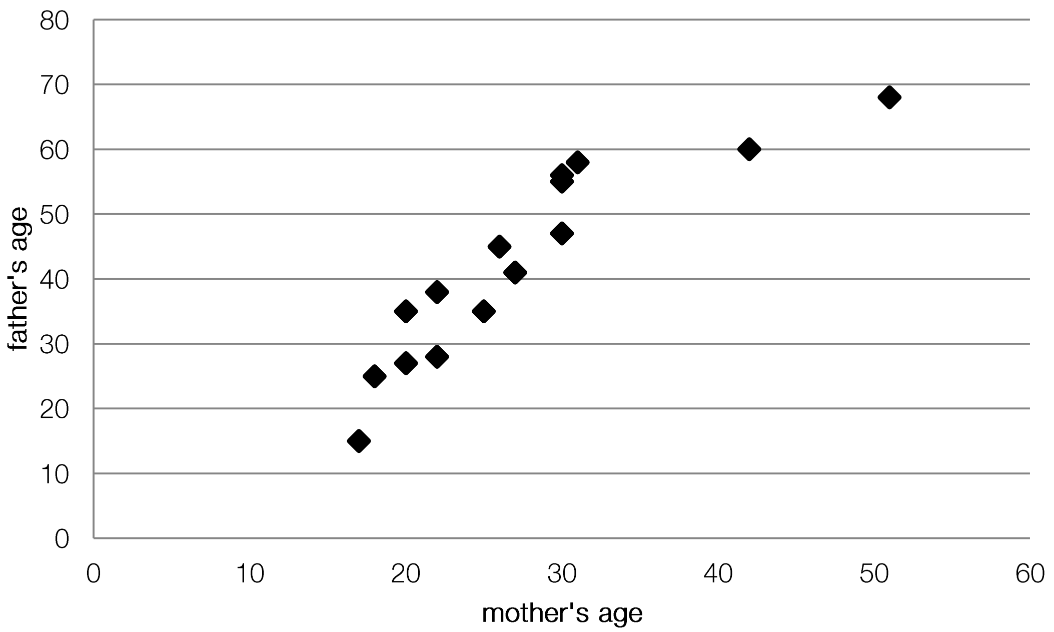
b. Does the data have any outliers? Explain.

2 marks

The above data is now compared with the age of fathers, which is shown in the table below.

| Mother's age | Father's age |
|---|---|
| 17 | 15 |
| 18 | 25 |
| 20 | 35 |
| 20 | 27 |
| 22 | 38 |
| 22 | 28 |
| 25 | 35 |
| 26 | 45 |
| 27 | 41 |
| 30 | 55 |
| 30 | 47 |
| 30 | 56 |
| 31 | 58 |
| 32 | 60 |
| 51 | 68 |

The following scatterplot is produced:



c. Which is the independent variable?

1 mark

d. If a linear regression line is fitted to this graph, what would you expect the residual plot to look like? Explain your answer.

2 marks

e. A data analyst wants to apply a transformation to this data. He is trying to choose between a $\frac{1}{x}$ transformation and a $log_{10}x$ transformation.

By comparing r and $r^2$ values, advise the analyst which transformation he should apply.

3 marks

f. Find the z-score of the mother whose partner was 55 years old when her first child was born.

2 marks

g. The analyst wants to compare the variability of the mother's and father's age. Which category is more variable?

2 marks

Total: 15 marks

# *Section B*

## *Module 1: Number patterns*

### Question 1

For her 14th birthday, Amy receives $20 from her aunt. She decides to save this money by opening a bank account with ZNA. Amy has a choice of two account types that she can open and a teller from ZNA uses sequences to explain how these accounts work. Assume that no other deposit is made after the initial $20 saved.

Account Type A adds an interest of 1.1%, of the existing amount of money saved, into the account at the end of each month.

a. Is the sequence describing the amount of interest added for Account Type A arithmetic or geometric?

1 mark

b. Find a rule for $I_n$ that represents only the amount of interest in the account after $n$ months.

1 mark

Let $C_n$ be the total amount of money saved in the account after $n$ months.

c. Find the difference equation that describes the total amount of money saved in the account after $n$ months.

1 mark

d. Therefore find the total amount saved in the account after two years, to the nearest cent.

1 mark

Account Type B adds $30 interest into the account at the end of each month. The teller uses a computer program to predict the amount of money in each type of account after a certain number of months.

e. If Amy only wants to save her $20 for twenty months, which account type should she choose if she wishes to get the most money back?

1 mark

f. When does the total amount interest from received Account Type A exceed the total amount of interest from received Account Type B, assuming that $20 is the initial deposit in both accounts?

1 mark

Total: 6 marks

## Question 2

Amy also receives a Bulldog puppy, which she names Oscar, from her parents. During Oscar's first visit to the vet, Amy notes that her puppy weighs 5 kg. Every subsequent year she makes a habit of remembering Oscar's weight.

a. If Oscar weighs 9 kg during his second vet visit and 12.2 kg during his third vet visit, what is the rule for the sequence that defines his weight gain, $G_n$, where $n$ = number of visits after the first vet visit?

1 mark

b. Using this sequence how much weight does Oscar gain between his fifth and sixth vet visit?

1 mark

c. What would you predict Oscar's total weight would be when he is fully-grown?

2 marks

**Total: 4 marks**

## Question 3

Amy had invited 48 guests to her 14th birthday party. For her 15th birthday party she invited half of those original guests plus thirty-two more new guests. She used this formula for inviting guests to her subsequent birthday parties.

a. Show that for her 16th birthday party Amy invited 60 guests.

1 mark

b. How many more people were invited to Amy's 19th birthday party than her 17th birthday party? Round your answer to the nearest whole number.

2 marks

On her 14th birthday party, Amy recalls that guests arrived in groups of four every 10 minutes. The party started at 12pm, with the first group of guests arriving at 12:10. The next group arrived at 12:20 and so on until all the guests had arrived.

c. How many guests had still not yet arrived by 1:30pm?

1 mark

d. At what time did the last group of guests arrive?

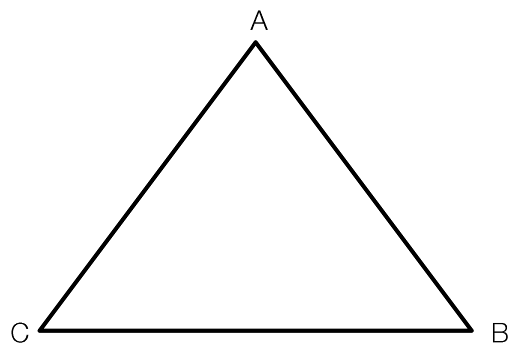
1 mark

Total: 5 marks

## *Module 2: Geometry and trigonometry*

### Question 1

ABC is an equilateral triangle, and is shown in the figure below:




a. What is the value of angle BAC?

1 mark

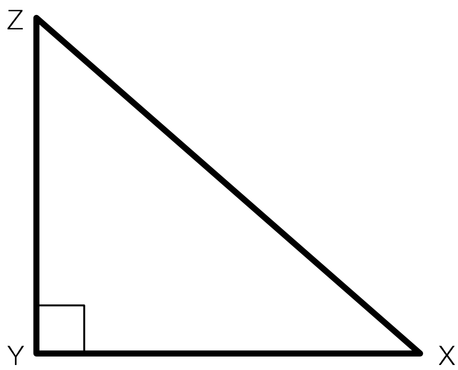
b. Given that the area of ABC is 1 $cm^2$, find the length of AB to 2 decimal places using Heron's formula.

4 marks

Total: 5 marks

## Question 2

XYZ is a right angled scalene triangle with the equal sides measuring 1 cm, as shown below.

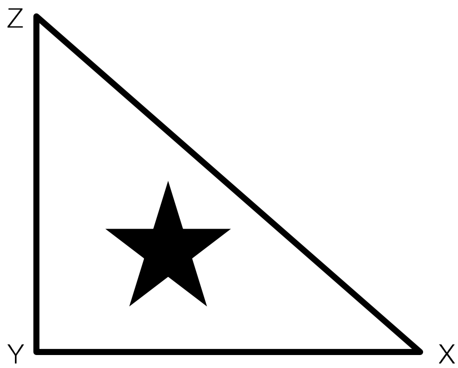



a. What is the area of XYZ?

______________________________________________

1 mark

Mark cuts a star out of XYZ such that the area of the star to the remaining area of the triangle is 1:3.




b. What is the area of the star?

______________________________________________

______________________________________________

______________________________________________

______________________________________________

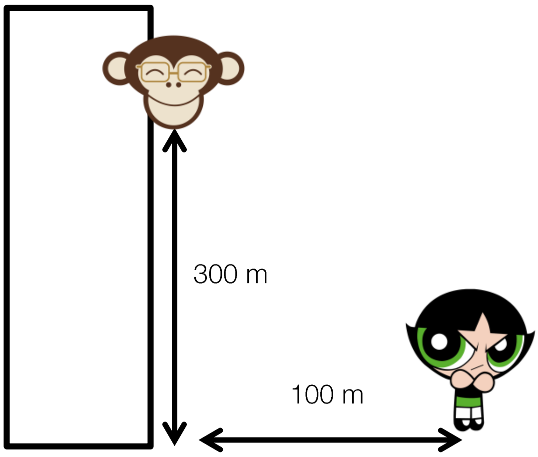
______________________________________________

3 marks

Total: 4 marks

### Question 3

King Kong is climbing the Empire State Building. He is 300 m from the ground when he spots Buttercup 100 m away.




a. What is King Kong's angle of depression to Buttercup, to 2 decimal places?

1 mark

b. What is King Kong's bearing from Buttercup in the diagram, to 2 decimal places?

1 mark

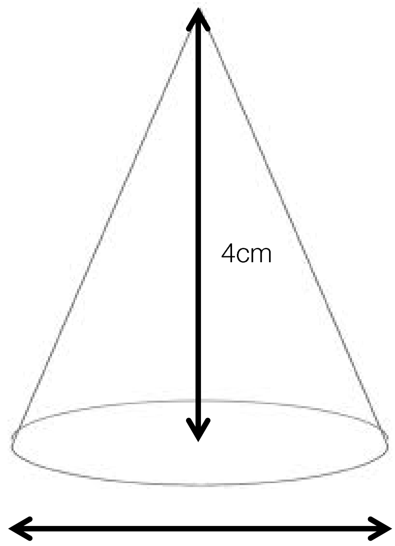
Kong moves a further 20 m up the building.

c. How far does Buttercup now have to fly to get to King Kong, to the nearest metre, assuming she flies directly from her position on the ground to King Kong?

1 mark

Total: 3 marks

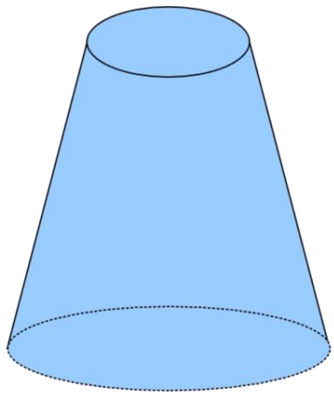
Question 4




a. What is the volume of the above cone, to 2 decimal places?

1 mark

The cone is now truncated such that the height of the truncated piece is ¾ of the original height.



b. What is the volume of the truncated cone, to 4 decimal places?

2 marks

Total: 3 marks

## *Module 3: Graphs and relations*

### Question 1

Lea Yen runs a chocolate cupcake stall. If $x$ represents the number of cupcakes sold, her revenue and cost equations each day are as follows:

$$R = 3.5x$$
$$C = 20 + 2.5x$$

a. How many cupcakes does Lea Yen have to sell break even?

1 mark

b. How much of a loss will Lea Yen make if she sells no cupcakes?

1 marks

Lea Yen decides that to increase the price of the cupcakes of $4.50. At the same time her fixed costs increase by $2.

c. What are her new revenue and cost equations?

2 marks

d. If Lea Yen sells 100 cupcakes each day for 3 weeks, how much of a profit or loss will she make at the end of the 3 weeks? Note: The fixed cost applies everyday.

2 marks

Total: 6 marks

**Question 2**

Lea Yen's cupcake stall is becoming very popular. She is now selling chocolate and vanilla cupcakes.

She still charges $4.50 for a chocolate cupcake ($x$), and $4 for a vanilla cupcake ($y$).

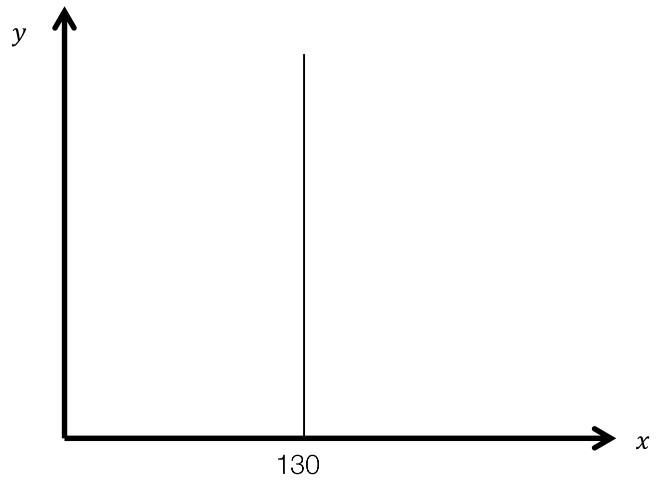
Her constraint equations are as follows:

$$x \leq 130$$
$$y \leq 150$$
$$x + y \leq 250$$

a. Explain what the last constraint equation means.

1 mark

b. Graph and label the remaining constraint equations.




2 mark

c. How many chocolate cupcakes and vanilla cupcakes does Lea Yen sell at the point where her revenue is a maximum?

3 mark

**Total: 6 marks**

### Question 3

Lea Yen soon outgrows the cupcake stand and moves into a shop. Her shop is 1.80 km from her house.

She leaves her house at 8am and rides her bike half way there at an average speed of 9km/hour. Her bike then breaks down.

a. How long did Lea Yen ride her bike for (in minutes)?

1 mark

b. Lea Yen walks the rest of the way to work at an average speed of 3km/h. If Lea Yen has to get to work by 8.30am, is she late?

2 marks

**Total: 3 marks**

## *Module 4: Business-related mathematics*

### Question 1

$7000 is borrowed, using a simple interest rate of 10% per annum. The total interest paid over the period of the loan is $2030.

a. What is the period of the loan? (rounding to 1 decimal place)

1 mark

b. Find the interest paid as a percentage of the original loan amount

1 mark

c. Find the interest paid after 2 years.

1 mark

Total: 3 marks

### Question 2

Jenna purchases an Xbox for $700 on a hire purchase plan. She pays a deposit of $100, and monthly instalments of $25 for 3 years.

a. What is the total amount of interest paid?

1 mark

b. What is the flat rate of interest? (rounding to 1 decimal place)

1 mark

c. What is the effective rate of interest? (rounding to the nearest percent)

1 mark

Total: 3 marks

## Question 3

Johnny goes to an office supply shop to purchase a new printer. First he is shown printer A, which costs $20,000, and depreciates at 6c per page printed.

a. Johnny prints 5000 pages per year, for 5 years. At the end of this period, how much is the printer worth?

2 marks

The salesman then shows Johnny printer B which costs $20,000, and depreciates at a flat rate of 1.51% per annum. He also shows Johnny printer C, which also costs $20,000, and uses reducing balance depreciation, at 1.55% per annum.

b. After 5 years, how much has printer B depreciated by?

1 mark

c. After 5 years, which printer has depreciated the most?

3 marks

**Total: 6 marks**

### Question 4

On Monday, a music shop is selling a guitar for $2000. On Tuesday, the guitar's price is raised by 34% (from Monday's price). On Wednesday, the guitar's price is lowered by 16% (from Tuesday's price).

Over these three days, by what percentage has the guitar's price increased/decreased overall? Give your answer to 2 decimal places.

2 marks

### Question 5

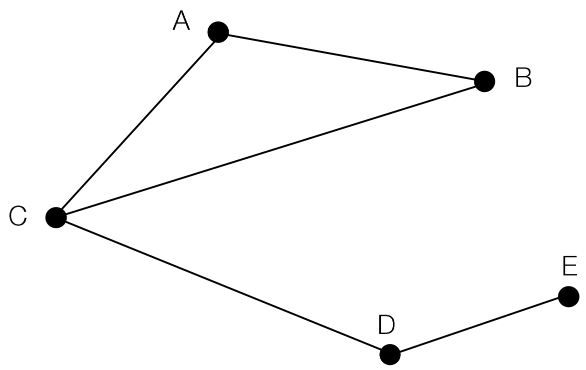
Johnny invests her inheritance of $1,000,000 in a perpetuity that pays 10% per annum compounding weekly. After 2 weekly payments, how much money remains invested in the perpetuity?

1 mark

## *Module 5: Networks and decision mathematics*

### Question 1

Anna, Brett, Carey, Dan and Emma enjoy playing table tennis. They decide to play a series of matches in round-robin style so that each player eventually plays every other player. The competition is represented by the unfinished network below, where each player is represented by a node corresponding to the first letter of their name and an edge between X and Y means 'X plays a match with Y'.




a. Complete the graph so that it represents the round-robin competition.

1 mark

b. How many matches were played?

______________________________________________________________________

______________________________________________________________________

1 mark

c. After all the matches were played, the results were collated as follows:

A defeated D and E
B defeated A
C defeated D and A and B
D defeated E and B
E defeated C and B

On the completed graph in part a., draw arrows to construct a directed network graph to represent the results, such that 'X defeated Y' is represented by an edge from X to Y.

1 mark

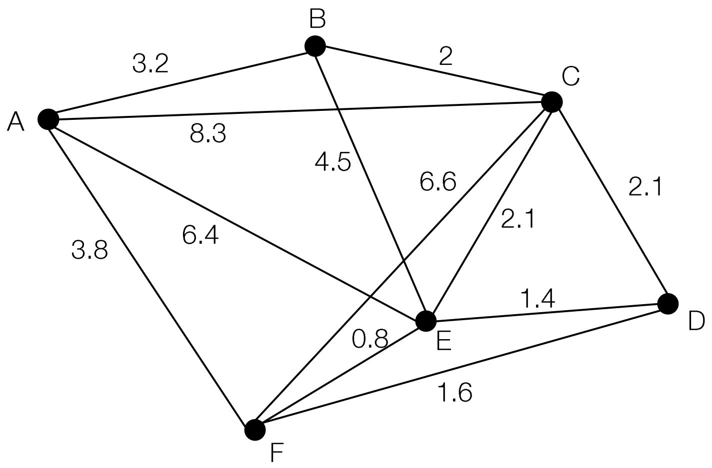
d. Construct an adjacency matrix, $M$ for your directed network diagram, where '1' represents 'defeated'. Hence, construct a dominance matrix, $D$ by adding all the elements in each of the rows of the adjacency matrix.

2 marks

Total: 5 marks

Question 2
A, B, C, D, E and F represent 6 major landmarks cities in the city of Gerryville.

The network shows the road connections between these landmarks and the distances between each (in km).



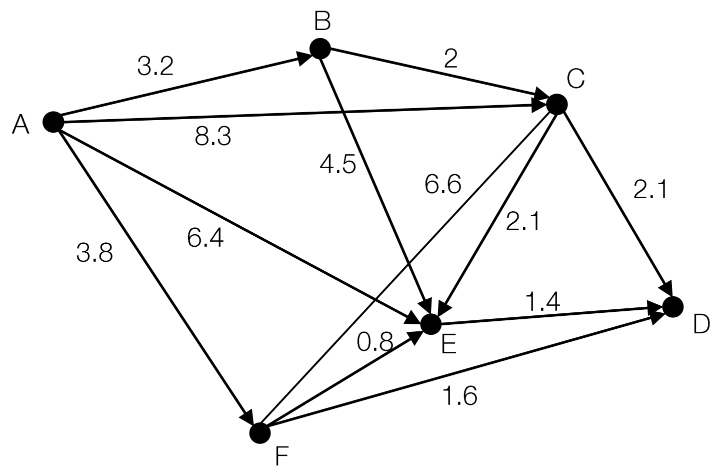

a. Write a Hamilton path for the journey from town A to B.

1 mark

b. Is it possible to start from A and traverse every single road only once? Why or why not? What is the name given to such a path?

1 mark

The city of Gerryville undergoes a major construction project, affecting the directionality of the roads. Now all cars may only travel in one direction along any road. As a result, each road now has a limit to how many cars can travel on the road before it becomes congested. The figures in the diagram now represent this and are shown with the units hundreds of cars per hour.



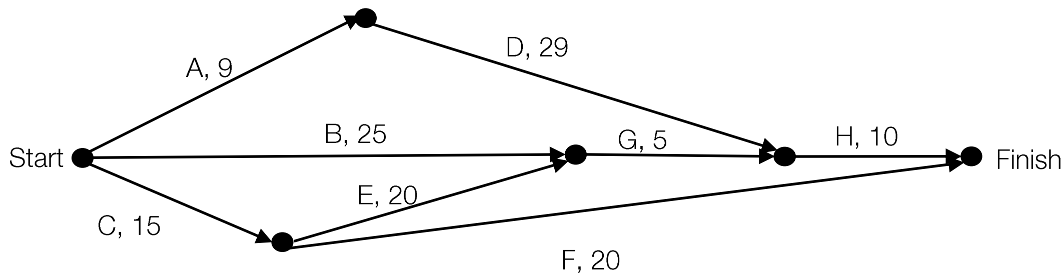

c. What is the maximum flow of traffic from A to D on average in an hour?

1 mark

Total: 3 marks

Question 3

Janet, a working mother of two children, must complete a series of tasks each morning before she can finally go to work herself. The directed network below shows the activities and their completion times in minutes.




a. Determine the minimum time Janet takes to complete her tasks and get to work and the critical path.

2 marks

b. Determine the slack time, in minutes, for activity D.

1 mark

c. What is the maximum time activity B be delayed by without affecting the minimum completion time?

1 mark

d. The project is to be crashed by reducing the completion time of **one** activity. Which activity or activities can be crashed and by how much? State the new completion time.

1 marks

Total: 5 marks

## Question 4

A company wishes to allocate four tasks to four of its employees so that the profit can be maximised. The table below summarises the profit each person (A, B, C and D) makes for the company for each task (W, X, Y and Z) on average.

| | W | X | Y | Z |
|---|---|---|---|---|
| A | 170 | 240 | 420 | 210 |
| B | 250 | 180 | 190 | 200 |
| C | 290 | 140 | 310 | 220 |
| D | 110 | 200 | 170 | 140 |

Find the optimal allocation if each task can only be allocated to one person and state the maximum profit.

2 marks

## *Module 6: Matrices*

### Question 1

A group of friends is going to Europe from Melbourne (M). They are going to Paris (P), Amsterdam (A) and Zagreb (Z).

The following matrix shows their flight path:

$$\text{to}\ \begin{matrix} A \\ M \\ Z \\ P \end{matrix} \overset{\displaystyle \begin{matrix} & \text{from} & \\ A & M & Z & P \end{matrix}}{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}$$

a. List the European countries in the order in which they are visited by the group.

____________________________________________________________

____________________________________________________________

1 mark

b. The group decides that it is too expensive to go to Amsterdam. They complete their trip as usual, just leaving Amsterdam out. Complete the matrix below, showing their new plans.

$$\begin{matrix} M \\ Z \\ P \end{matrix} \overset{\displaystyle \begin{matrix} M & Z & P \end{matrix}}{\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}}$$

2 marks

**Total: 3 marks**

Question 2

While in Paris the group is deciding whether or not to do a cruise of the Seine River. They know that 20% of people who have never done the cruise, will do the cruise at some point; and 40% of people who have done the cruise will do it again.

a. Use the information above to complete the matrix below:

$$\begin{array}{r} \\ \text{will go} \\ \text{won't go} \end{array} \begin{array}{c} \begin{array}{cc} \text{have} & \text{have not} \end{array} \\ \begin{bmatrix} 0.40 & 0.20 \\ & \end{bmatrix} \end{array}$$

1 mark

b. Given that all 5 of have never been on the cruise, how many of them can be expected to go on the cruise?

1 mark

Total: 2 marks

Question 3

When the group returns they have to go to back to university. As they have been away, they did not check their marks from their last set of exams.

Sunny is very nervous about his grades.

He knows that:

Of the people who average a pass grade:

- 40% receive a pass again
- 30% receive a credit
- 20% receive a distinction

Of the people who average a credit grade:

- 10% receive a pass
- 40% receive a credit
- 30% receive a HD

Of the people who average a distinction grade:

- 5% receive a pass
- 10% receive a credit
- 60% receive a distinction

Of the people who average a HD grade:

- 3% receive a pass
- 10% receive a credit
- 70% receive a HD

a. Use the information above to complete the matrix below:

$$\begin{array}{ccc} & & \text{last semester} \\ & & \begin{array}{cccc} P & C & D & HD \end{array} \\ \text{this semester} & \begin{array}{c} P \\ C \\ D \\ HD \end{array} & \begin{bmatrix} \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \end{bmatrix} \end{array}$$

3 marks

b. If Sunny averaged a HD last semester, what are the chances that he will get a pass average this semester?

---

1 mark

In semester 1 2013, 1000 students at the university received a pass grade, 3000 received a credit grade, 2000 received a distinction grade and 500 received a HD grade.

c. Draw the state matrix for the university.

1 mark

d. Assuming that no new students joined and no one left the university, how many students receive a credit in semester 1 2014 if every semester there is exams? (t = 2)

2 marks

e. If everyone who received a pass grade dropped out of university, draw the new transition matrix.

1 mark

f. Based on the condition in part e, how many students will drop out after semester 1 2013?

1 mark

g. In the long term, what will majority of students get as their grade? (Using the original transition matrix)

1 mark

**Total: 10 marks**

# *Formula Sheet*

## Core: Data analysis

| | |
|---|---|
| Standardised score: | $z = \dfrac{x - \bar{x}}{s_x}$ |
| Least squares line: | $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$ |
| Residual value: | $\text{residual value} = \text{actual value} - \text{predicted value}$ |
| Seasonal index: | $\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$ |

## *Module 1: Number patterns*

| | |
|---|---|
| Arithmetic series: | $a + (a + d) + \cdots + (a + (n - 1)d) = \dfrac{n}{2}[2a + (n - 1)d] = \dfrac{n}{2}(a + l)$ |
| Geometric series: | $a + ar + ar^2 + \cdots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$ |
| Infinite geometric series: | $a + ar + ar^2 + ar^3 + \cdots = \dfrac{a}{1 - r}, \lvert r \rvert < 1$ |

## *Module 2: Geometry and trigonometry*

| | |
|---|---|
| Area of a triangle: | $\dfrac{1}{2}bc \sin A$ |
| Heron's formula: | $A = \sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$ |
| Circumference of a circle: | $2\pi r$ |
| Area of a circle: | $\pi r^2$ |
| Volume of a sphere: | $\dfrac{4}{3}\pi r^3$ |
| Surface area of a sphere: | $4\pi r^2$ |
| Volume of a cone: | $\dfrac{1}{3}\pi r^2 h$ |
| Volume of a cylinder: | $\pi r^2 h$ |
| Volume of a prism: | $\text{area of base} \times \text{height}$ |
| Volume of a pyramid: | $\dfrac{1}{3}\text{area of base} \times \text{height}$ |
| Pythagoras' theorem: | $c^2 = a^2 + b^2$ |
| Sine rule: | $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ |
| Cosine rule: | $c^2 = a^2 + b^2 - 2ab \cos C$ |

## Module 3: Graphs and relations

### Straight line graphs

Gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation: $y = mx + c$

## Module 4: Business-related mathematics

Simple interest: $I = \dfrac{PrT}{100}$

Compound interest: $A = PR^n$, where $R = 1 + \frac{r}{100}$

Hire purchase: $\text{effective rate of interest} \approx \dfrac{2n}{n+1} \times \text{flat rate}$

## Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

## Module 6: Matrices

Determinant of a 2 x 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Inverse of a 2 x 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$

End of Booklet

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