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YEAR 12 *Trial Exam Paper*

2015

FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:

- correct solutions with full working
- tips

SECTION A

Core: Data analysis

Question 1

Answer is A

Worked solution

Four households had six mobile phones and one household had seven mobile phones.

Therefore, $\frac{5}{20} \times 100 = 25\%$.

Question 2

Answer is B

Worked solution

Upper fence $= Q3 + 1.5 \times IQR$
$= 60 + 1.5 \times 20$
$= 90$

Because the top score of 95 is above the upper fence, it will be shown as an outlier.

Question 3

Answer is E

Worked solution

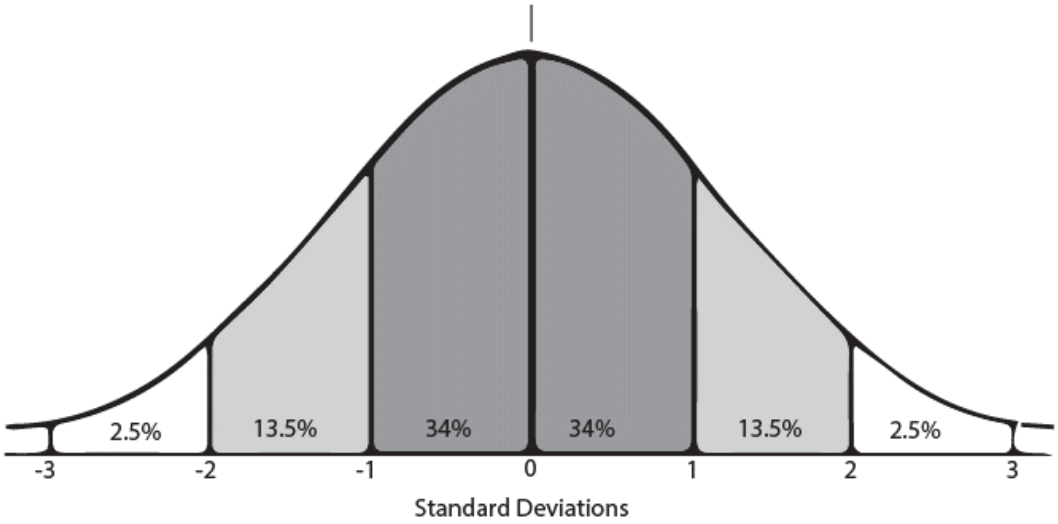
A is incorrect when all values in the dataset are the same.
B is incorrect when there are only 1 or 2 values in the dataset.
C is incorrect when the distribution is uniform and no mode exists.
D is incorrect when the data is skewed.
E is always correct.

Question 4

Answer is D

Worked solution

From the information given in the question, the standard deviation is 4. Therefore, 86 g is one standard deviation below the mean and 98 g is 2 standard deviations above the mean.



Using the graph above, the percentage of eggs sold to the public is 34% + 34% + 13.5%. Therefore, 81.5% of Billy Fowler's eggs are sold to the public.

Question 5

Answer is E

Worked solution

84 + 36 + 45 + 92 = 257

Question 6

Answer is C

Worked solution

$$\frac{84}{(84+36)} \times 100 = 70\%$$

$$\frac{45}{(45+92)} \times 100 = 33\%$$

Of the males who completed the survey, 70% liked external exams, whereas of the females surveyed, 33% liked external exams.

Tip:

- *For two-way frequency tables, it is convention to let the categories of the dependent variable define the rows of the table and the categories of the independent variable define the columns. To use percentages to support a contention that there is a relationship between the variables, percentages need to be calculated by dividing elements by the column totals.*

Question 7

Answer is C

Worked solution

Use the calculator to calculate r^2. Swimming time is the dependent variable and water temperature is the independent variable.

A temp	B time
20	45
19	35
12	5
13	9

LinRegMx *temp*,*time*,1: CopyVar *stat.Regl*

"Title"	"Linear Regression (mx+b)"
"RegEqn"	"m·x+b"
"m"	4.18450184502
"b"	-41.8782287823
"r²"	0.956505763405
"r"	0.978011126422
"Resid"	"{...}"

The coefficient of determination is $r^2 \approx 0.96$. Therefore, 96% of the variation in swimming time can be explained by variation in temperature.

Question 8

Answer is E

Worked solution

The equation of the least squares regression line is given by $y = a + bx$, where

the gradient, b, is given by $b = r\frac{s_y}{s_x}$

and

the y intercept, a, is given by $a = \bar{y} - b\bar{x}$

where

r is Pearson's product moment correlation coefficient

s_x and s_y are the standard deviations of x and y respectively

$\bar{x}$ and $\bar{y}$ are the means of x and y respectively

$$b = \frac{rs_y}{s_x}$$
$$= \frac{0.9362 \times 1.8}{1.1}$$
$$= 1.53$$

$$a = \bar{y} - b\bar{x}$$
$$= 12.7 - 1.53 \times 6.3$$
$$= 3.06$$

Therefore, the equation is $y = 3.06 + 1.53x$

Question 9

Answer is C

Worked solution

residual value = actual value – predicted value

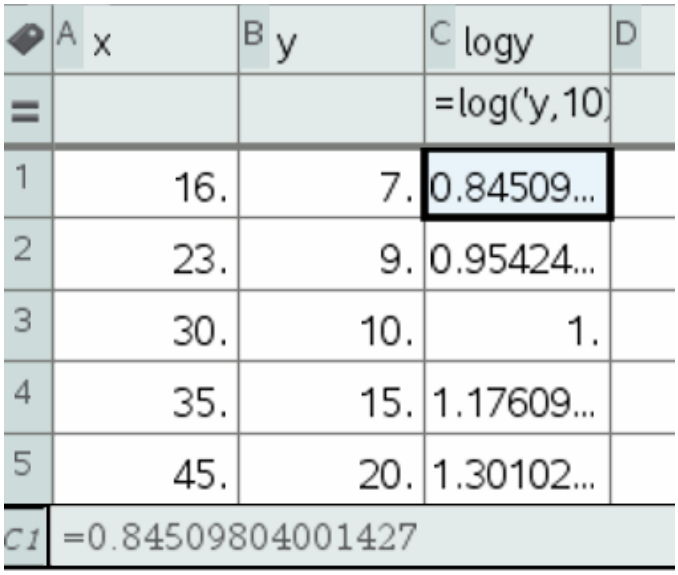
$0.75 = b - 2(3) + 1$
$b = 0.75 + 7$
$b = 7.75$

Question 10

Answer is B

Worked solution

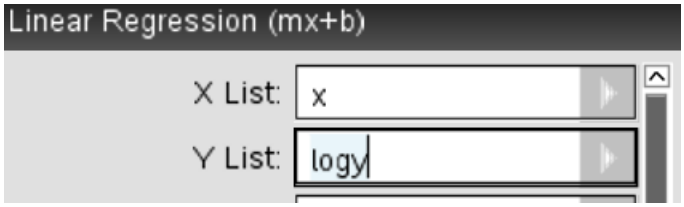
Enter the data into the calculator and then create a column with log *y* values.

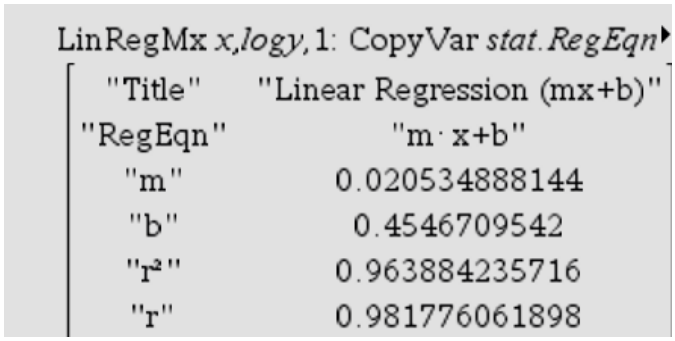


	A x	B y	C logy	D
=			=log('y,10)	
1	16.	7.	0.84509...	
2	23.	9.	0.95424...	
3	30.	10.	1.	
4	35.	15.	1.17609...	
5	45.	20.	1.30102...	

c1 =0.845098040014 27

Then find the equation of the least squares regression line using log *y* as the *y* variable.





LinRegMx *x,logy*,1: CopyVar *stat.RegEqn*

"Title"	"Linear Regression (mx+b)"
"RegEqn"	"m·x+b"
"m"	0.020534888144
"b"	0.4546709542
"r²"	0.963884235716
"r"	0.981776061898

The equation of the transformed data is $\log y = 0.02x + 0.45$.

Question 11

Answer is D

Worked solution

Mean monthly caravan sales
$= (25 + 28 + 12 + 8 + 6 + 3 + 4 + 8 + 12 + 23 + 34 + 52) \div 12$
$= 17.9167$

February seasonal index $= 28 \div 17.9167$
$= 28 \div 17.9167$
$= 1.56$

Question 12

Answer is B

Worked solution

Calculate the seasonal index for May.

May seasonal index $= 12 - (1.2 + 1.3 + 1.1 + 1.0 + 0.8 + 0.7 + 0.9 + 0.9 + 1.0 + 1.0 + 1.2)$
$= 0.9$

actual value = deseasonalised value × seasonal index
$= 25\,120 \times 0.9$
$= 22\,608$

Tip:

- *When working towards an answer that requires 2 or more steps in which a calculation from a previous step is to be used, do not round off numbers until the final answer is reached.*

Question 13

Answer is C

Worked solution

The median of 6, 4, 2, 5 and 3 is 4.

Tip:

- *When calculating the median (middle number) of a set of numbers, put them in numerical order first.*

SECTION B

Module 1: Number patterns

Question 1

Answer is E

Worked solution

The sequence is arithmetic with $a = 37$ and $d = -4$. We want to find the term t_n that is less than zero.

$$\begin{aligned} t_n &< 0 \\ a + (n-1)d &< 0 \\ 37 + (n-1) \times -4 &< 0 \\ 41 - 4n &< 0 \\ 41 &< 4n \\ 10.25 &< n \end{aligned}$$

n is greater than 10.25
Therefore, the first negative number is the 11th term.

Alternatively, continue to write out the sequence by subtracting 4 each time until you get a negative term.

37, 33, 29, 25, 21, 17, 13, 9, 5, 1, –3 …

The first negative term is –3, and it is the 11th term in the arithmetic sequence.

Question 2

Answer is D

Worked solution

This is an example of an arithmetic sequence because the next term is obtained by adding the common difference, $d = 10$, to the previous term. The rule below can be used to calculate the sum, S_n, of $n = 12$ terms in an arithmetic sequence when $a = 80$ is the first term.

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$
$$S_n = \frac{12}{2}\left(2 \times 80 + (12-1)10\right)$$
$$S_n = 1620$$

Alternatively, write out the first 12 terms and add them up.

80 + 90 + 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 + 180 + 190 = 1620

Question 3

Answer is C

Worked solution

For a sequence to be arithmetic there must be a common difference between each term. This cannot be true when a previous term is multiplied by any factor other than 1 (not –1). C is the only equation with a common difference (–3).

Question 4

Answer is D

Worked solution

To find b, solve $14 = 5 \times 2 - b$
$14 = 10 - b$
$b = -4$

SECTION B

Question 5

Answer is B

Worked solution

This is a geometric sequence. The first term, a, is 2 and the common ratio, r, is

$$\frac{0.5}{2} = \frac{0.125}{0.5} = 0.25$$

The sum of the first 9 terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{2(1-0.25^9)}{1-0.25}$$

$$S_n = 2.67$$

Question 6

Answer is A

Worked solution

The value of the first term, a, is 12 and the common ratio, r, is 0.9.

The sum of an infinite geometric sequence is given by

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{12}{1-0.9}$$

$$S_\infty = 120$$

Question 7

Answer is E

Worked solution

Answers A to D are true. E could be true, and would be for a Fibonacci sequence, but it is not necessarily true.
For example, the sequence could be $-3,\ 2,\ -1,\ 1,\ 0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34 \ldots$

SECTION B

Question 8

Answer is B

Worked solution

$s_1V_1 = s_2V_2$

$50 \times V_1 = 10 \times 3$

$V_1 = 0.6$

Question 9

Answer is C

Worked solution

Because the inheritance is to be divided into the ratio 3:2:1, let the total inheritance be 6*x*. The initial split is 3*x*:2*x*:*x*

Because Christine gives half of her inheritance ($\frac{x}{2}$) to each of Axel and Barry, Axel's total inheritance is \$52 500.
Therefore,

$3x + \frac{x}{2} = 52\,500$

$\frac{7x}{2} = 52\,500$

$x = 15\,000$

Barry's total inheritance will be $2x + \frac{x}{2} = \frac{5x}{2}$

Because *x* = 15 000, Barry's inheritance is \$37 500.

SECTION B

Module 2: Geometry and trigonometry

Question 1

Answer is C

Worked solution

All 3 angles in an equilateral triangle are equal and will add up to 180°. Each angle is 60°.

Question 2

Answer is C

Worked solution

The length of *DC* must be 5 and triangle *BDC* must be a right-angled triangle. The length of *BD* can be calculated using Pythagoras' rule

$$a^2 + b^2 = c^2$$
$$BD^2 + DC^2 = BC^2$$
$$BD^2 + 5^2 = 10^2$$
$$BD = \sqrt{100 - 25}$$
$$BD = 8.66$$

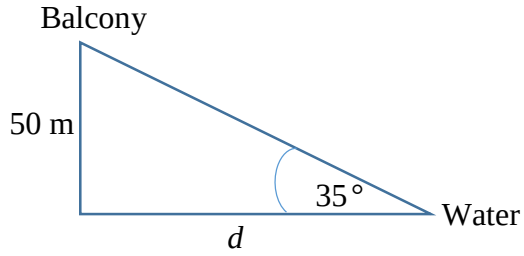
SECTION B

Question 3

Answer is D

Worked solution

Draw a diagram.



We are looking for distance *d*.

$$\text{ADJ} = \frac{\text{OPP}}{\tan x}$$

$$d = \frac{50}{\tan 35}$$

$$d = 71.41$$

Tip:

- *Check your calculator settings to make sure the angle setting is in degree mode.*

SECTION B

Question 4

Answer is B

Worked solution

Using the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin x}{58} = \frac{\sin 18}{25}$$
$$x = \sin^{-1}\left(\frac{58 \times \sin 18}{25}\right)$$
$$x = 45.8$$

Question 5

Answer is E

Worked solution

The volume ratio is equal to the size ratio cubed.
The volume ratio is
12 000:1500
8:1
The size ratio is
2:1
Therefore, the height of the small can is 75 cm.

Question 6

Answer is A

Worked solution

The angle at vertex *F* would be 50 + 60 = 110.
Use the cosine rule to find the length from *T* to *H*.

$$a^2 = b^2 + c^2 - 2bc \times \cos A$$
$$TH^2 = 180^2 + 140^2 - 2 \times 180 \times 140 \times \cos 110$$
$$TH^2 = 69\,237.815$$
$$TH = 263.13$$
$$TH \approx 263$$

SECTION B

Question 7

Answer is B

Worked solution

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

The rise is 30 m, but we need to calculate the run (the horizontal distance) using Pythagoras' theorem.

$$a^2 + b^2 = c^2$$
$$30^2 + h^2 = 120^2$$
$$h = 116.1895$$
$$\text{slope} = \frac{30}{116.1895}$$
$$\text{slope} = 0.2582$$
$$\text{slope} \approx 0.26$$

Question 8

Answer is D

Worked solution

volume = area of cross-section $\times$ height

$$= \frac{1}{2} \times a \times b \times \sin x \times \text{height}$$
$$= \frac{1}{2} \times 8 \times 8 \times \sin 60 \times 20$$
$$= 554.26$$
$$\approx 554$$

SECTION B

Question 9

Answer is B

Worked solution

Let's call the direct distances *Dc* and *Df* (for closer direct distance and further direct distance respectively) and let the altitude of each plane be *H*.

$$\text{Hyp} = \frac{\text{Opp}}{\sin x}$$

$$Dc = \frac{H}{\sin 30} \text{ and } Df = \frac{H}{\sin 9}$$

Then $Dc : Df$

$$\frac{H}{\sin 30} : \frac{H}{\sin 9}$$

$$\frac{H}{\sin 30} \times \frac{\sin 30}{H} : \frac{H}{\sin 9} \times \frac{\sin 30}{H}$$

$$1 : \frac{\sin 30}{\sin 9}$$

$$1 : 3.20$$

SECTION B

Module 3: Graphs and relations

Question 1

Answer is C

Worked solution

First, find the gradient, *m*, between the 2 points.

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{8 - {}^{-}2}{{}^{-}2 - 8}$$
$$= -1$$

Then use $y = mx + c$ to find the *y* intercept.

Using $y = mx + c$, substitute the point (–2, 8) into *x* and *y*:

$$8 = -1(-2) + c$$
$$c = 6$$

Therefore, $y = -x + 6$ or $y + x - 6 = 0$

Question 2

Answer is B

Worked solution

Substitute any (*x*, *y*) pair of numbers except for (0, 0), into the equation and solve for *k*.

$$y = k^2$$
$$k = \frac{y}{x^2}$$
$$k = \frac{4.8}{4}$$
$$k = 1.2$$

SECTION B

Question 3

Answer is E

Worked solution

Surfboard hire for 5 hours costs $35, 0.5 hours costs $15 and 2.5 hours costs $30. Total for 3 days is $80.

Tip:

- *Solid dots are included, i.e., 1 hour costs $15, not $25.*

Question 4

Answer is D

Worked solution

Using the TiNspire to solve the system of linear equations.

$$\text{linSolve}\left(\begin{cases} 4\cdot x+2\cdot y=7 \\ 3\cdot x-2\cdot y-14=0 \end{cases}, \{x,y\}\right)$$
$$\{3.,-2.5\}$$

Tip:

- *The solution to a pair of simultaneous equations is the coordinates of the point that lies on both lines.*

SECTION B

Question 5

Answer is B

Worked solution

B is incorrect because the *y*-intercept of the cost equation is 500, not 300.

Tip:

- *The profit equation can be found by subtracting the cost equation from the revenue equation.*

Question 6

Answer is A

Worked solution

Bruce's athletes always do at least 40 minutes of track work $(x \geq 40)$ and at least 30 minutes of field work $(y \geq 30)$ during each training session. The athletes spend a maximum of 120 minutes training in any session $(x + y \leq 120)$ and athletes must do track work for at least twice as long as they do field work $(x \geq 2y)$.

Question 7

Answer is C

Worked solution

Substituting the vertices into the objective function we get

$P = 0$ at $(0,0)$
$P = 58$ at $(10,14)$
$P = 80$ at $(20,10)$
$P = 72$ at $(24,0)$

SECTION B

Question 8

Answer is D

Worked solution

Dinkum's cost equation is $C = 200 + 0.5p$, where p is the number of pies made and sold.

Dinkum's revenue equation is $R = xp$, where x is the price of a pie.

Dinkum's profit equation is $P = R - C$

$$2000 = xp - 200 - 0.5p$$

Aussie's cost equation is $C = 300 + 0.4p$, where p is the number of pies made and sold.

Aussie's revenue equation is $R = xp$, where x is the price of a pie.

Aussie's profit equation is $P = R - C$

$$2000 = xp - 300 - 0.4p$$

Using the tiNspire to solve the 2 profit equations to find x and p.

$$\text{solve}\left(\begin{cases} 2000 = x \cdot p - 200 - 0.5 \cdot p \\ 2000 = x \cdot p - 300 - 0.4 \cdot p \end{cases}, \{x, p\}\right)$$

$$x = 2.7 \text{ and } p = 1000.$$

We find that each company sold 1000 pies at \$2.70.

Question 9

Answer is A

Worked solution

$$y = kx^2$$

$$1 = k2^2$$

From the original graph $k = \frac{1}{4}$

$$y = \frac{1}{4}x^2$$

When $x^2 = 16$, $y = 4$.

SECTION B

Module 4: Business-related mathematics

Question 1

Answer is D

Worked solution

$$\text{Interest} = \frac{Prt}{100}$$

$$\text{Interest} = \frac{5000 \times 7.6 \times 0.5}{100}$$

$$\text{Interest} = 190$$

$$\text{Investment} = 5000 + 190$$

$$\text{Investment} = 5190$$

Question 2

Answer is C

Worked solution

A is the total value of the investment, P is the principal invested, r is the interest rate, n is the number of compounding periods per year and t is the number of years.

$$A = P\left(1 + \frac{r}{100 \times n}\right)^{n \times t}$$

$$A = 2500 \times \left(1 + \frac{3.7}{100 \times 52}\right)^{52 \times 3}$$

$$A = 2793.38$$

Interest earned = \$2793.38 – \$2500 = \$293.38

SECTION B

Question 3

Answer is C

Worked solution

Let the original price be x.

After a 25% discount, the price becomes $x \times 0.75$.

After a further 15% discount, the price becomes $x \times 0.75 \times 0.85$, which equals the final sale price of $76.50.

Using $\text{solve}(x \cdot 0.75 \cdot 0.85 = 76.5, x) \quad x = 120.$

Question 4

Answer is B

Worked solution

Complete the balance column in the table.

Date	Transaction details	Credit	Debit	Balance
1 June 2014	Opening balance			$ 1256.50
8 June 2014	Withdrawal		$87.50	$1169.00
15 June 2014	Deposit	$76.00		$1245.00
30 June 2014				$1245.00

The minimum balance for the month is $1169.00.

Interest earned is $0.0035 \times 1169 = \$4.09$

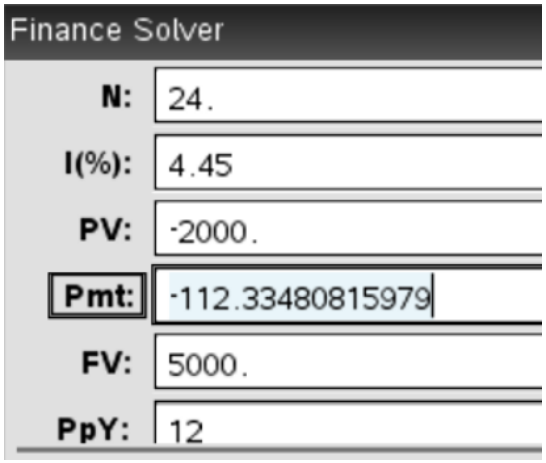
SECTION B

Question 5

Answer is A

Worked solution

This is an 'adding to an investment' problem for which we need to use the finance solver.



The monthly payment is $112.33.

Tip:

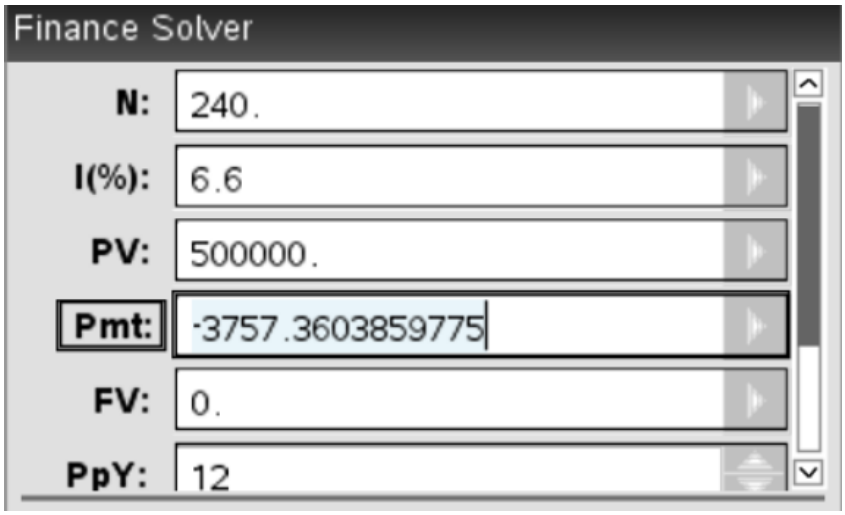
- *When using the finance solver, any amount given to the bank such as an initial deposit or a regular payment is entered as a negative number.*

Question 6

Answer is B

Worked solution

Using the finance solver to find pmt:



Answer B

SECTION B

Question 7

Answer is B

Worked solution

Total repayments = deposit + balance borrowed + interest on balance borrowed at 15% for 3 years.

$$= \$500 + \$2200 + \$2200 \times 0.15 \times 3$$

This means that Harvey pays $3690 in total.
That is $990 extra.

Tip:

- *The principal in the calculation is $2200, the amount borrowed, not $2700, the total price.*

Question 8

Answer is D

Worked solution

Total depreciation over 5 years is $40 000. Each year this is $8000.
Depreciation = depreciation rate per kilometre $\times$ number of kilometres

$$8000 = \text{depreciation rate} \times 50\,000$$

$$\text{Depreciation rate} = 0.16$$

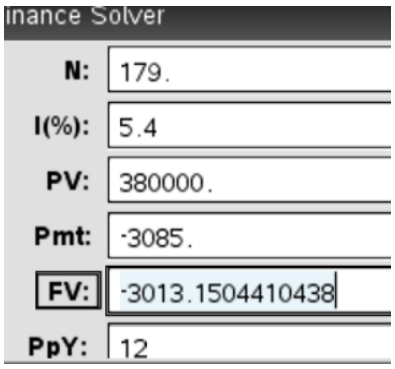
SECTION B

Question 9

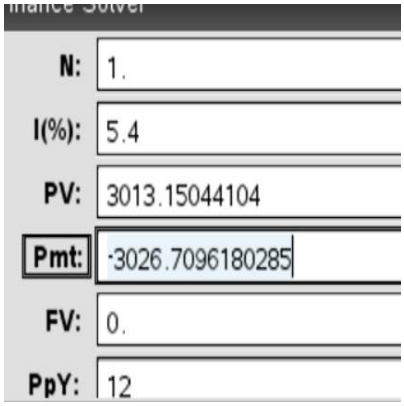
Answer is D

Worked solution

This question involves using the TVM solver.
First, calculate the balance owing after 179 payments.



Now transfer the amount owing into the present value so that we can calculate the final payment which will include 1 month's interest on this PV. Change N back to 1 and FV to 0.



The final payment will be $3026.71.

SECTION B

Module 5: Networks and decision mathematics

Question 1

Answer is D

Worked solution

$$\text{The number of edges in a connected graph} = \frac{\text{the sum of the vertex degrees}}{2}$$
$$= \frac{(4\times 3)+(3\times 2)+(2\times 1)}{2}$$
$$= \frac{20}{2}$$
$$= 10$$

Question 2

Answer is B

Worked solution

4 + 2 + 2 = 8

SECTION B

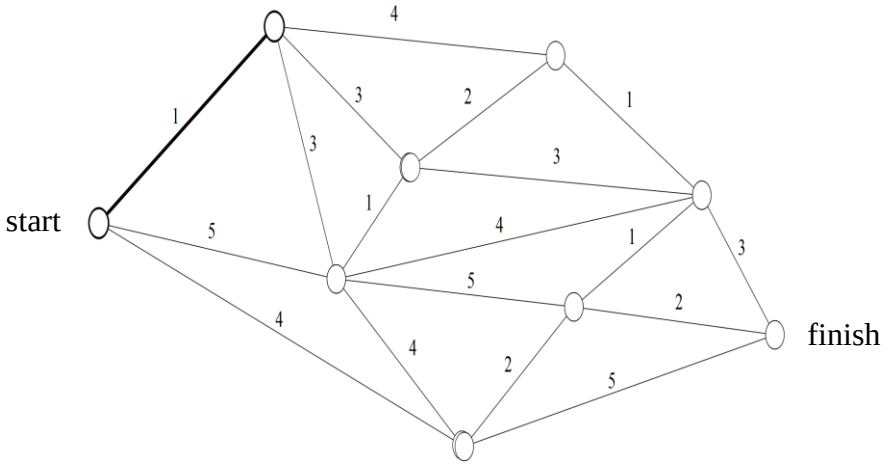
Question 3

Answer is E

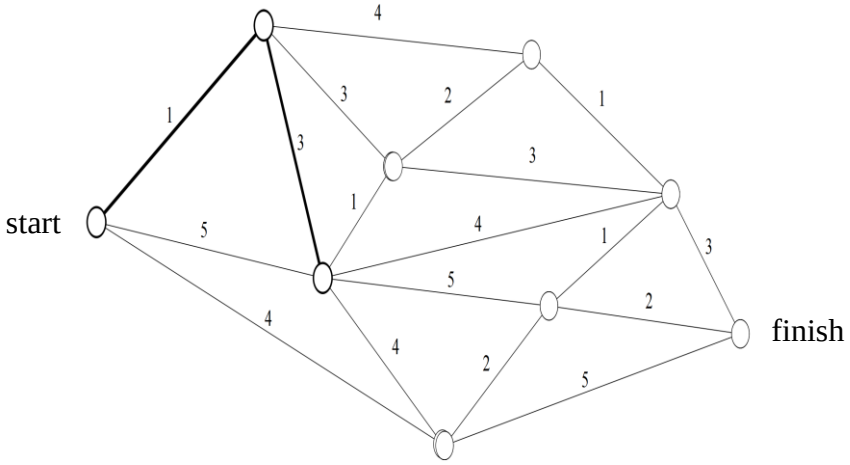
Worked solution

To find a minimum spanning tree we can use Prims algorithm.

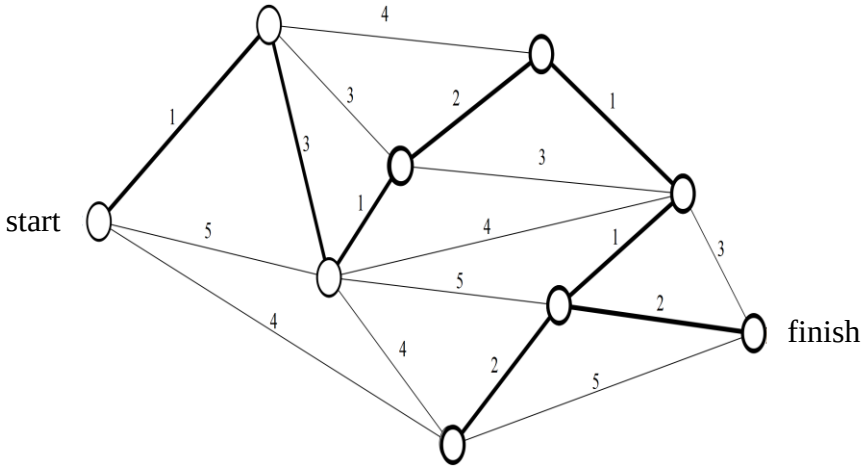
Step 1. Choose any vertex and highlight the lowest edge coming from that vertex. It won't matter which vertex you start at.



Step 2. Considering the vertices that are now joined by highlighted edges, choose the lowest edge coming from any of these vertices. If two edges are equally low it won't matter which of the two you choose.



Step 3. Continue to choose the lowest edge from the already connected vertices, but do not choose an edge which forms a cycle, until all vertices are connected.



SECTION B

Question 4

Answer is D

Worked solution

For a graph to be complete it will have $\frac{n(n-1)}{2} = 6$ edges (where n = number of vertices).

Therefore, the graph could not be complete. All other statements are possible.

Question 5

Answer is B

Worked solution

The critical paths through the network are AXFIL, AXFKM and AXGHM, each with a length of 12 weeks. When activity C takes 7 weeks, the new critical path becomes CHM, which has a length of 13 weeks.

Therefore, preparation is delayed by 1 week.

Question 6

Answer is D

Worked solution

Activities L and M are on critical paths so we need to spend $200 on each of these to have them completed on time. Activity J has 1 week of slack time so we need only pay $100 so that it is delayed by only 1 week.

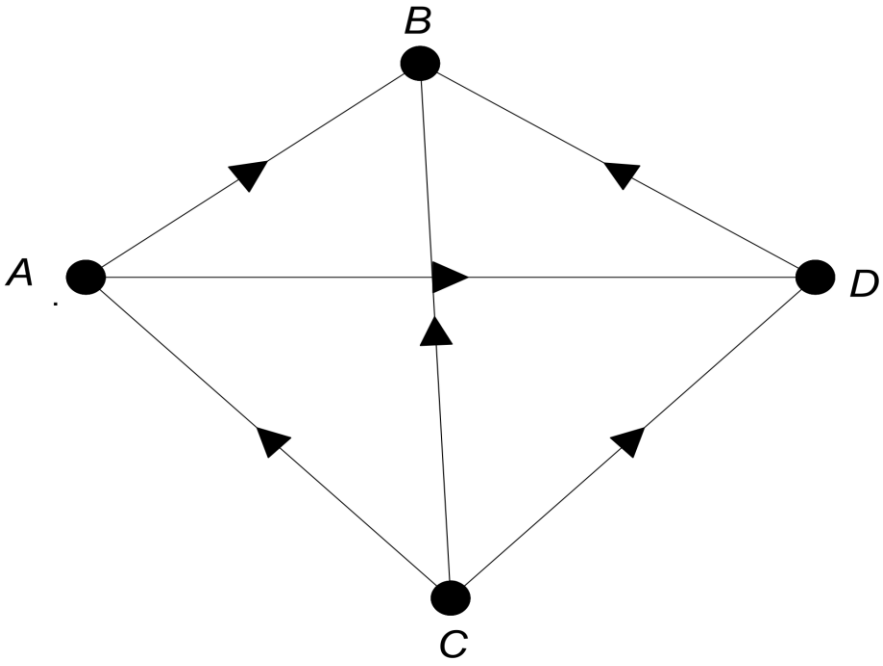
SECTION B

Question 7

Answer is C

Worked solution

The best way to solve this is to draw a directed graph.

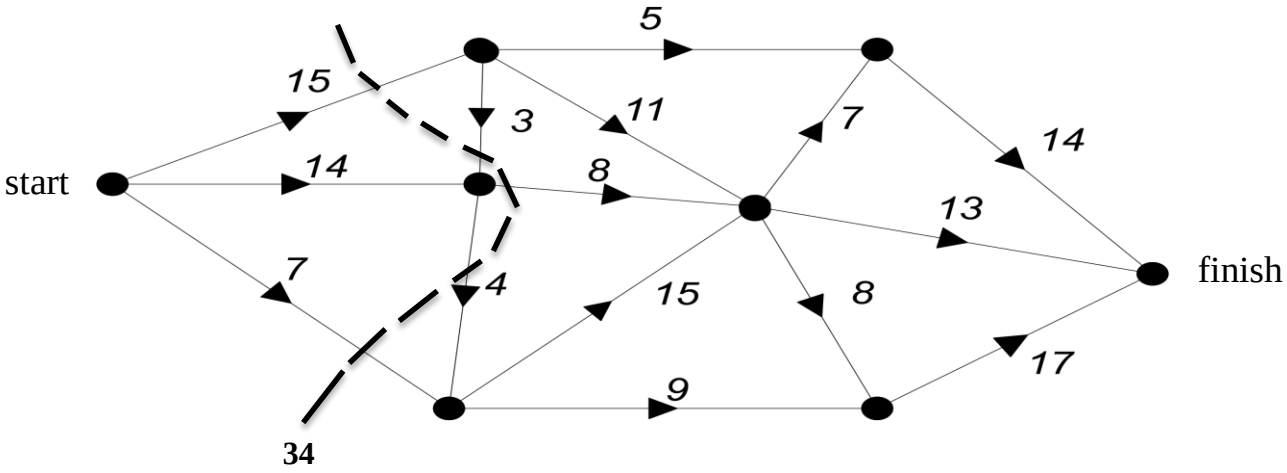


SECTION B

Question 8

Answer is C

Worked solution



The minimum cut is 34.

Tip:

- *Edges that cross a cut going from the finish side of the cut to the start side of the cut are not counted in the total of the cut. In the above cut, the edge of weight 3 is not counted.*

Question 9

Answer is E

Worked solution

Euler's rule can be used for a connected planar graph. All statements are false.

SECTION B

Module 6: Matrices

Question 1

Answer is A

Worked solution

$a_{3,2}$ is the element in matrix A in row 3 column 2.

Tip:

- *When working with matrices, it is useful to remember rows first columns second. This applies to many things such as element position, order, multiplying matrices, etc.*

Question 2

Answer is E

Worked solution

This matrix calculation is easily done on the calculator.

$$\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \rightarrow a \qquad \begin{bmatrix} 3. & -2. \\ 2. & -2. \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ -3 & -1 \end{bmatrix} \rightarrow b \qquad \begin{bmatrix} 2. & 1. \\ -3. & -1. \end{bmatrix}$$

$$a \cdot (a-b) \qquad \begin{bmatrix} -7. & -7. \\ -8. & -4. \end{bmatrix}$$

Tip:

- *The store function is a useful calculator tool when working with matrices.*

SECTION B

Question 3

Answer is D

Worked solution

I	$(A \times B) + C$	Product of $(A \times B)$ is a 2×2 matrix, which cannot be added to C, which is a 2×1 matrix.
II	$C + A \times B$	Matrices follow laws of BODMAS ($A \times B$) first cannot be added to C.
III	$A + B \times C$	The product of $B \times C$ is a 2×1 matrix, which cannot be added to A.
IV	$A \times B \times C$	$A \times B$ is a 2×2 matrix, which can be multiplied by C, a 2×1 matrix.
V	$(A + B) \times C$	$(A+B)$ is a 2×2 marix, which can be multiplied by C, a 2×1 matrix.

Alternatively store each matrix and let your calculator do the work.

Question 4

Answer is D

Worked solution

$$1 \times 3 \qquad\qquad 3 \times 3 \quad = \quad 1 \times 3$$

$$\begin{bmatrix} 20.35 & 18.60 & 12.90 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40.70 & 55.80 & 12.90 \end{bmatrix}$$

Question 5

Answer is C

Worked solution

A is a singular matrix (i.e. no inverse and $\det(A) = 0$) when $a = 2$. For all other values of a, the $\det(A) \neq 0$ and an inverse exists.

SECTION B

Question 6

Answer is C

Worked solution

The 2nd pair has a determinant of 2, and the 3rd pair has a determinant of –4. So these 2 sets have a unique solution.

The 1st pair and the 4th pair have a determinant of zero. The 1st pair is inconsistent (zero solutions – parallel lines) but the 4th pair is dependent (infinite solutions).

Question 7

Answer is E

Worked solution

The transition matrix is $\begin{matrix} L & S \\ \begin{bmatrix} 0.65 & 0.15 \\ 0.35 & 0.85 \end{bmatrix} & \begin{matrix} L \\ S \end{matrix} \end{matrix}$

Question 8

Answer is C

Worked solution

Reading from top down and then across we get Banana, Apple, Eggfruit, Dates and Cherries.

SECTION B

Question 9

Answer is A

Worked solution

$(a \times 3) + (3 \times 1) = 6$

$3a + 3 = 6$

$a = 1$

and

$(a \times 2) + (3 \times b) = -1$

$2a + 3b = -1$

Substituting

$(2 \times 1) + 3b = -1$

$3b = -3$

$b = -1$

END OF WORKED SOLUTIONS

SECTION B

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