The Mathematical Association of Victoria

Trial Exam 2015

FURTHER MATHEMATICS

Written Examination 2

STUDENT NAME: ....

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
2	2	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 32 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

This examination consists of core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer all questions with the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

		Page
Core		3
Module		
Module 1:	Number patterns	8
Module 2:	Geometry and trigonometry	12
Module 3:	Graphs and Relations	18
Module 4:	Business-related mathematics	22
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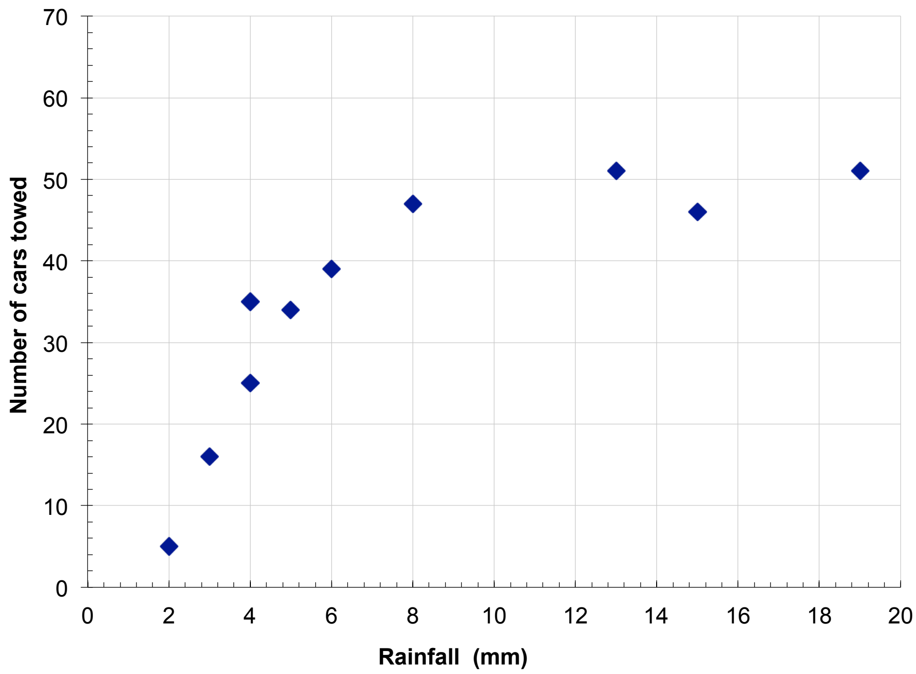
Core

Question 1 (11 marks)

The number of cars requiring a tow truck in the city of Melbourne is recorded against the rainfall for 10 weekdays.

A scatterplot of the variables is shown below.



a. For the data shown on the scatterplot

i. Which one of the two variables is discrete? 1 mark

ii. State the independent variable. 1 mark

Core – Question 1 – continued
TURN OVER

b. For the data representing the rainfall Q_1 = 4 mm and Q_3 = 13 mm. A particular day has a rainfall of 25.5 mm. Is this rainfall considered to be an outlier for this set of data? Justify your answer with calculations. 2 marks

c. The rainfall for the data shown on the scatterplot is given in the following table.

Rainfall (mm)	2	3	4	4	5	6	8	13	15	19

Complete the table below by calculating the mean and the standard deviation of this data correct to two decimal places. 1 mark

	Mean	Standard deviation
Rainfall (mm)		
Number of cars towed	34.90	15.47

c. Pearson's correlation coefficient for this data is 0.7984. Using this and the statistics in the table above, show that the equation of the least squares regression line is

$$\text{Number of cars towed} = 2.137 \times \text{rainfall} + 18.018$$

2 marks

Core - continued

e. The slope of the least squares regression line suggests that for an increase of 10 mm in daily rainfall the number of cars towed, to the nearest whole number, is expected to increase by how many? 1 mark

f. Use the least squares regression line to calculate the expected rainfall, correct to the nearest mm, if 42 cars were towed on a particular day. 1 mark

g. Explain why the least squares regression line will **not** reliably predict the number of cars towed when the rainfall is 25.5 mm. 1 mark

Question 2 (4 marks)

It was decided that the data shown on the scatterplot in **Question 1** is not linear. In the attempt to transform the data to linearity, a reciprocal transformation of the independent variable was undertaken.

a. State one other type of transformation that would be appropriate in an attempt to transform the data to linearity. 1 mark

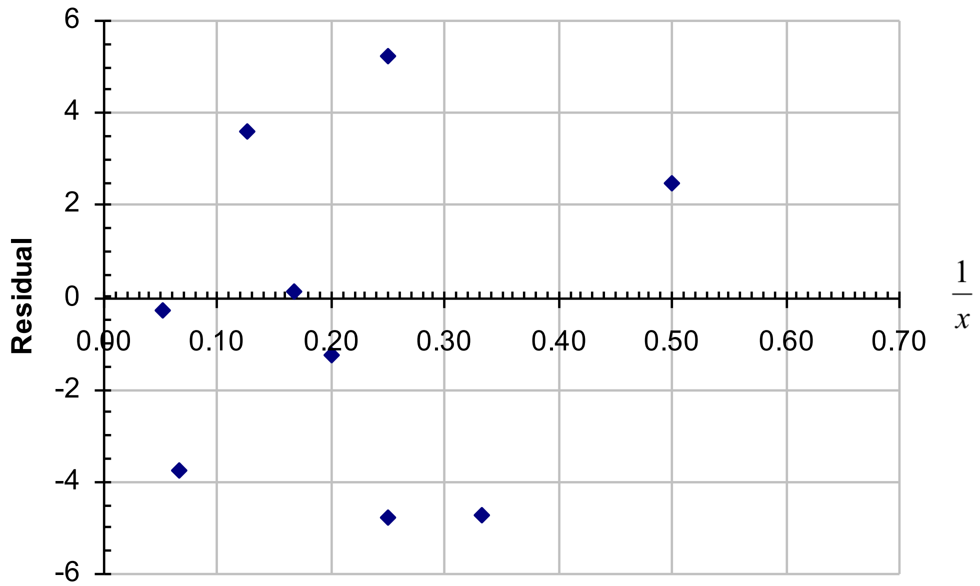
Core – Question 2 – continued
TURN OVER

b. The equation of the least squares regression line for the transformed data was found to be

$$\text{Number of cars towed} = \frac{-108.96}{\text{rainfall}} + 57.02 \quad \text{where } r = -0.97$$

i. According to this model, find the expected number of cars that will require towing when 13 mm of rainfall is recorded.
Give your answer correct to the nearest whole number. 1 mark

A residual plot resulting from fitting the transformed model, given above, to the transformed data was undertaken. The incomplete residual plot is shown below.



The residual for the point (13, 51) is missing on the residual plot.

ii. Find the residual value for (13, 51).
Give your answer correct to the nearest whole number. 1 mark

iii. Plot the missing point on the residual plot. 1 mark

Core – Question 2 – continued

c. Use the residual plot to explain why the reciprocal rainfall transformation is an appropriate model for the data. 1 mark

END OF CORE
TURN OVER

Module 1: Number patterns

Question 1 (6 marks)

Regular maintenance of an underground road tunnel has revealed a crack with leaking water. Andrew, a representative from the Victorian Road Authority, has monitored the leak for three hours. The amount of water leakage, in litres, is shown in the table below.

1st hour	**2nd hour**	**3rd hour**
1.8	2.4	3

The amount leaking continues to increase in this arithmetic sequence.

a. How many litres water will leak in the fifth hour? 1 mark

b. Write an expression, in terms of n, that determines the amount of leaking water, A_n, in the n^{th} hour. 1 mark

c. The tunnel will have to be closed if the amount leaking first exceeds 1 litre per minute (60 litres per hour). During which hour will this occur? 1 mark

d. How much water in total will leak in the first 24 hours? 1 mark

Module 1 – Question 1 – continued

e. The expression, in terms of n, that specifies the total amount of water leakage, T_n, in n hours is given by

$$T_n = an+bn^2.$$

Find the values of a and b. 2 marks

Question 2 (3 marks)

Andrew tells Brian, the foreman, about the leak and Brian decides to conduct his own investigation. At the end of each hour Brian measures the **total** amount of water that has leaked, in litres.

End of 1st hour	**End of 2nd hour**	**End of 3rd hour**
5	7	9.8

Using Brian's results and assuming the pattern will continue

a. Show that these figures form the first three terms of a geometric sequence. 1 mark

b. Write an expression, in terms of n, that determines the total amount of water that has leaked, B_n, at the end of n hours. 1 mark

c. At the end of which hour will the total amount of water leaked first exceed 1000 litres? 1 mark

Question 3 (3 marks)

Brian eventually consults with the engineer, Carol, who also conducts her own investigation of the leak and obtains the following results.

1st hour	**2nd hour**	**3rd hour**
20 litres	17 litres	14.45 litres

a. Calculate the percentage decrease in the amount of water leaking from the first to the second hour. 1 mark

b. Find the value of a if the amount leaking, C_n, in the nth hour from the time Carol started her investigation, is specified by the difference equation

$$C_{n+1} = a\, C_n \quad \text{where} \quad C_1 = 20$$

1 mark

Carol concludes that the amount leaking is decreasing and that no action needs to be taken to stop the leak.

c. If Carol's pattern continues, find the total volume of water that will leak from the crack from the time Carol first started her investigation.

Give your answer to the nearest litre. 1 mark

Question 4 (3 marks)

The local Geologist, Linda, discovers that the salinity in the water is affecting the crack in the road tunnel. A more accurate model describing the amount of water, L_n, leaking in the nth hour is given by

$$L_{n+1} = 0.9L_n + 1.2, \quad L_1 = 5$$

a. Find the amount of water (to the nearest litre) that leaks in the 20th hour. 1 mark

b. Explain why there will never be more than 12 litres of water leaking in any hour. Use a calculation to support your explanation. 1 mark

c. It is found that the water leakage will in fact stabilise to 15 litres per hour. If the model describing this situation is now given as

$$L_{n+1} = cL_n + 1.2, \quad L_1 = 5$$

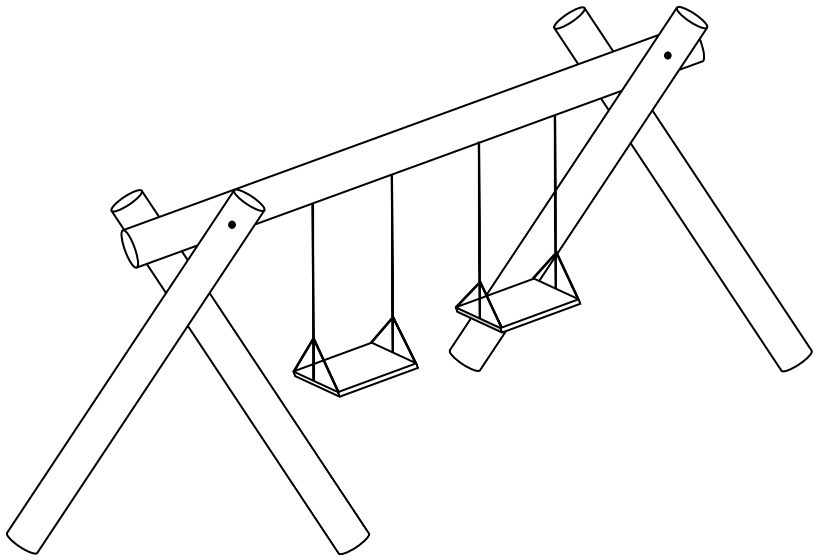
Find c. 1 mark

END OF Module 1
TURN OVER

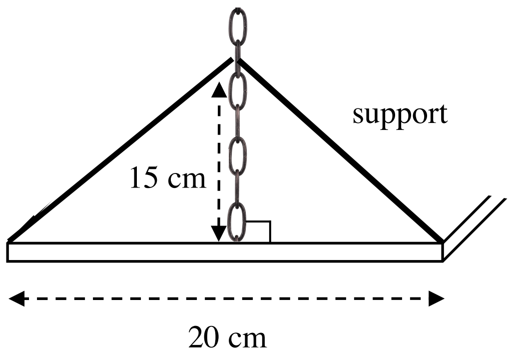
Module 2: Geometry and Trigonometry

Question 1

A swing made out of treated pine logs is to be built at a playground in a large suburban park. A diagram of the swing is shown below.



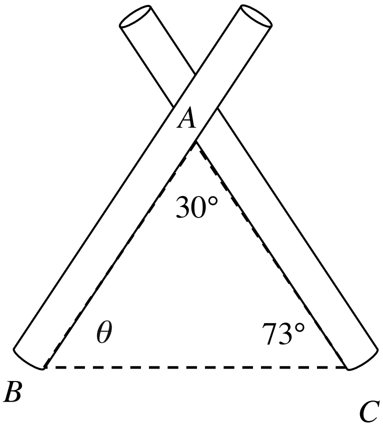
Each 20 cm wide swing seat has a swing chain mounted at right angles to the seat exactly **halfway** along the end of the seat. A small support is attached to the swing chain as shown below to keep the seat stable. The support is connected to the chain 15 cm above the swing seat.



a. Calculate the length of the support to the nearest centimetre. 1 mark

Module 2 – Question 1 – continued

The builder decides to join the end-supports of the swing at point A as shown below.



b. Write down a calculation to show that the value of θ is 77°. 1 mark

The builder measures the length of AB at 1.80 metres.

c. The distance BC that separates the bases of the swing supports can be calculated using the sine rule.

Fill in the missing numbers in the equation below. 1 mark

$$BC = \frac{\sin \boxed{}^\circ \times \boxed{}}{\sin \boxed{}^\circ}$$

d. Calculate the distance BC that separates the bases of the swing supports, in metres correct to two decimal places. 1 mark

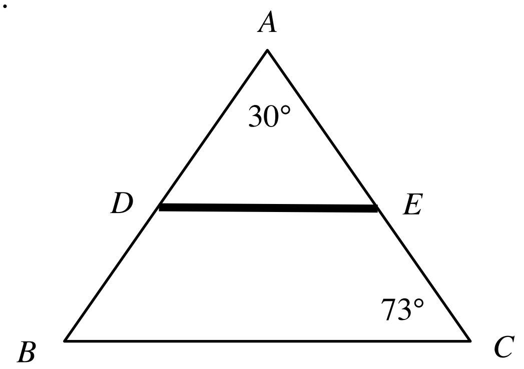
Module 2 – Question 1 – continued
TURN OVER

To brace the swing set the builder considers using a triangular sheet of structural plywood with the same dimensions as triangle *ABC*.

The builder measures the length of *AC* and finds that it is 1.83 metres.

e. Find the area of the triangular sheet of structural plywood. Give your answer in square metres correct to two decimal places. 1 mark

Instead of using a triangular piece of structural plywood to brace the swing set, the builder decides to use a single bracing beam attached to the supports at *D* and *E* as shown below. She measures *BD* at 0.60 metres.

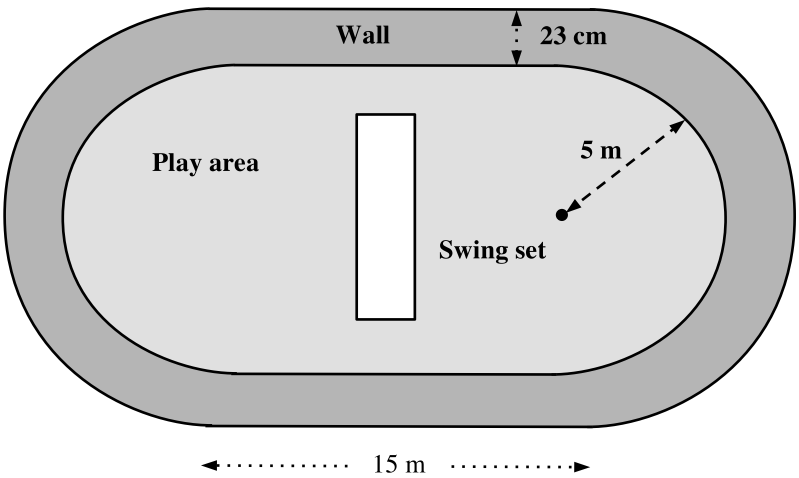


f. Calculate the length of the brace, *DE*, in metres correct to two decimal places. 2 marks

Question 2
The builder of the swing set wants to paint the swing supports in bright colours. The swing supports are made of treated pine logs whose cylindrical shape is similar to that of a newly designed paint tube. The surface area of a tube of paint is one-sixteenth of the surface area of each swing support.
The volume of each swing support is 19000 cubic centimetres

Calculate the volume of paint in each tube of paint, correct to the nearest cubic centimetre. 2 marks

Question 3
To contain a play area the swing set is to be surrounded by a short concrete wall as shown below. The interior of the wall is made of two semi-circles each of radius 5 m and two parallel lines each in 15 m length. The wall is 23 cm wide at all points.



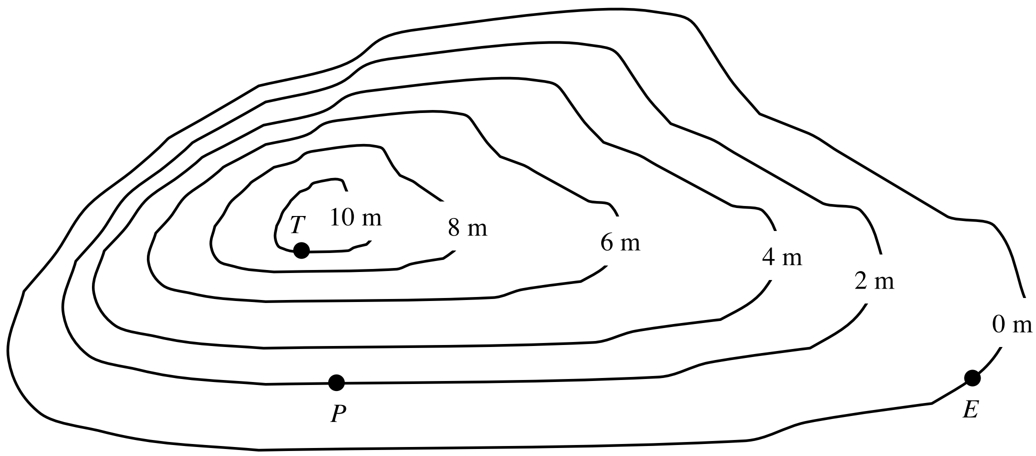
a. Calculate the area of the play area, correct to the nearest square metre. 1 mark

Module 2 – Question 3 – continued
TURN OVER

b. Calculate the volume of concrete required to make a 40 cm high wall surrounding the play area, correct to the nearest cubic metre. 2 marks

Question 4

A toilet block is situated at point *T* on a hill away from the playground at point *P* as shown in the diagram below. An entrance gate to the park is situated at point *E*.



a. Determine the difference in height between the playground at point *P* and the toilet bock at point *T*. 1 mark

Module 2 – Question 4 – continued

A surveyor measures the horizontal distance between the playground and the toilet block at 38 m.

b. Determine the angle of elevation from the playground to the toilet block, correct to the nearest degree. 1 mark

The surveyor locates the entrance at 50 m due east of the playground and measures the bearing of point T from point E at 290°.

c. Determine the bearing of the toilet block from the playground to the nearest degree. 2 marks

END OF Module 2
TURN OVER

Module 3: Graphs and relations

Question 1 (6 marks)

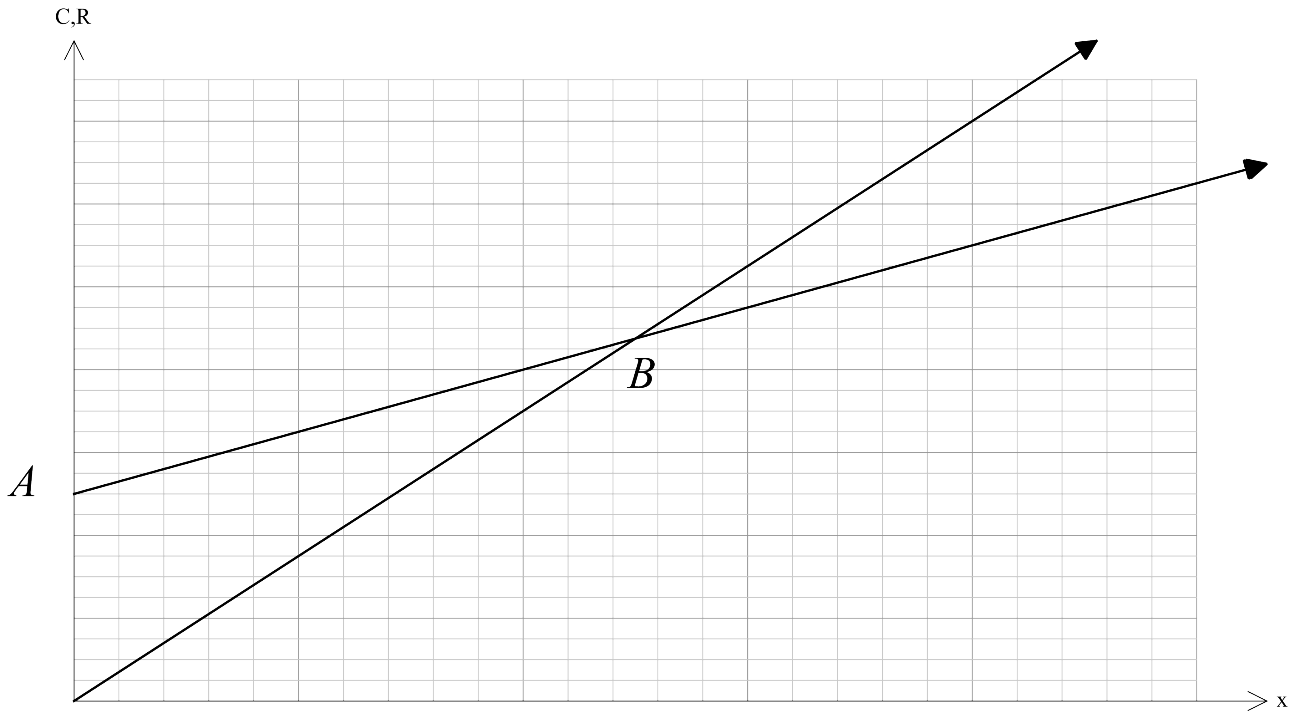
Stan runs a small catering company. For a Christmas function he has fixed costs of $500 in addition to a cost of $15 per meal provided.

a. Write down an expression for the total cost, C dollars, incurred by Stan in providing meals to x guests. 1 mark

The guests at the function will paying $35 per head.

b. Write down an expression for the revenue, R dollars, that will result from x guests. 1 mark

The graph below shows the lines representing both the revenue and cost.



c. Write down the coordinates of point A and point B. 2 marks

d. Stan is hoping to make a profit of at least \$650 from the function.
Determine the minimum number of guests that will be required to achieve this. 2 marks

Question 2 (9 marks)

Stan's catering service provides two types of hampers with either hot food or cold food.

Let x the number of hampers of hot food prepared in one day
y the number of hampers of cold food prepared in one day

It takes 30 minutes to prepare a hot food hamper and 15 minutes to prepare a cold food hamper
There are 300 minutes available each day to prepare all the hampers

The information can be summarised by the following inequalities

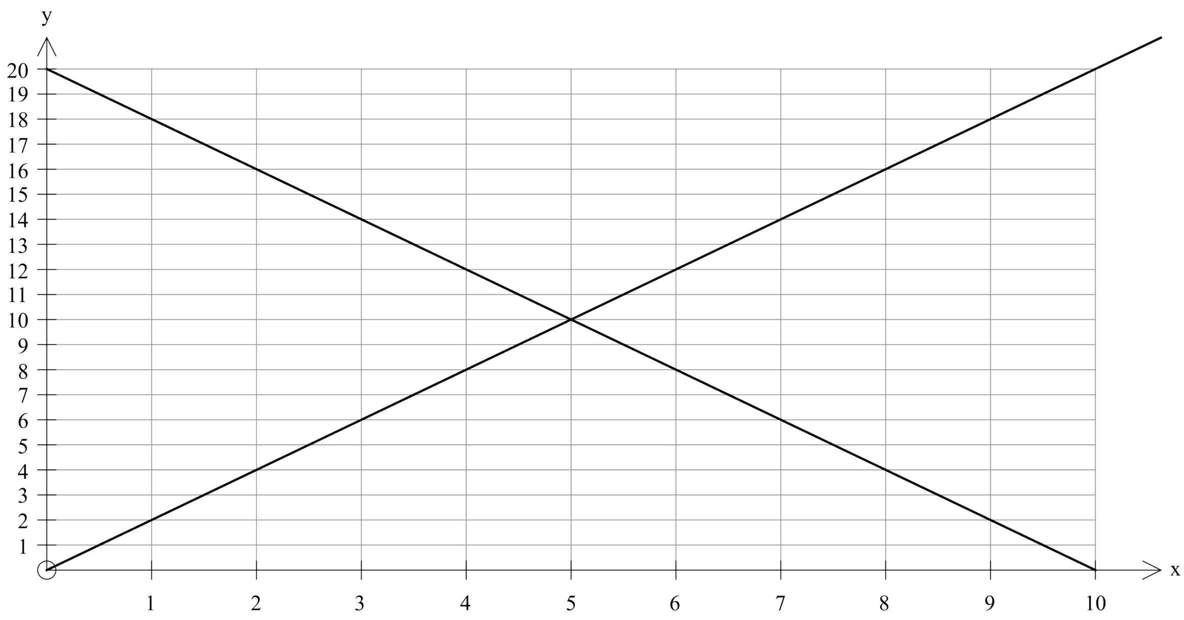
Inequality 1: $x \geq 0$
Inequality 2: $y \geq 0$
Inequality 3: $30x + 15y \leq 300$

One further inequality also applies

Inequality 4: $y \geq 2x$

a. Describe, in practical terms, the constraint described by Inequality 4. 1 mark

The graph below has the lines of the relevant equations drawn.



b. On the graph clearly **draw over all lines representing the boundaries** of the region which represents the four inequalities. 1 mark

c. On Monday Stan prepared eight cold food hampers. What is the maximum number of hot food hampers he could have prepared on this day? 1 mark

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The profit made by Stan is \$60 for each hot food hamper and \$20 for each cold food hamper.

d. Write down an equation for the profit, P, in terms of x and y. 1 mark

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e. Determine the maximum profit that Stan can make in any one day.
How many hampers of each type will he need to prepare and sell? 2 marks

Stan decides to increase the price of the cold food hamper.
As a result he can now expect a profit of $30 for each cold food hamper.
The profit on each hot food hamper remains $60.

f. Determine all possible solutions for the number of each hamper that he can prepare that will maximise his profit on any one day. 2 marks

g. Determine the profit made by Stan under this revised arrangement. 1 mark

END OF Module 3
TURN OVER

Module 4 : Business-related mathematics

Question 1 (2 marks)
Ronith is planning to purchase a new computer game. The game is advertised at a price of \$352 to which a 10% Goods and Services Tax (GST) will be added.

a. Determine the total amount that Ronith will pay for the computer game. 1 mark

Another computer game is also advertised at \$352 but this includes the 10% GST.

b. Determine the price of this computer game before the GST was added.

1 mark

Question 2 (6 marks)
Ronith has paid \$2500 for new computer equipment.

a. It is assumed that inflation will average 3% per annum over the next five years. If the equipment Ronith bought today cost \$2500, calculate the cost of similar items in five years time adjusted for inflation. Give your answer correct to the nearest dollar. 1 mark

Ronith plans to depreciate the value of the \$2500 equipment over the next four years. Ronith believes the equipment will have a useful life of four years and will have a scrap value of \$500.

b. Using the flat rate depreciation method, determine the annual depreciation rate that will result in this scrap value figure. 2 marks

Module 4 – Question 2 – continued

c. Using the reducing balance depreciation method, determine the annual depreciation rate that will result in this scrap value figure. Give your answer correct to two decimal places.
2 marks

d. Give a reason why the reducing balance rate from part c) is higher than the flat rate depreciation rate from part b).
1 mark

Question 3 (3 marks)

Ronith's father is looking to invest in a perpetuity at an interest rate of 4.8% per annum.
He will invest the amount required that will provide him with a monthly payment of $2000.

a. Determine the amount he would need to invest to invest. 2 marks

b. For what length of time will Ronith's father continue to receive $2000?
Explain your answer briefly. 1 mark

Module 4 – continued
TURN OVER

Question 4 (4 marks)

Ronith has won \$20 000 in a competition and is looking to invest the money.
He invests the money at an interest rate of 4.8% per annum compounding monthly for six years.

a. The value of Ronith's investment after six years is correctly calculated to be
$20000 \times R^{72}$

Write down the value of R. 1 mark

b. Determine the value of Ronith's investment after six years. Give your answer correct to the nearest dollar. 1 mark

Ronith is looking to maximise the amount of interest he earns from his investment.
He decides that at the end of each month for the six year period, he will deposit an extra \$50 after interest has been paid.

c. Determine how much **additional interest** he will earn over the six years of the investment by making these monthly deposits. Give your answer correct to the nearest dollar. 2 marks

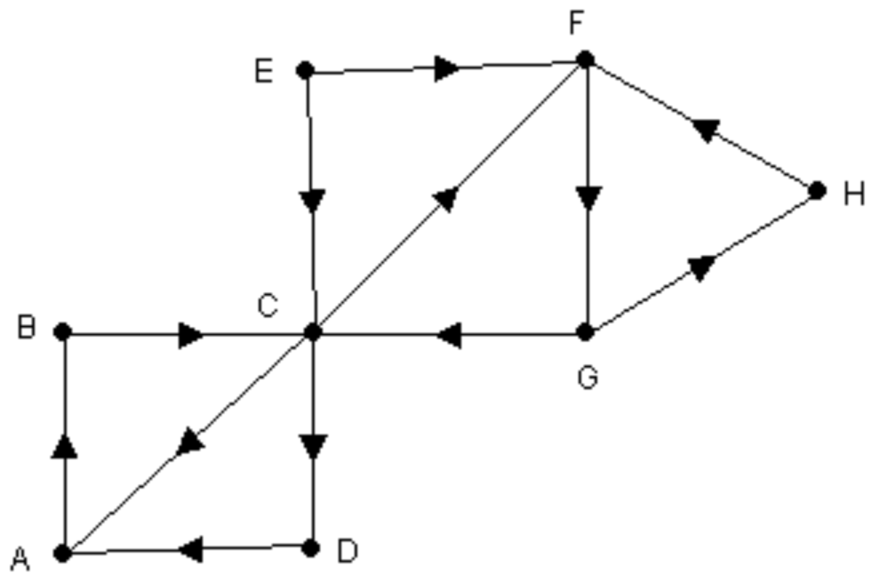
END OF Module 4

Module 5 : Networks

Question 1 (3 marks)

Nodes A to H in the following directed graph represent eight people who are standing in an election. The names of the people are Archie, Bob, Craig, Deidre, Elaine, Frederick, Gai and Hugo. Each person was asked to name one or more of the other candidates that they thought had a good chance of winning.

For example a directed arrow from A to B means that Archie believes Bob has a good chance of winning.



a. Which person in the group in not regarded as a winning chance by any of the others? 1 mark

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b. Which candidates are most highly regarded going into the election? 1 mark

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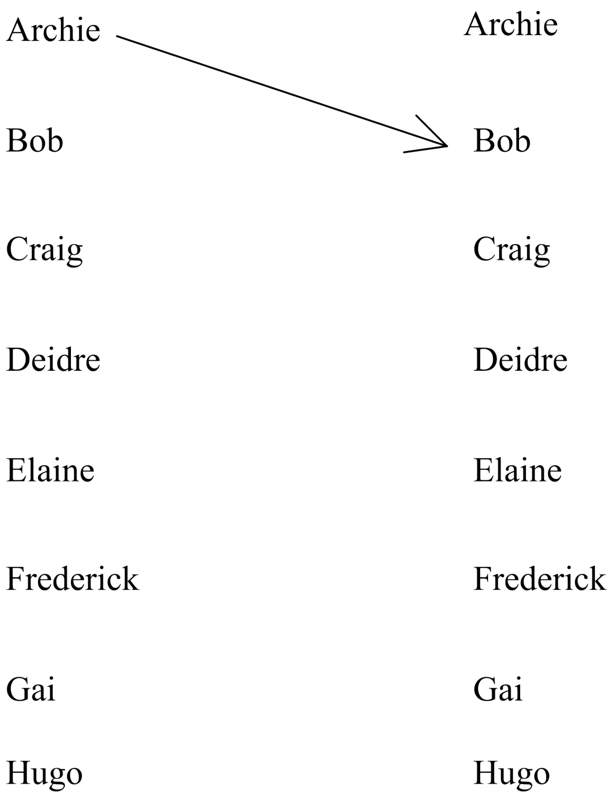
Module 5 – Question 1 – continued
TURN OVER

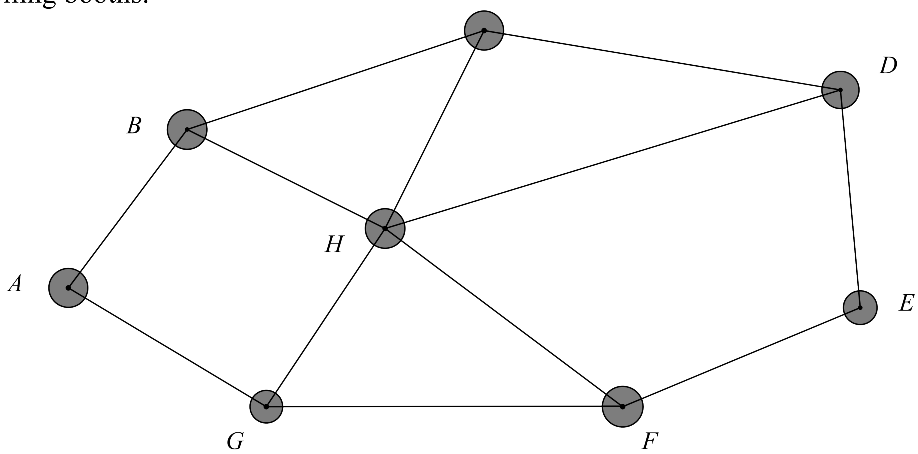
c. Complete the following bipartite graph that also demonstrates the candidates that are considered a winning chance by each individual. 1 mark



Question 2 (5 marks)

Within the electorate there are eight polling booths in various locations. These are denoted as vertices A to H on the network diagram below. Edges on the network represent roads that link the eight polling booths.

C



a. Write down all the vertices that have a degree of 3. 1 mark

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Module 5 – Question 2 – continued

b. **i.** An election supervisor, Mugu, drives along the journey AGFHBCDE
Give a mathematical term to describe Mugu's journey. 1 mark

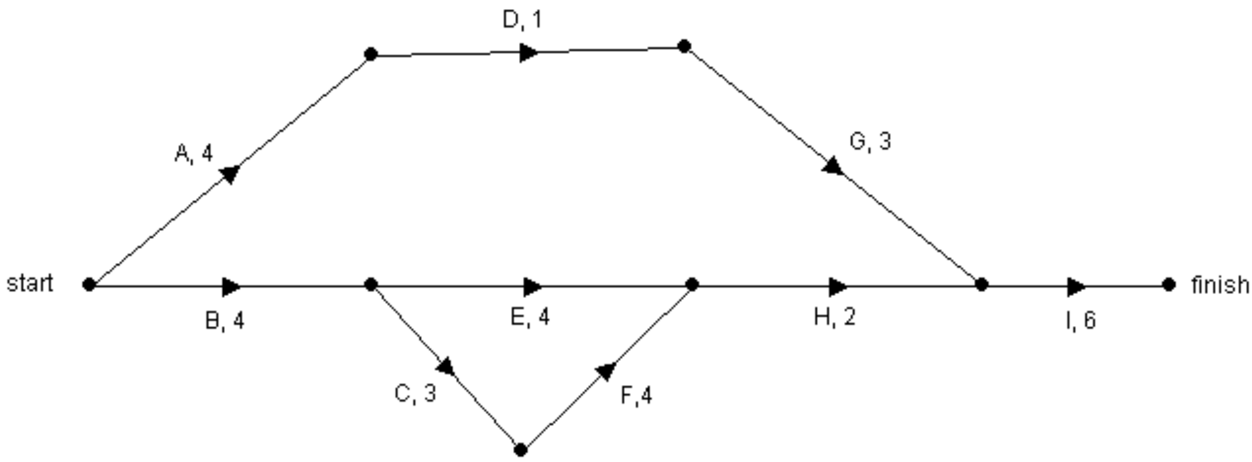
ii. Write down a second example of a journey of this type that starts at A and ends at E. 1 mark

c. **i.** A second supervisor, Deepa, wishes to drive along every road once and return to the polling booth that she started from. Explain why this is not possible for this network. 1 mark

ii. What is the minimum number of new roads that would be required in order for Deepa to be able to achieve her aim? 1 mark

Question 3 (7 marks)

To prepare for the election, the organisers at each polling booth have identified nine activities that must be completed. The directed network shows these activities with completion times in hours.



Module 5 – Question 3 – continued
TURN OVER

a. Determine the earliest start time, in hours, for activity H. 1 mark

b. Write down the critical path for this project. 1 mark

c. What is the minimum time in which this project can be completed? 1 mark

d. Determine the float time, in hours, for activity E. 2 marks

e. Three activities all have the same float time. Write down these three activities. 1 mark

A tenth activity, J, is to be added to the project which does not affect the critical path.
Activity J is of duration 2 hours, has an earliest start time of 4 and a latest start time of 9.

f. Add activity J to the network diagram above. 1 mark

END OF Module 5

Module 6 : Matrices

Question 1 (3 marks)
In 2014 four football teams Antelopes, Beavers, Crabs and Drakes played each other once in a competition.

In the following results matrix R_{2014} a '1' represents "defeated".

For example $r_{2,1} = 1$ denotes Antelopes defeated Beavers.

$$\begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ R_{2014} = & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

a. Explain the meaning of element $R_{2,4}$ 1 mark

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b. Matrix R contains one row of zeros. What does this tell us? 1 mark

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In 2015, the four teams will play each other once again.

c. Complete the results matrix R_{2015} below with possible results if Beavers, Crabs and Drakes all win two of their three games. 1 mark

$$\begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ R_{2015} = & \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

Module 6 – continued
TURN OVER

Question 2 (6 marks)

During the season 500 supporters of the Antelopes were asked each week if they believed the team would make the finals in 2015. In week 1, 250 said yes, 150 said no and 100 were undecided.

a. Complete the initial state matrix, S_1, representing the proportions of supporters in each category in week 1.. 1 mark

$$S_1 = \begin{array}{c} \\ \\ \\ \end{array}\begin{bmatrix} \\ \\ 0.2 \end{bmatrix}\begin{array}{l} Y \\ N \\ U \end{array}$$

The opinions of supporters change from week to week according to the following transition matrix.

$$T = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} Y & N & U \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0.4 \\ 0.1 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.4 \end{bmatrix} \end{array}\begin{array}{l} \\ Y \\ N \\ U \end{array}$$

b. What information does the figure of 0.1 in the matrix above provide? 1 mark

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c. How many of the 500 supporters changed their opinion from week 1 to week 2? 1 mark

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Module 6 – Question 2 – continued

d. Evaluation of the expression $k \times T^n S_1$ enables the calculation of the number of supporters with each opinion in any particular week.

Write down the values of k and n required to determine the number of supporters in each of the three categories in week 3. 2 marks

$k = \boxed{}$ $n = \boxed{}$

e. The season goes for 20 weeks. How many supporters are undecided about the team's finals chances in the last week? 1 mark

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Question 3 (6 marks)

At Beavers' home games, patrons can purchase beer and wine at bar prices.

Andrew purchased 4 beers and 3 wines at a total cost of \$46.50, whereas Fiona purchased 2 beers and 5 wines at a total cost of \$49.50.

Matrices can be used to determine the individual cost of each beer and each wine.

a. Complete the following simultaneous equations in matrix form where b represents beer and w represents wine. 1 mark

$$\begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} b \\ w \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

b. Explain why these simultaneous equations will have a unique solution. 1 mark

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Module 6 – Question 3 – continued
TURN OVER

c. Peter bought 1 beer and 1 wine. Determine how much Peter was charged. 1 mark

Question 4

The Antelopes begin to sell team jumpers and caps to supporters as a way of raising funds.
The number of jumpers and caps sold and the total cost in each of the first two months were written as two simultaneous linear equations.
Using these simultaneous equations, the individual cost of each jumper, J , and each cap, C , can be found by evaluating the matrix product

$$\begin{bmatrix} J \\ C \end{bmatrix} = \begin{bmatrix} \frac{1}{25} & -\frac{7}{150} \\ -\frac{1}{50} & \frac{2}{75} \end{bmatrix} \begin{bmatrix} 16250 \\ 12750 \end{bmatrix}$$

a. Calculate the cost of each jumper and each cap. 1 mark

b. Determine the two simultaneous equations that were originally used. 2 marks

END OF QUESTION AND ANSWER BOOK