Neap:

Trial Examination 2015

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

Core

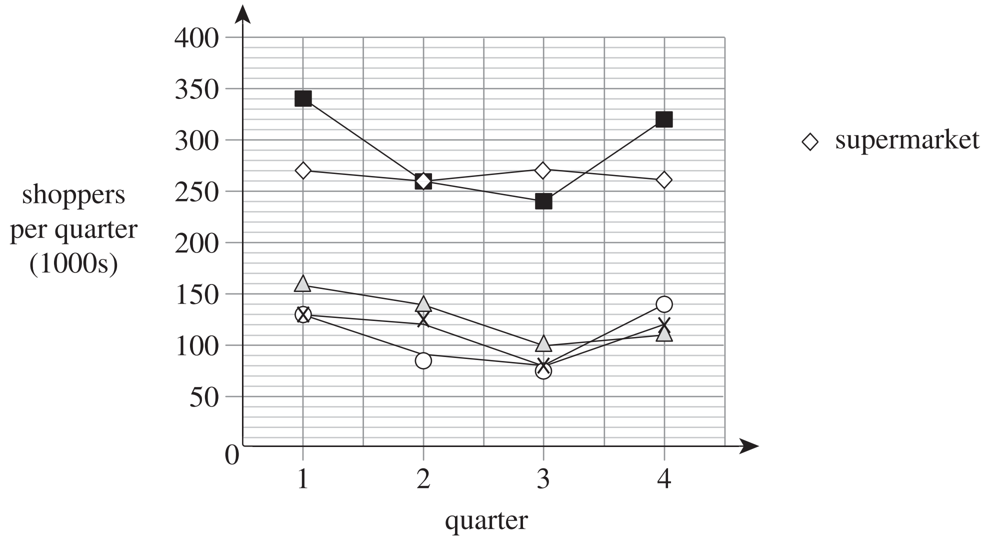
Question 1 (4 marks)

a. 100 000 — 1 mark

Remember the figures are in thousands.

b. **i.** All four categories show the same seasonal pattern, with two decreases followed by an increase. — 1 mark

ii.



1 mark

iii. The supermarket sales are steady and do not display the seasonal trend of the other data sets. — 1 mark

Question 2 (5 marks)

a. $$\frac{\frac{2550}{1975}+\frac{2700}{2133}+\frac{30\ 000}{2275}}{3}=1.29$$ 1 mark

b. 2826 — 1 mark

This is found by dividing the raw figure by the seasonal index.

c. The completed table is as follows:

2008 Q_1	2550	
2008 Q_2	1800	1800
2008 Q_3	1650	1800
2008 Q_4	1900	1900
2009 Q_1	2700	2000
2009 Q_2	2000	2000
2009 Q_3	1750	2000
2009 Q_4	2100	

2 marks

d. The moving mean shows a positive trend. 1 mark

Question 3 (6 marks)

a. average shoppers per day = 280 + 1485 × (years of operation of the attraction) 2 marks

1 mark for each equation entry

b. There is a pattern, so the residuals do not support the assumption of linearity. 1 mark

c. Yes, the transformation improves the fit of the data to the model. 1 mark

For the transformed data $r = 0.99$, but for the original data $r = 0.98$. After the transformation, the new equation is:

average shoppers per day = 2010 + 247.7 × (years of operation)^2 1 mark

d. $\frac{4200 + 6000 + 8200}{3} = 6133$ 1 mark

Module 1: Number patterns

Question 1 (4 marks)

a. The sequence is geometric with common ratio $\frac{2}{3}$ and first term 9.

$a = 9$

$r = \frac{2}{3}$

$t_5 = ar^4$

$= 9 \times \left(\frac{2}{3}\right)^4$

$= \frac{16}{9}$ A1

b. This is the seventh sum:

$S_7 = 9 \times \dfrac{1 - \left(\frac{2}{3}\right)^7}{1 - \frac{2}{3}}$ M1

$= 27\left(1 - \frac{128}{2187}\right)$

$= 27 - \frac{128}{81}$

$= \frac{2059}{81}$

$\cong 25.42$ A1

c. It is the infinite sum:

$S_\infty = \frac{a}{1 - r}$

$= \dfrac{9}{\frac{1}{3}}$

$= 27$ A1

Question 2 (7 marks)

This time the sequence is arithmetic.

$a = 9$

$d = -1$

a. $t_5 = a + (n - 1)d$ *identifies sequence type* M1

$= 9 + 4 \times -1$

$= 5$ A1

b. $S_7 = \frac{7}{2}[2 \times 9 + 6 \times -1]$

$= \frac{7}{2}(12)$

$= 42$ A1

c. The pattern can continue until the next length is zero or negative.

Thus we will have 9 lines comprising the pattern.

$S_9 = \frac{9}{2}[2 \times 9 + 8 \times -1]$

$= 45$ A1

d. We need the total to be 36. We do not know the number of terms, but since the terms decrease progressively by 1 each time, the number of terms must be equal to the first term value.

Thus $n = a$. M1

$\frac{n}{2}[2n - (n - 1)] = 36$

$n(n + 1) = 72$

If trial and error is used, the result $n = 8$ is eventually found. M1

OR

$n^2 + n - 72 = 0$

$(n + 9)(n - 8) = 0$

$n = 8$ M1

Thus the first line has length 8 cm. A1

Question 3 (4 marks)

a. We need to show that the even terms are all the same and that the odd terms are also all identical to each other. Thus we simply must show that the sequence alternates between two different values.

$t_3 = \frac{1}{2}(3 - 1) + 2 = 3$

$t_4 = \frac{1}{2}(1 - 3) + 2 = 1$ A1

Thus terms three and four are a repeat of terms one and two. In fact, terms five and six must repeat these again, as each term depends only on the preceding two. If the two preceding terms repeat, the resulting term must repeat also. A1

b. $t_3 = \frac{1}{2}(t_1 - t_2) + 2$

Thus $3 = \frac{1}{2}(a - b) + 2$

$2 = a - b$ M1

Also:

$2 = \frac{1}{2}(3 - b) + 2$

$b = 3$

Thus $a = 5$. A1

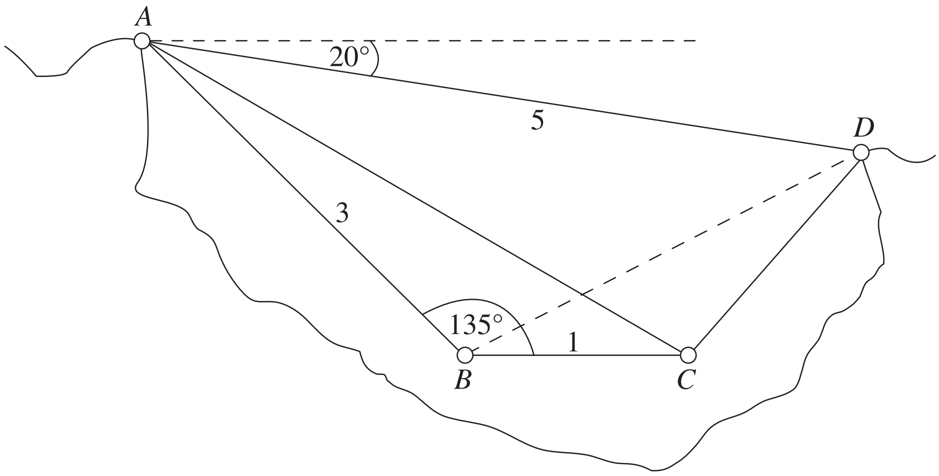
Both terms must be given for full marks.

Module 2: Geometry and trigonometry

Question 1 (5 marks)

a. distance east $= 3\cos 45$

$= 2.12$ km A1



b. Use cosine rule.

$d^2 = 3^2 + 1^2 - 3 \times 2\cos 135$

$d = \sqrt{14.2426}$

$= 3.774$ km A1

Find angle *BAC* using the sine rule.

$\frac{\sin\theta}{1} = \sin\frac{135}{3.774}$

$\theta = 10.8°$

Hence *C* is on a bearing 124°T from *A*. A1

c. Angle *BAD* is $45 - 20 = 25°$ M1

$a^2 = 3^2 + 5^2 - 2 \times 3 \times 5\cos 25$

$= 2.61$ km A1

Question 2 (3 marks)

a. Students can calculate the gradients for each section, or just see by inspection, that *EF* is the steepest. A1

$m = \frac{25}{100}$

$= 0.25$ A1

b. The rise is 25 m and the horizontal distance is 500 m.

$d = \sqrt{500^2 + 25^2}$

$= 500.6$ m A1

Question 3 (7 marks)

a. **i.** At the centre point, C, there are five angles which sum to 360°.

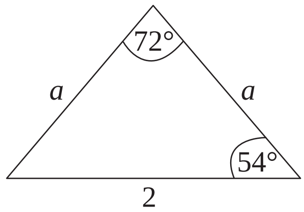
Thus each is $\frac{360}{5} = 72°$. A1

ii. The internal angle sum for any triangle is 180°. We already have a single angle of 72°.

Thus the sum of the other two angles must be $180 - 72 = 108°$.

Each angle is thus $\frac{108}{2} = 54°$. A1

b. Each of the 5 sides of the pentagon must be of length $\frac{10}{5} = 2$ km. M1



$$\frac{a}{\sin 54} = \frac{2}{\sin 72}$$

$$a = \frac{2\sin 54}{\sin 72}$$

$$= 1.701 \text{ km}$$ A1

c. Students must find the area of the pentagon.

The pentagon consists of 5 triangles. Students must determine the triangle area and multiply by 5. M1

$$\text{area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 1.701 \times 1.701 \times \sin 72$$

$$= 1.376 \text{ km}^2$$

$$\text{total area of pentagon} = 5 \times 1.376$$

$$= 6.88 \text{ km}^2$$ A1

d. All angles in the model will be identical to those on the real track. The shape is the same. Only the lengths (and thus areas and volumes) change.

The angle is 72°. A1

Module 3: Graphs and relations

Question 1 (7 marks)

a. **i.** Use points (0, 400) and (100, 900).

$$m = \frac{900 - 400}{100 - 0}$$

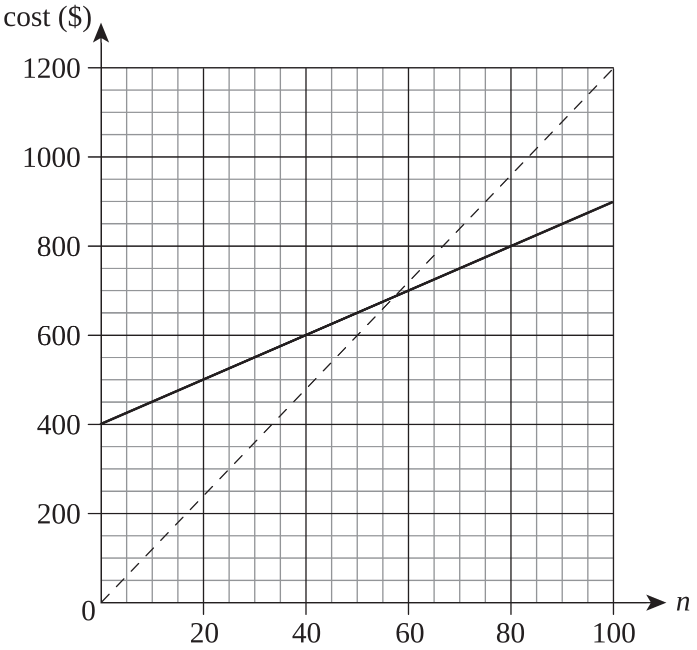
$$= 5$$ A1

ii. y-intercept = 400, gradient = 5

Thus $C = 5n + 400$. A1

b. **i.** $R = 8n$ A1

ii.



A1

iii. $12n = 5n + 400$

$7n = 400$

$n = 57.14$

The minimum number that allows a profit is 58 boxes. A1

c. The cost of making 80 boxes is $400 + 5 \times 80 = 800$. M1

Thus we need revenue of $800 from 80 boxes.

$$\text{selling price per box} = \frac{800}{80}$$

$$= \$10$$ A1

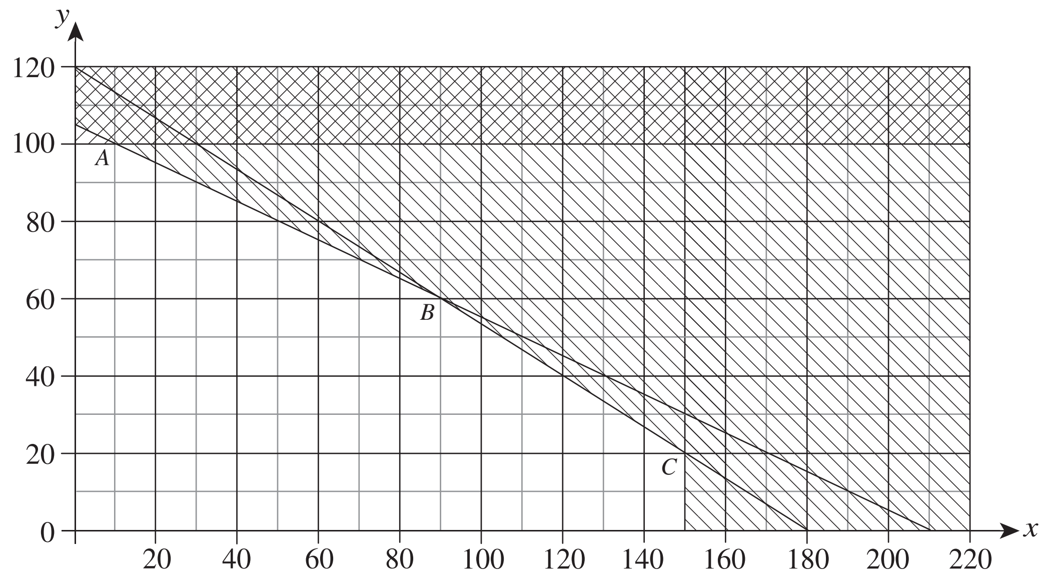
Question 2 (8 marks)

a. **i.** Each standard box takes 2 hours to make, and thus x boxes will take $2x$ hours. Each deluxe box takes 3 hours and thus y boxes will take $3y$ hours. That makes a total of $2x + 3y$ hours. This time must not exceed the 360 hours allocated. A1

ii. total cost $\leq$ 840

$4x + 8y \leq 840$ A1

b.



A2

1 mark for correct position of both lines

1 mark for correct shading

c. The result can be seen in the graph or found algebraically. The more severe restriction in this case is the less steep line $4x + 8y = 840$.

When $x = 70$, $280 + 8y = 840$

$y = 70$ A1

d. A (10, 100)

$R = 9 \times 10 + 14 \times 100 = 1490$

B (90, 60)

$R = 90 \times 9 + 14 \times 60 = 1650$

C (150, 20)

$R = 8 \times 150 + 14 \times 20 = 1480$

Thus the greatest revenue is obtained from making 90 standard and 60 deluxe boxes.

revenue calculation method M1

correct calculation A1

e. As we saw from part **d.**, the maximum weekly revenue is \$1650. A1

Module 4: Business-related mathematics

Question 1 (3 marks)

a. $0.25 \times \$780 = \$195$ A1

b. $\$780 - \$195 = \$585$ A1

c. $\frac{\$585}{5} = \$117$

$3 \times \$117 = \$351$

$\$585 - \$351 = \$234$ A1

Question 2 (6 marks)

a. **i.** $\$32\,000 \times \frac{7.5}{100} = \$2400$ A1

$\$32\,000 - 2 \times \$2400 = \$27\,200$ A1

$\$32\,000 - \$20\,000 = \$12\,000$ M1

$\frac{\$1200}{\$2400} = 5 \text{ years}$ A1

b. **i.** $BV = \$32\,000 \times \left(1 - \frac{6.5}{100}\right)^2$

$= \$27\,975.20$ A1

ii. depreciation amount $= \$32\,000 \times \left(1 - \frac{6.5}{100}\right)^2 - \$32\,000 \times \left(\frac{1 - 6.5}{100}\right)^3$

$= \$1818.38$ A1

Question 3 (2 marks)

a. $N = 12$

$I\% =$ **4.4** A1

$PV = 32\,000$

$Pmt = -117.33$

$FV = -32\,000$

$P_PY = 12$

$C_PY = 12$

b. $32 000 A1

The amount owing on an interest-only loan remains constant.

Question 4 (4 marks)

a. $N = 25 \times 12$

$I\% = 5.3$

$PV = 690\ 000$

$Pmt =$ **$4155.19** A1

$FV = 0$

$P_PY = 12$

$C_PY = 12$

b. $N = 50$

$I\% = 5.3$

$PV = 690\ 000$

$Pmt = -4155.19$

$FV =$ **$628 176** A1

$P_PY = 12$

$C_PY = 12$

c. $N = 100$

$I\% = 5.3$

$PV = 690\ 000$

$Pmt = -4155.19$

$FV =$ **$551 112** M1

$P_PY = 12$

$C_PY = 12$

principal owing = $690 000 – 100 × 4155.19 = $274 481

interest = amount owing – principal owing

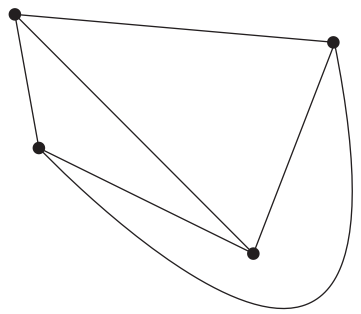
= $551 112 – $274 481

= $276 631 A1

Module 5: Networks and decision mathematics

Question 1 (4 marks)

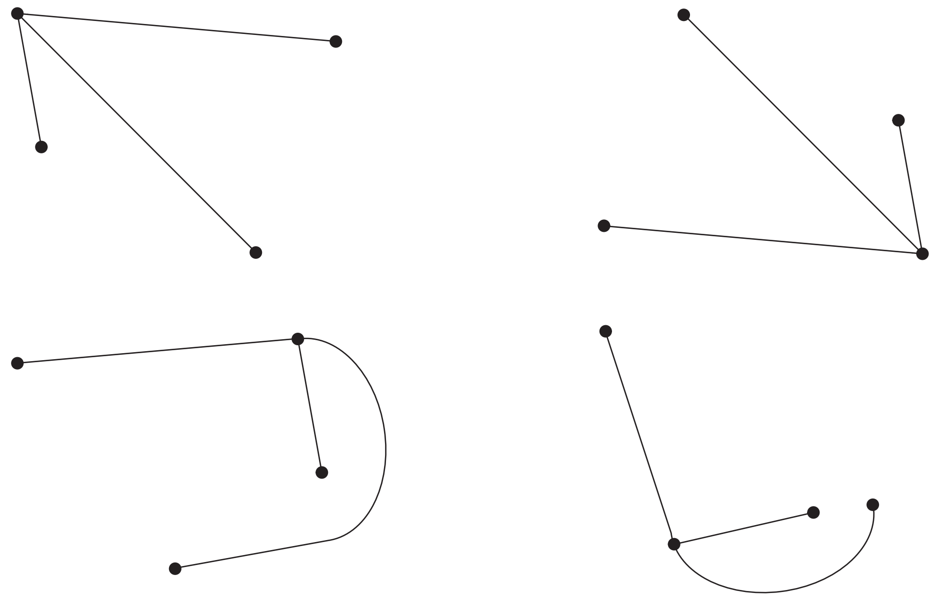
a.



A1

b. 6 A1

c. Given a complete planar graph with 4 vertices, a subgraph with one vertex of degree 3 and no vertices of degree 2 can be formed by removing 3 edges that form a circuit. This can be done in four different ways:



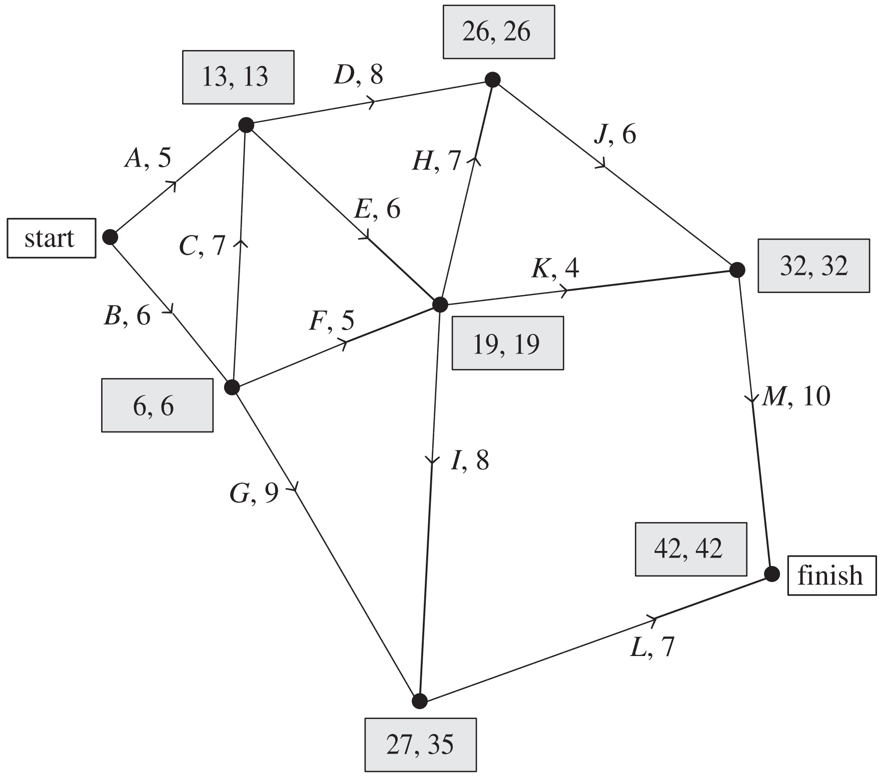
A1

d. A network with no edges or loops will have all vertices with degree zero and hence the total of the degrees will be zero.

A loop in a network adds **two** to the degree of a vertex, and each edge that is not a loop adds one to the degree of the vertex at each of its ends. Each time an edge is added, the total of the degrees of all the vertices will therefore increase by 2. Hence the total of all the degrees of the vertices will always be twice the number of edges. The constant C is thus 2. A1

Question 2 (6 marks)

a.



M2

b. B, C, E, H, J, and M A1

c. 6 + 7 + 6 + 7 + 6 + 10 = 42 days A1

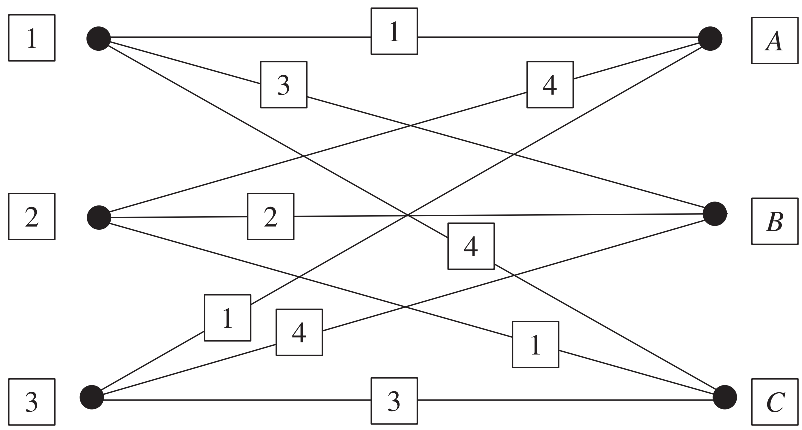
d. The answer is task G, since it must start on day 6 but can be completed as late as day 35 without delaying the project.

Since G has a duration of 9 days, the float time is (35 – 6) – 9 = 20 days. A1

e. If task M is reduced by 8 days to 2 days, the overall time for the project will be reduced to 34 days. If task M is reduced by more than 8 days, the critical path will become B, C, E, I, L and the project will still require 34 days. Hence the maximum reduction is 8 days. A1

Question 3 (3 marks)

a.



A1

b.

	Project *A*	**Project *B***	**Project *C***
Company1	$1 million	$3 million	$4 million
Company 2	$4 million	$2 million	$1 million
Company 3	$1 million	$4 million	$3 million

1	3	4
4	2	1
1	4	3

0	2	3
3	1	0
0	3	2

0	2	3
3	1	0
0	3	2

0	1	3
3	*0*	*0*
0	2	2

0	0	2
4	*0*	*0*
0	1	1

The project allocations are as follows:

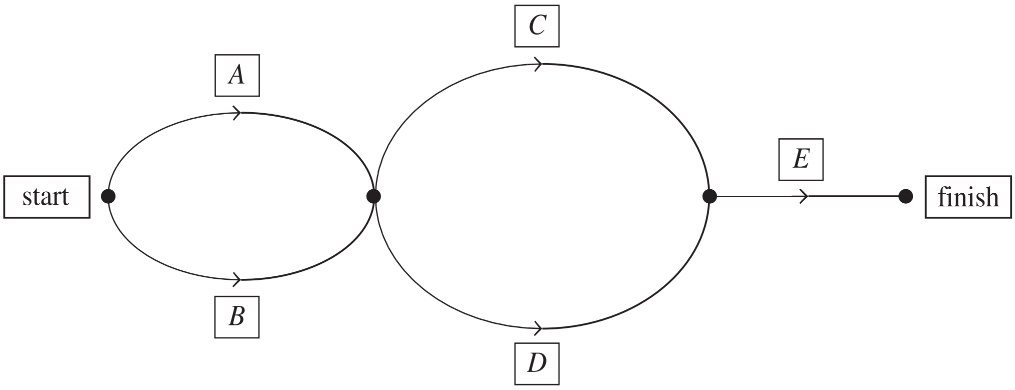
	Project *A*	**Project *B***	**Project *C***
Company	3	1	2

A1

c. $1 million + $3 million + $1 million = $5 million A1

Question 4 (2 marks)

a.



A1

b. The critical paths could be:

- A, C, E
- A, D, E
- B, C, E
- B, D, E

A1

Module 6: Matrices

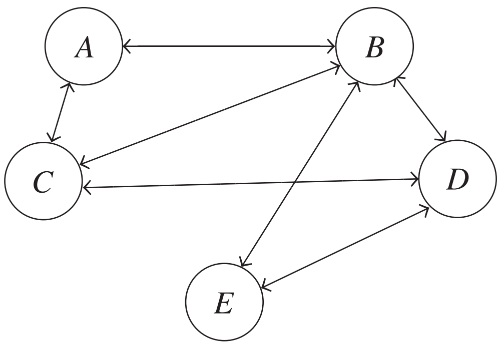
Question 1 (8 marks)

a. The matrix has a cable between *B* and *E,* but the diagram does not. This is the missing cable. A1

b. The sum is 4. A1

c. Row 2 represents town *B*. Each element in the row refers to the cable to another town.
The sum of 4 represents that town *B* is directly linked by cabling to four other towns. A1

d.



A1

e. The row sums are as follows:

Row	1	2	3	4	5
Sum	2	3	2	2	2

These are the numbers of cables going to the respective towns. A1

f.

$$F^2 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}^2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 2 & 0 \\ 1 & 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

A1

g. Matrix *G* needs to add all of the elements in each row. This can be achieved by multiplying each by 1 and then adding them. The matrix multiplication process will automatically add, so a matrix of 1s is correct, as shown below.

$$G = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

A1

h. $F^2G = \begin{bmatrix} 5 \\ 6 \\ 5 \\ 5 \\ 5 \end{bmatrix}$

Town *B* is least vulnerable as it has the most connections. A1

Question 2 (7 marks)

a. From the matrix, we can see the relevant figure is 0.02. This is 2%. A1

b. The data in the table can be used to form a state matrix.

$$S_0 = \begin{bmatrix} 3200 \\ 2000 \\ 1800 \\ 2500 \\ 500 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0.93 & 0.01 & 0 & 0 & 0 \\ 0.01 & 0.92 & 0 & 0.01 & 0.05 \\ 0.02 & 0 & 0.97 & 0.01 & 0.01 \\ 0.02 & 0 & 0 & 0.83 & 0.02 \\ 0.02 & 0.07 & 0.03 & 0.15 & 0.92 \end{bmatrix} \begin{bmatrix} 3200 \\ 2000 \\ 1800 \\ 2500 \\ 500 \end{bmatrix}$$

$$= \begin{bmatrix} 2996 \\ 1922 \\ 1840 \\ 2149 \\ 1093 \end{bmatrix}$$

The population of *E* in 2015 would be 1093. A1

c. Students need to calculate the population state matrix for each year. This is easy if they use a graphing calculator. In 2016 the population in *D* would still be larger (1865 versus 1578), but in 2017 this is reversed (1636 to 1975). A2

d. $T^{300} = \begin{bmatrix} 403 \\ 2822 \\ 1895 \\ 556 \\ 4323 \end{bmatrix}$ to the nearest whole numbers are the long-term populations. A2

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e. The key to this question is to recall what happened in the first year.

The 2015 populations were $\begin{bmatrix} 2996 \\ 1922 \\ 1840 \\ 2149 \\ 1093 \end{bmatrix}$.

A lost 204 people, *B* lost 78, *C* gained 40, *D* lost 351 and *E* gained 593 people. The total population of the area comprising the five towns was unchanged at 10 000.

To achieve the council's objectives, students need to simply reverse these changes. 204 people must be moved to *A*, 78 to *B* and 351 to *D*. *C* needs to lose 40 people and *E* must lose 593. A1