The Mathematical Association of Victoria

Trial Examination 2016

FURTHER MATHEMATICS

Written Examination 1 - SOLUTIONS

Core **Data Analysis**

1. **C** Sum of frequencies is 45. The median score will be the 23rd value. The 23rd value is 14 (frequencies of columns from the left are 2+3+1+5+3+2+7 =23).

2. **D** The data is categorical (a quality rather than a quantity).
Since the categories are in a ranked order, the data is ordinal categorical.

3. **C** Modal (most frequent) lies between 4 and 5 on the log scale – i.e. between \$10 000 and \$100 000.

4. **A** Sum of frequencies is 34.
Value less than \$1 000 000 is 6 or less on log scale.

Percentage $= \dfrac{31}{34} \times 100 = 91.176\ldots$

5. **D** Let x represent *size* and y represent *price* in $y = a + bx$.

$$b = r\frac{S_y}{S_x} = 0.68 \times \frac{350}{25} = 9.52$$

$$a = \overline{y} - b\overline{x} = 1080 - 9.52 \times 120 = -62.4$$

6. **B** The correlation coefficient is positive and moderate – strong, indicating that as one variable increases, the other variable tends to increase as well.

7. **D** We need to quote values for the same medication use category across all three age groups present.

8. **B** There are six points above the zero line with two pairs of points directly above one another.

9. **A** Response variable is TV sets so eliminate options B and D.
Calculate gradient using (0, 30) and (5, 630)

Gradient $= \dfrac{630 - 30}{5 - 0} = \dfrac{600}{5} = 120$

10. **A** When the explanatory variable has a zero value, the value of the response variable can be determined from the vertical axis intercept.

Core **Data Analysis**

11. **C** Any option claiming a causal link must be ignored – hence eliminate options A, B and E. While coincidence may apply in some cases, the common cause of a country's wealth would be more correct.

12. **A** Data will need to be accurately entered and the logarithm of the GDP values calculated before finding the regression constants.

13. **C** *TV sets* = – 338 + 0.0983 × *life expectancy*^{2}

So 502 = – 338 + 0.0983 × *life expectancy*^{2}

Life expectancy = $\sqrt{\dfrac{502+338}{0.0983}}$ = 92.440…

14. **D** There are peaks (more than a year apart) with a decreasing trend.

15. **C** Need median of values at years 3, 4, 5, 6 and 7.
The child mortality value at year 7 is the median in the vertical direction, approximately 185.

16. **D** Need two calculations – one for years 4, 5, 6 and 7 and another for years 5, 6, 7, and 8.

Value (4, 5, 6, 7) = $\dfrac{157+151+175+159}{4}$ = 160.5

Value (5, 6, 7, 8) = $\dfrac{151+175+159+202}{4}$ = 171.75

Centred value = $\dfrac{160.5+171.75}{2}$ = 166.125

Core — Recursion and Financial Modelling

17. B Decay arithmetically means to decrease by the same amount each time, i.e. subtracting a constant amount only.
Note that option C is a combination situation and options D and E involve geometric growth or decay.

18. C Value depreciated = \$27 500 – \$10 000 = \$17 500

Kilometres travelled $= \dfrac{17500}{0.35} = 50\,000$

19. A 6.5 % compounded quarterly is effectively 6.6601…%.
6.5 % compounded weekly is effectively 6.7115…%.
The difference is (6.711 – 6.6601) = 0.0514 %

20. A Interest rate per month as a decimal $= \dfrac{7.2}{1200} = 0.006$

The interest is calculated using 1.006, and the payment will be – 498.

21. B Balance = previous balance – reduction in loan balance
= 29 116.92 – 446.18
= 28 670.74

22. E Interest charged for ninth payment $= \dfrac{8.4}{1200} \times 26\,392.54$ = \$184.75

Reduction in loan balance = 650.00 – 184.75 = \$465.25

Percentage reduction $= \dfrac{465.25}{650.00} \times 100 = 71.576\ldots$

23. A Fraction reduction in value $= \dfrac{20000}{25000} = 0.80$

This is geometric decay with each year's value 80% of the previous year's value.

24. C Use Finance Solver to find the *FV* after four years of investing
$N = 48,\ I = 5.4,\ PV = -380000,\ Pmt = -2500,\ PpY = CpY = 12;$
solve for *FV* (\$605 002.19)
Use Finance Solver to find the *FV* after five years of retirement
$N = 60,\ I = 5.4,\ PV = -605002.19,\ Pmt = 4500,\ PpY = CpY = 12;$
solve for *FV* (\$482 880.22)

Module 1 Matrices

1. **C** AB is NOT defined (4 × 3 × 2 × 4)
BA IS defined (2 × 4 × 4 × 3).
The outer pair of numbers give the order of the product matrix.

2. **D** When a matrix is transposed, the elements in the first row become the elements in the first column.
A 2 × 3 matrix will thus transpose to become a 3 × 2 matrix.

3. **D** Reading from the permutation matrix :
The second letter becomes the first (D)
The third letter becomes the second (I)
The first letter becomes the third (E)
The fourth letter remains the fourth letter (T)

4. **B** 700 + 900 = 1600, so we are looking for $\begin{bmatrix} 800 \\ 800 \end{bmatrix}$ as the final matrix.

$$S_7 = T^7 S_0 = \begin{bmatrix} 0.96 & 0.05 \\ 0.04 & 0.95 \end{bmatrix}^7 \begin{bmatrix} 700 \\ 900 \end{bmatrix} = \begin{bmatrix} 791.278... \\ 808.721... \end{bmatrix} = \begin{bmatrix} 791 \\ 809 \end{bmatrix}$$

$$S_8 = T^8 S_0 = \begin{bmatrix} 0.96 & 0.05 \\ 0.04 & 0.95 \end{bmatrix}^8 \begin{bmatrix} 700 \\ 900 \end{bmatrix} = \begin{bmatrix} 800.063... \\ 799.936... \end{bmatrix} = \begin{bmatrix} 800 \\ 800 \end{bmatrix}$$

$$S_9 = T^9 S_0 = \begin{bmatrix} 0.96 & 0.05 \\ 0.04 & 0.95 \end{bmatrix}^9 \begin{bmatrix} 700 \\ 900 \end{bmatrix} = \begin{bmatrix} 808.057... \\ 791.942... \end{bmatrix} = \begin{bmatrix} 808 \\ 792 \end{bmatrix}$$

5. **C**

$$S_{15} = T_n^5\left(T^{10} S_0\right) = \begin{bmatrix} 0.96 & 0.03 \\ 0.04 & 0.97 \end{bmatrix}^5 \begin{bmatrix} 0.96 & 0.05 \\ 0.04 & 0.95 \end{bmatrix}^{10} \begin{bmatrix} 700 \\ 900 \end{bmatrix}$$

$$= \begin{bmatrix} 775.888... \\ 824.111... \end{bmatrix} = \begin{bmatrix} 776 \\ 824 \end{bmatrix}$$

6. **C** From

$$S_{n+1} = TS_n + B$$
$$TS_n + B = S_{n+1}$$

i.e. $$\begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix} S_0 + \begin{bmatrix} 25 \\ 20 \end{bmatrix} = \begin{bmatrix} 130 \\ 115 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix} S_0 = \begin{bmatrix} 130 \\ 115 \end{bmatrix} - \begin{bmatrix} 25 \\ 20 \end{bmatrix} = \begin{bmatrix} 105 \\ 95 \end{bmatrix}$$

$$S_0 = \begin{bmatrix} 0.75 & 0.30 \\ 0.25 & 0.70 \end{bmatrix}^{-1} \begin{bmatrix} 105 \\ 95 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

7. **E** The two equations are $3d + 2h = 12$ and $5d + 3h = 19$.

In matrix form $\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} d \\ h \end{bmatrix} = \begin{bmatrix} 12 \\ 19 \end{bmatrix}$,

Hence $\begin{bmatrix} d \\ h \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ 19 \end{bmatrix}$ and finally $\begin{bmatrix} d \\ h \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 12 \\ 19 \end{bmatrix}$.

8. **D** If S can be added to PQR, then PQR is a 5×2 matrix.

In general terms, where the orders are P $m \times m$, Q $o \times p$ and R $n \times n$

$PQR = m \times m \times o \times p \times n \times n$

But, for multiplication to be defined, $m = o$ and $p = n$,

giving PQR = $m \times m \times m \times n \times n \times n$, where $m = 5$ and $n = 2$

Module 2 Networks & decision mathematics

1. **E** The graph is planar (edges meet only at vertices), but not simple (it contains loops and multiple edges between vertices).
The graph is undirected (no directions on edges), but not degenerate (the vertices are not all isolated)
The graph is connected (there is a path between each pair of vertices), but not complete (there are not edges connecting all pairs of vertices)
The graph is neither simple nor complete (see above)
The graph is planar and connected.

2. **B** From Euler's formula where $v - e + f = 2$, if $v = e$, then $f = 2$.

3. **B** This network cannot have any cycles or circuits (eulerian or hamiltonian) because they cannot start and finish at the same vertex.
A hamiltonian path can exist (D – C – E – F – A – B)
An eulerian trail can exist (D – C – E – F – A – F – B – A – E – B – C)

4. **D** This is a complete graph situation where every vertex has an edge to every other vertex.
The total number of edges is given by $\frac{n(n-1)}{2}$.
Solve $\frac{n(n-1)}{2} = 105$, gives $n = 15$.

5. **E** Checking each option,
Options A and C fail because Eamon does not like Boost or Dairy Milk
Options B and D are allocations that work.
Given Options B and D are successful, but not unique, allocations, option E is the answer.

6. **C** To obtain the latest starting time of activity C we also need to take into account its duration, so options A, B and D are incorrect
Where there are two options for the LST calculations, take the one that leads to the smallest value.

7. **C** First, identify the critical path (Start – A – C – D – F – H – Finish).
Looking at activities NOT on the critical path :
Activity B has an EST of 5 and a LST of 13, so a 3 hour delay can be tolerated.
Activity E has an EST of 12 and a LST of 13, so a 3 hour delay cannot be tolerated.
Activity G has an EST of 16 and a LST of 22, so a 3 hour delay can be tolerated.
Only activities B and G can be delayed by 3 hours without affecting the overall completion time of the project.

8. **D** Activity F has an EST of 16 hours. Activity E has an EST of 12 hours and waiting for its completion (6 hours) means that 18 hours will have elapsed.
Starting activity F after activity E has been completed means that two (2) extra hours will have elapsed, so the project will be delayed by two (2) hours.

Module 3 Geometry & Measurement

1. **E** Other base angle of the isosceles triangle is 67º
θ is external angle equal to sum of opposite interior angles
θ = 67º + 67º = 134º
OR Third angle = 180º – 67º – 67º = 46º
θ = 180º – 46º = 134º (Angle in straight line = 180º)

2. **C** Closest to South Pole precludes any city NORTH of the Equator
Closest to South Pole will have largest latitude angle
Mildura at 34.1º is slightly closer than the others.

3. **C** Hobart (angle TO South Pole) = 90º – 43º = 47º
Puntas Arenas (angle TO South Pole) = 90º – 53º = 37º

Hobart (distance to South Pole) = $\dfrac{6400\times\pi}{180^\circ}\times 47^\circ$ = 5250 km

Puntas Arenas (distance to South Pole) = = 4133 km
Total distance = 5250 + 4133 = 9383 ≈ 9400 km

4. **C** Radius end = 1.6 ÷ 2 = 0.8 m
Area = 2 hemispherical ends + roof + floor

$$= 2\times\frac{1}{2}\times\pi\times0.8^2 \quad + \quad \frac{1}{2}\times2\times\pi\times0.8\times3.2 \quad + \quad 1.6\times3.2 \quad = 15.17$$

5. **C** Volume large hut = $\frac{1}{2}\times H\times W\times L \;=\; \frac{1}{2}\times HWL$

$$V = \frac{1}{2}\times\frac{H}{2}\times\frac{W}{2}\times L \;=\; \frac{1}{2}\times\frac{HWL}{4}$$

So $4V = \frac{1}{2}\times HWL$ = Volume large hut

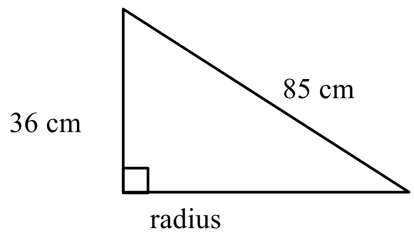
6. **D** If the bearing of A from C is 270º,
then the bearing of C from A is 270º – 180º = 90º
If the bearing of B from A is 025º, then remaining angle to 90º is 65º.
From Cosine Rule,

BC = $\sqrt{1648^2+1950^2-2\times1648\times1950\times\cos(65^\circ)}$ = 1949.91… ≈ 1950 m

7. **D** The semiperimeter of 1648, 1950 and 1950 is 2774, so NEITHER options A or B are correct.
If the bearing of B from A is 025º, then remaining angle to 90º is 65º, the angle between the sides 1648 and 1950, to be used in the Sine Area Rule.

8. **C** Construct a triangle to connect centre of hemisphere face, water edge and centre of water.
Distance from centre of water surface to hemisphere centre
= radius hemisphere – water depth = 85 – 49 = 36 cm
From Pythagoras,

radius = $\sqrt{85^2 - 36^2}$
= 77 cm



Area = $\pi \times 77^2$
= 18 626.5 cm^2
= 1.86 m^2.

Module 4 Graphs and relations

1. **A** Using the points (3, 7) and (1, 1), $m = \frac{7-1}{3-1} = \frac{6}{2} = 3$

2. **D** Using the points (0, 10) and (6, 0), $m = \frac{10-0}{0-6} = -\frac{10}{6} = -\frac{5}{3}$

Y-axis intercept is (0, 10), hence equation is (from y = mx + c)

$y = -\frac{5}{3} \times x + 10$ giving $5x + 3y = 30$

3. **E** Draw a line on the graph to represent "We deliver" charges.
Key points : (0, 0), (1, 2), (2, 4) and (2.5, 5).
So parcels weighing MORE than 1 kg and up to 2 kg inclusive are cheaper by "Deliveries R Us"
ALSO parcels weighing MORE than 2.5 kg will be cheaper by "Deliveries R Us"

4. **C** If b represents the cost of a bracelet and c represents the cost of a charm,
The equations are $b + 7c = 435$ and $3b + 12c = 900$.
Using the SOLVE function, gives $b = 120$ and $c = 45$
Hence Timothy's cost will be 2 × 120 + 8 × 45 = $600.

5. **A** The first 15 days are charged at $3.50 per day so the gradient is 3.5. After 15 days the gradient changes to 2.2 and extrapolating back to the y intercept the first 15 days have been overcharged by $1.30 per day, meaning a y intercept of 19.5.
The rules last for a calendar year so there is an upper limit on days at the cheaper rate, eliminating option E.

6. **E** The cost equation is 235 + 1.20 n where n is the number of coffees sold.
The revenue equation is 4.50n. The breakeven point is 71.2 coffees sold.

Option A : breakeven point at 72 coffees sold	TRUE
Option B : 4.50 × 150 – (235 + 1.20 ×150) = $260	TRUE
Option C : less than 72 coffees for loss	TRUE
Option D : 4.50 × 254 – (235 + 1.20 × 254) = $603.20	
Selling 253 gives a profit of $599.80 – which is < $600	TRUE
Option E : 235 + 1.20 n = 3.90 n, gives n = 87.03	NOT TRUE

(87 coffees lead to a 10¢ loss)

7. **D** At least implies 2-seaters will be higher than this.
So half the number of 2-seaters must be equal to or greater than the number of 3-seater lounges.

8. **E** To draw an objective function line that maximises at point B only requires drawing a line between that passing through A and B ($g = h$) and a line passing through B and C ($g = 1.5h$)
Option E is the one that meets these criteria.