2017
VCE
Further Mathematics
Trial Examination 1
Suggested Answers

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Section A Core – Data Analysis

Question 1 C
Number of families surveyed
$= 5 + 15 + 30 + 25 + 10 + 5$
$= 90$
Number that consumed more than 15 cans of beans $= 25 + 10 + 5 = 40$
$\frac{40}{90} \times 100 = 44.4\%$

Question 2 D
Level of agreement is an ordinal variable because there is an order from high to low.
Political orientation is a nominal variable because the numbers stand for categories with no order.

Question 3 C
There are 20 ages. The median will lie between the 10^{th} and 11^{th} ages. That is between 33 and 34. Half way between 33 and 34 is 33.5

Question 4 A
The mean of B is 30, so we want the probability that the piece selected has an elasticity greater than or equal to 30. For A, 30 is 2 standard deviations greater than the mean. 95% lie within 2 standard deviations either side of the mean. This leaves 5% outside. 2.5% less than 10 and 2.5% greater than 30.
$2.5\% = 2.5 \div 100 = 0.025$

Question 5 E
E is not true because the total area under any standard normal curve is 1.
A is true because
$$z = \frac{x - \bar{x}}{s} = \frac{15 - 20}{5} = -1$$
B is true because
$$2 = \frac{x - 30}{10}, x = 50$$
C is true because $84\% = 50\% + 34\%$
34% is half of 68%, so 37 must be one standard deviation less than 40. $40 - 37 = 3$.
D is true because the standard deviation measures the spread and B has a larger standard deviation than A

Question 6 D
The right hand whisker represents 25% of the population surveyed. 25% of 400 = 100.

Question 7 E
Here, the three teams are the explanatory variable and are categorical data and the response variable, the number of goals scored, is numerical data.
We have 3 teams, so cannot use back-to-back stem and leaf, which could only be used for two teams.
Scatter plots cannot be used when the explanatory variable is categorical.
You would need to compare three dot plots so would need parallel dot plots.
Time series always have time for the explanatory variable.
Parallel box plots would be good to use when comparing three categorical explanatory variables where the response variable is numerical.

Section A Core – Data Analysis

Question 8 D

The *x* axis is a log scale.

$0.1 = 10^{-1}$

$\log_{10}(0.1) = \log_{10}(10^{-1}) = -1$

Percentage of animals with weight greater than 0.1 kg = 30 + 22 + 14 + 2 = 68%

Question 9 B

A is true.

Using the calculator you get r = -0.92. This indicates that there is a strong negative correlation between the two variables, so D is true.

r^2 = 0.85. This is the coefficient of determination so E is true.

85% of the variation can be attributed to hours spent watching TV so 100 – 85 = 15% can be attributed to other factors. So C is true.

B is false because the gradient is -0.86. The *y* intercept is 23.3

Question 10 B

The gradient of the graph as given by the calculator is -0.86. This tells us that for every increase of 1 in the explanatory variable, there is a decrease of 0.86 in the response variable.

Question 11 E

Not A: It only has nine points instead of ten.

Not B or D: The first point of the residual should be negative not positive.

Not C: The fifth point should be positive not negative.

Question 12 C

$\log_{10} Q = 3.5 - 0.27$

$\log_{10} Q = 3.23$

$Q = 10^{3.23}$

$Q = 1698$

Question 13 E

There is no increasing trend. It is not seasonal because the distance between the peaks and troughs is not regular over a period of 1 year. Cycles are periodic movements over a period greater than one year, which is the case here.

Question 14 B

Mean for Winter 2015 and Spring 2016
= (990 + 1320) ÷ 2 = 1155.

Mean for Spring 2016 and Summer 2016
= (1320 + 1430) ÷ 2 = 1375.

Average of these two means
= (1155 + 1375) ÷ 2 = \$1265.

Question 15 D

$$Deseasonalised = \frac{Actual}{Seasonal\ Index}$$

$$= \frac{4728}{1.2} = 3940$$

Question 16 A

$$0.4 \text{ increase on } 0.6 = \frac{0.4}{0.6} \times 100 = 66\frac{2}{3}\%$$

Section A Core – Recursion and financial modelling

Question 17 E
Use calculator to generate the first 6 terms.
These are 4, -1, 9, -11, 29 -51 so V_5 is -51.

Question 18 B
$1+\frac{4}{100}=1.04$
$V_{n+1}=1.04V_n-20, \quad V_0=5000$

Question 19 C
10% of 4000 = 400. So the equipment decreases in value by $400 each year. The graph is a decreasing graph so not A or B or E.
It is linear so not D.

Question 20 E
Monthly interest = 1.0043 – 1
= 0.0043 = 0.43%
Annual interest rate = 0.43 × 12 = 5.16%

Question 21 D
Quarterly interest rate = 8 ÷ 4 = 2%
2% of 25000 = $500
Of the $3760 paid $500 goes in interest and the remainder goes to reduce the principal.
Principal is reduced by 3760 – 500= $3260

Question 22 A
N =
I = 4
PV = 250000
PMT = -2000
FV = 0
P/Y = 12
C/Y = 12
This gives N = 161.968 months = 13.5 years.

Question 23 C
N = 48
I = 6
PV = 24000
PMT = -200
FV =
P/Y = 12
C/Y = 12
This gives FV = $19672.17342

N = 36
I = 6
PV = 19672.17342
PMT =
FV = 0
P/Y = 12
C/Y = 12
This gives PMT = $598.47

Question 24 B
Depreciation over 12 years = 42000 – 5976
= 36024
Average depreciation = 36024 ÷12 = 3002
Depreciation of $3002 for 15000 km
= 3002 ÷ 15000 = 20 cents per km.

Module 1 – Matrices

Question 1 E

a_{23} is the element in the second row third column $= 4$

a_{12} is the element in the first row second column $= -2$

$2 \times 4 - 3 \times -2 = 14$

Question 2 C

To transpose a matrix you interchange the rows and the columns.

$$X^T = \begin{bmatrix} 2 & -3 & 5 \\ 1 & 4 & 0 \\ 3 & -1 & -2 \end{bmatrix}$$

Then the transpose of this takes us back to the original matrix.

$$\begin{bmatrix} 2 & 1 & 3 \\ -3 & 4 & -1 \\ 5 & 0 & -2 \end{bmatrix}$$

Question 3 A

$$4X = \begin{bmatrix} -2 & 0 & 6 \\ 8 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 4 & 2 \\ 8 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 8 \\ 16 & 10 & 4 \end{bmatrix}$$

$$X = \frac{1}{4}\begin{bmatrix} 2 & 4 & 8 \\ 16 & 10 & 4 \end{bmatrix} = \begin{bmatrix} 0.5 & 1 & 2 \\ 4 & 2.5 & 1 \end{bmatrix}$$

Question 4 D

$$\text{Inverse of } P = -\frac{1}{2}\begin{bmatrix} 3 & -7 \\ -2 & 4 \end{bmatrix}$$

$$P^{-1}PQ = P^{-1}R$$

$$IQ = Q = -\frac{1}{2}\begin{bmatrix} 3 & -7 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 3a - 7c & 3b - 7d \\ 4c - 2a & 4d - 2b \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 7c - 3a & 7d - 3b \\ 2a - 4c & 2b - 4d \end{bmatrix}$$

Question 5 D

To multiply matrices, the number of columns of the first matrix must equal the number of rows of the second matrix, so not B or C.

E gives the total value of oranges at store B, so not E. The total value of all the fruit would be a 1×1 matrix, but A gives a 3×3 matrix, so not A.

D gives $0.65 \times 180 + 0.40 \times 200 + 4.00 \times 100$, which is the total price of apples at B + the total price of oranges at B + the total price of mangoes at B. when added this gives the total price of all the fruit sold at B.

Question 6 D

First order dominance by adding up the rows:

$P = 2$, $Q = 1$, $R = 2$, $S = 1$ $T = 1$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Second order dominance by adding up the rows in the squared matrix gives

$P = 2$, $Q = 1$, $R = 3$, $S = 2$, $T = 1$

Adding first and second order for each team gives $P = 4$, $Q = 2$, $R = 5$, $S = 3$, $T = 2$

So Roosters, Panthers, Swallows.

Module 1 – Matrices

Question 7 B

$$\begin{matrix} & M & S \\ M \\ S \end{matrix}\begin{bmatrix} 0.6 & 0.28 \\ 0.4 & 0.72 \end{bmatrix}^2\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}=\begin{bmatrix} 0.421 \\ 0.579 \end{bmatrix}$$

Sara's chance of winning the third game is closest to 58%

Question 8 B

Since C is multiplied by itself it must be a square matrix.

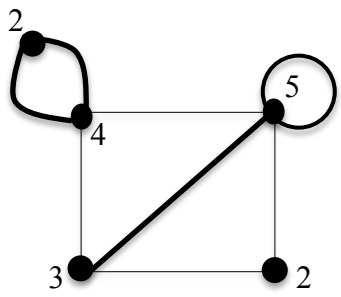
Since CD is defined, C must have 4 columns, so C is a 4×4 matrix. So BA must be a 4×4 matrix. So A has 4 columns. Since BE is defined, B must have 2 columns.

B must be a 4×2 matrix and A a 2×4 matrix.

A is 2×4.

Module 2 – Networks and decision mathematics

Question 1 E



The sum of the vertices = 2 + 4 + 5 + 2 + 3 = 16

Question 2 D

A complete graph has every vertex connected to every other vertex.
For 7 vertices where $n = 7$

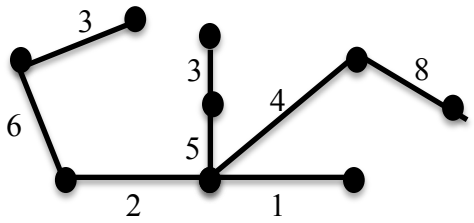
$$\text{Number edges} = \frac{n(n-1)}{2} = \frac{7 \times 6}{2} = 21$$

There are already 5 edges there.
Require 21 – 5 = 16 more edges.

Question 3 B

A Hamiltonian path goes through every vertex once and only once and does not start and end on the same vertex. Option A is a circuit.
Options C, D and E all visit a vertex twice.

Question 4 C



We need the minimum spanning tree, which is shown above.
Length of pipeline
= 1 + 2 + 4 + 5 + 3 + 6 + 3 + 8 = 32 km

Question 5 D

3 people did Karate and 2 people did Hockey, so A is true.
Only one person did Judo. More than one person did each of the other sports, so B is true.
4 people participated in the Iron Person, but Ben did not, so C is true.
Dan and Ellie participated in Karate, Iron Person and Golf. Charles and Ann participated in Hockey, Iron Person, Karate and Golf, so D is not true.
Ben did Hockey and Judo. Dan did Karate and Iron Person, so E is true.

Question 6 E

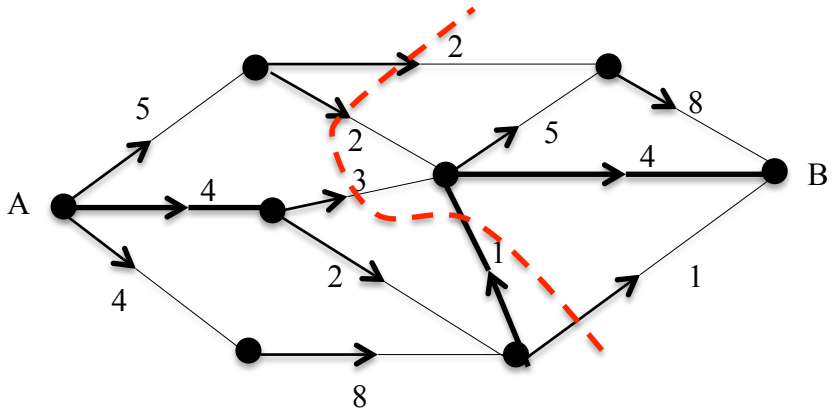
B – I – K = 4 + 3 + 2 = 9 days
B – F – J – K = 4 + 9 + 2 + 2 = 17 days
A – D – H – K = 3 + 5 + 7 + 2 = 17 days
C – G – J – K = 5 + 6 + 2 + 2 = 15 days
B – E – H – K = 4 + 7 + 7 + 2 =20 days.
The critical path is the longest path.

Question 7 D

Earliest start time for J = 13
Latest start time for J = 16
Float time = 16 – 13 = 3 days

Module 2 – Networks and decision mathematics

Question 8 C



The maximum flow = 1 + 1 + 3 + 2 + 2 = 9

Module 3 – Geometry and measurement

Question 1 C

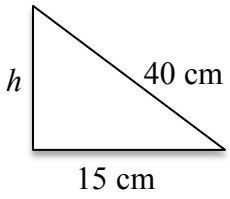
$\angle CBA = 180 - (52 + 23) = 105^0$

$\frac{320}{\sin 23^0} = \frac{AC}{\sin 105^0}$

$AC = 791.07$

AC is closest to 791 m

Question 2 B



$h^2 + 15^2 = 40^2$

$h = 37$ cm

Question 3 E

Q is further east than P, so Q will be ahead in time, so not A or C.

Difference in longitude from 6^0 W to 38^0 E is 44^0

44^0 change in longitude gives $44 \times 4 = 176$ minutes change in time.

$176 \div 60 = 2.93333$

= 2 hours 56 minutes.

Question 4 E

Angle in large sector = $360 - 95 = 265$

Radius of large circle = 8 + 4 = 12

Area of large sector of large circle

$= \pi \times 12^2 \times \frac{265}{360} = 333$

Area of large sector of small circle

$= \pi \times 8^2 \times \frac{265}{360} = 148$

Area of shaded region = $333 - 148 = 185$

185 cm^2

Question 5 D

$\tan \angle DAB = \frac{5}{4}$

$\angle DAB = 51.34^0$

$\tan \angle DAC = \frac{2}{4}$

$\angle DAC = 26.57^0$

$\theta = 51.34 - 26.57 = 24.77^0$

This is closest to 25^0

Question 6 A

Since $AB = AE$ and $\angle EAB = 60^0$, then triangle AEB is equilateral. So, $DC = EB = AB = AE = 4$

Let $BC = x$.

$2x + 4 + 4 + 4 = 24$

$x = 6$

Area of rectangle = $6 \times 4 = 24$.

Let height of triangle = y.

$y^2 + 2^2 = 4^2$

$y = 3.464$

Area of triangle = $4 \times 3.464 \div 2 = 6.9$

Total area = 24 + 6.9 = 30.9 which is closest to 31 m^2

Module 3 – Geometry and measurement

Question 7 E

Triangles *PDQ* and *ADC* are similar (A.A.A.)

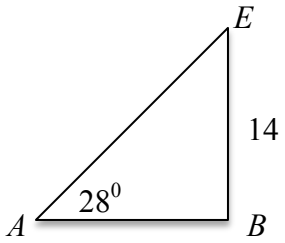
$\therefore \frac{AD}{PD} = \frac{CD}{QD}$

$\frac{5}{3} = \frac{CD}{4}$

$CD = 6\frac{2}{3}$

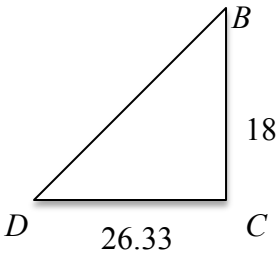
$QC = 6\frac{2}{3} - 4 = 2\frac{2}{3}$

Question 8 A



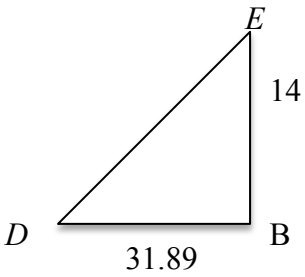
$\tan 28^0 = 14 \div AB$

$AB = 26.33 = DC$



$BD^2 = 26.33^2 + 18^2$

$BD = 31.89$



$\tan \angle EDB = 14 \div 31.89$

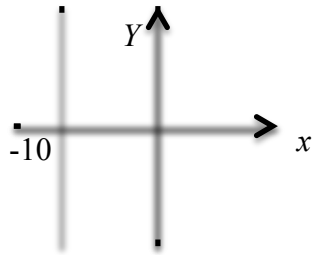
$\angle EDB = 23.7^0$

This is closest to 24^0

Module 4 – Graphs and relations

Question 1 E

The line passes through the point (-10, 0)



$x = -10$ is a vertical line and so the gradient is undefined.

Question 2 B

To find the point of intersection solve simultaneous equations on calculator for the given equations

$3x + y - 9 = 0$ and $x + 3y - 11 = 0$

This gives $x = 2$ and $y = 3$.

The sum of these values is 2 + 3 = 5

Question 3 E

Speed = Distance ÷ Time

For *P* Speed = 60 ÷ 2 = 30 km/hr.

For *Q* Speed = 40 ÷ 1 = 40 km/hr.

For *R* Speed = 0 ÷ 4 = 0 km/hr.

For *S* Speed = 20 ÷ 2 = 10 km/hr.

For *T* Speed = 80 ÷ 1 = 80 km/hr.

Question 4 A

$$\text{Rate of flow} = \frac{3-12}{15-9} = -1.5$$

The negative indicates that the water is decreasing at 1.5 litres per minute. A decrease of -1.5 would be a double negative.

Question 5 D

$$\text{Gradient of } BC = \frac{6-3}{12-0} = \frac{1}{4}$$

BC is parallel to *AD* (Opposite sides of a rectangle), so gradient of AD = $\frac{1}{4}$

$$\text{Equation of } AD: y = \frac{1}{4}x + c$$

$$\text{When } x = 2, y = 0$$

$$0 = \frac{1}{2} + c$$

$$c = -\frac{1}{2}$$

$$y = \frac{1}{4}x - \frac{1}{2}$$

$$4y = x - 2$$

$$4y - x + 2 = 0$$

Question 6 A

The dotted line is parallel to the line *BC*. *B* and *C* will be the last points that the dotted line will touch as you slide the line to the right. Hence, any point on the line segment *BC* will give the maximum for the objective function.

Module 4 – Graphs and relations

Question 7 C	**Question 8 A**
$y = kx^3$ $108 = k \times 27$ $k = 4$ When plotting y against x^3 all the y values will be 4 times the x^3 values and the graph will be linear.	Distance = speed × time = area under graph 1360 = Area of triangle + area of rectangle + area of triangle $1360 = (10 \times 80 \div 2) + (10 \times 80)$ $+ (20 - p) \times 40 \div 2$ $p = 12$ hrs.

End of Suggested Solutions 2017 VCE Further Mathematics Trial Examination 1

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