The Mathematical Association of Victoria

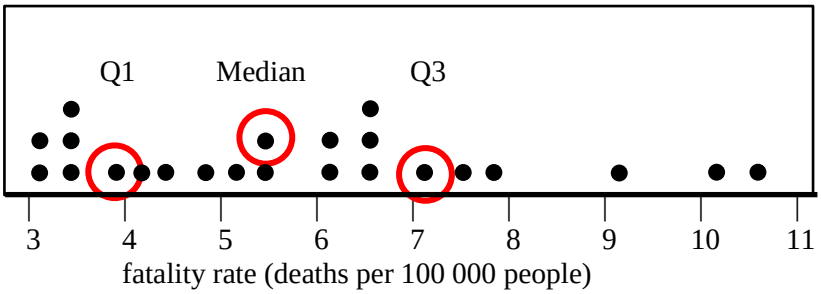
Trial Examination 2017

FURTHER MATHEMATICS

Trial Written Examination 2 - SOLUTIONS

SECTION A
Core – Data Analysis

Q1
(a)



Q1, Q3 A1

Median A1

(b)

Any four columns correct height, placement A1

Other three columns correct height, placement A1

Q2

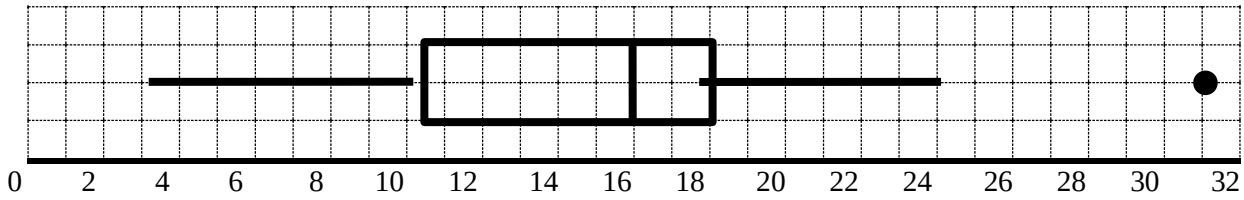
(a) From univariate analysis :

Min : 3.6 Q1 : 10.5 Median : 15.95 Q3 : 18.1 Max : 31.2

IQR = 18.1 – 10.5 = 7.6

Upper limit = Q3 + 1.5 × IQR = 18.1 + 1.5 × 7.6 = 29.5 A1

(b)



Min, Q1, Median, Q3 A1

Whisker end (24.5), outlier A1

(c) In the first quartile (whisker to the left), as the value (5.4) lies between 3.6 and 10.5. A1

Q3

(a) (Approximately) symmetrical. A1

(b) There is a negative association between average income and road fatality rates – i.e. as income rises, fatality rate falls. A1

The median for low income is approximately 25, whereas for medium income it is approximately 18 and for high income it is approximately 7. A1

(Note that **three (3)** values MUST be quoted)

Q4

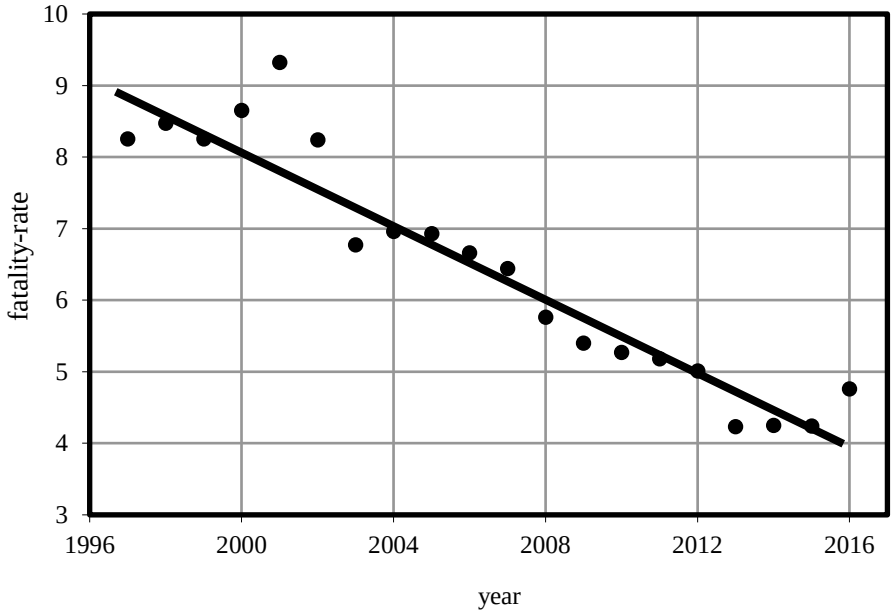
(a) Strong, negative, linear A1

(b) *Fatality rate* = $9.24 - 0.265 \times$ *year number* A1

(c) Correct calculations for two points MUST be shown

For example : *fatality rate* (1996) = $9.24 - 0.265 \times 0 = 9.24$

fatality rate (2016) = $9.24 - 0.265 \times 20 = 3.94$



correct line H1

(d) The *fatality rate* DECREASES by 0.265 for each 1 increase in the *year* value. A1

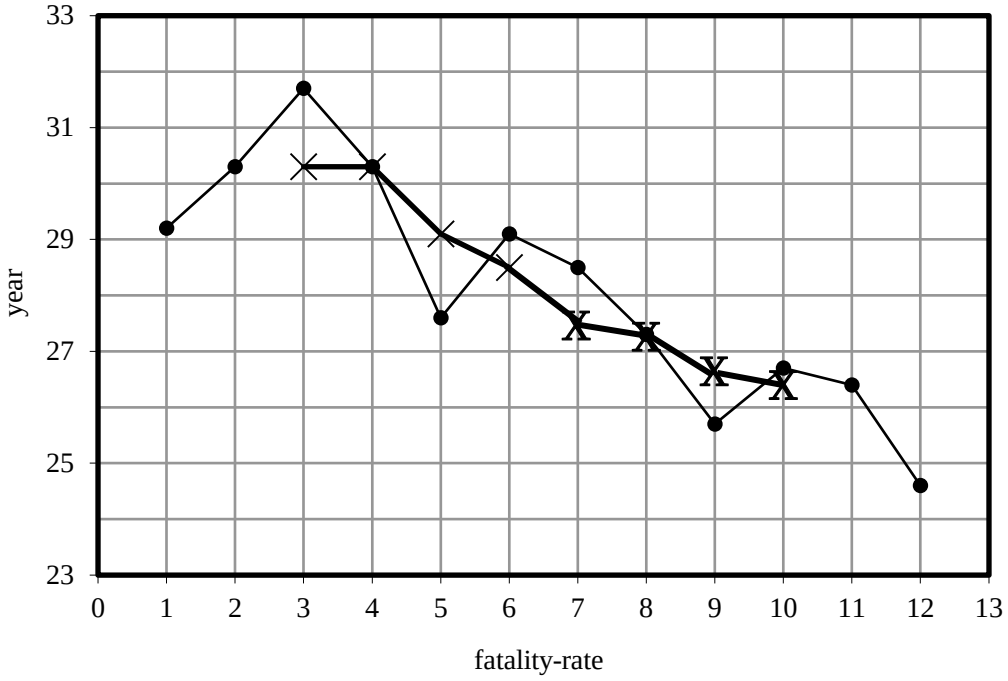
(e) *fatality rate* (1994) = $9.24 - 0.265 \times (-2) = 9.77$ A1

(f) Residual = actual – predicted = $8.45 - 9.77 = -1.32$ A1

(g) The actual *fatality rate* was LOWER than that predicted by the model. A1

(h) Prediction is an extrapolation (outside the range used to construct the regression equation). A1

Q5



ALL FOUR two points correctly placed A1

Q6

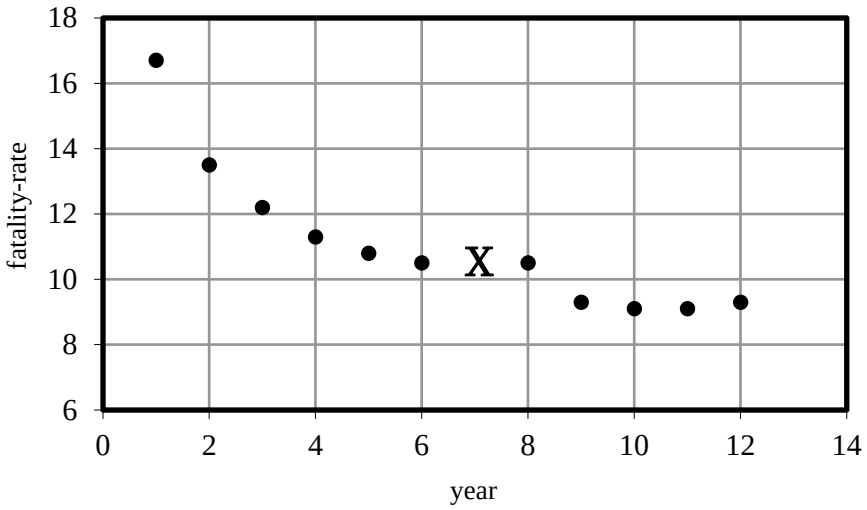
(a) $\frac{1}{\textit{fatality rate}} = 0.067 + 0.004 \times \textit{time}$ 0.067 A1

+ 0.004 A1

(b) $\frac{1}{\textit{fatality rate}} = 0.067 + 0.004 \times 7$

$\textit{fatality rate} = \frac{1}{0.067 + 0.004 \times 7} = 10.5$ A1

(c)



Correctly plot the value calculated in (b) H

Core – Recursion & Financial Modelling

Q7

(a) 3.5% A1

(b) H1 = $1.035 \times 22\,000$ = \$22 770
H2 = $1.035 \times 22\,770$ = \$23 566.95
OR H2 = $1.035 \times 1.035 \times 22\,000$ = \$23 566.95 A1

(c) $H_n = (\ldots 1.035 \ldots)^n \times (\ldots 22\,000 \ldots)$ A1

(d) SOLVE(30 000 = 1.035^N × 22 000,N) gives N = 9.0157… ≈ 10 years A1
OR Carefully repeated manual recursion

8	28 969.80	9	29 983.74	10	31 033.17

(e) Using Finance Solver
I% = 3.5, PV = – 22000, Pmt = 0, FV = 30000, PpY = 12, CpY = 12
Solve for N, N = 106.493… ≈ 107 months
OR
SOLVE(30 000 = (1+3.5/1200)^N 22 000,N) gives N = 106.493… ≈ 107 months
120 – 107 = 13 months sooner A1

(f) Using the calculator app/function
Effective interest = eff(3.5,12) = 3.556… ≈ 3.56% A1

Q8

(a) Interest-only payment = $\dfrac{4.2 \times 370000}{100 \times 12}$ = \$1295.00 A1

(b) Using Finance Solver
N = 276, I% = 4.2, PV = 370000, FV = 0, PpY = 12, CpY = 12
Solve for Pmt, Pmt = 2092.9119… ≈ \$2092.91 A1

(c) Using Finance Solver
N = 300, I% = 4.2, PV = 370000, FV = 0, PpY = 12, CpY = 12
Solve for Pmt, Pmt = 1994.0865… ≈ \$1994.09 A1

(d) With interest-only option, total repaid = \$1295.00 × 24 + \$2092.91 × 276 = \$608 723.16 A1
Repay loan plus interest, total repaid = \$1994.09 × 300 = \$598 227.00
Difference = \$608 723.16 – \$598 227.00 = \$10 496.16 ≈ \$10 500 A1

(e) The interest only period means that the balance remains at \$370 000 until the end of 2 years whereas paying some principal reduces the balance over this period. As compound interest is calculated on the current balance and this remains higher under the interest only arrangement, the overall interest is greater. A1

SECTION B : MODULES

Module 1 – Matrices

Q1

(a) 1×3 A1

(b) $R = C \times M = \begin{bmatrix} 1050 & 350 & 675 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 1050 \times 1 + 350 \times 4 + 675 \times 7 \end{bmatrix} = \begin{bmatrix} 7175 \end{bmatrix}.$

Full correct calculation shown M1
Correct answer A1

(c) $R = C \times M = \begin{bmatrix} 1050 & 350 & 675 \end{bmatrix} \times \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 9275 \end{bmatrix}.$ A1

Q2

(a) *A* v *C* and *B* v *D*. A1

(b) Barry's team (*B*) A1

(c) If Chris' team and Barry's team win their games, then

$$\text{D1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix},\ \text{D2} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ \text{D1} + \text{D2} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} = \begin{matrix} 3 & A \\ 5 & B \\ 4 & C \\ 2 & D \end{matrix}$$ A1

Q3

(a) $S_1 = \begin{bmatrix} 23 \\ 19 \\ 36 \\ 42 \end{bmatrix}$ A1

(b) $0.10 \times 20 + 0.10 \times 20 + 0.70 \times 40 + 0.10 \times 40 = 2 + 2 + 28 + 4 = 36$ A1

(c) Calculating $T^n S_0$ for large n, say $n = 30$ gives $\begin{bmatrix} 27 \\ 17 \\ 30 \\ 47 \end{bmatrix} \begin{matrix} B \\ G \\ R \\ Y \end{matrix}$ rounded to nearest integer, showing

Yellow will be the most popular colour. A1

Q4

$$P_{2017} = \begin{bmatrix} 0.55 & 0.05 & 0.05 & 0.05 \\ 0.15 & 0.65 & 0.10 & 0.05 \\ 0.10 & 0.15 & 0.65 & 0.20 \\ 0.20 & 0.15 & 0.20 & 0.70 \end{bmatrix} \times \begin{bmatrix} 140 \\ 160 \\ 160 \\ 200 \end{bmatrix} + \begin{bmatrix} 20 \\ 25 \\ 25 \\ 40 \end{bmatrix} = \begin{bmatrix} 123 \\ 176 \\ 207 \\ 264 \end{bmatrix}$$ A1

$$P_{2018} = \begin{bmatrix} 0.55 & 0.05 & 0.05 & 0.05 \\ 0.15 & 0.65 & 0.10 & 0.05 \\ 0.10 & 0.15 & 0.65 & 0.20 \\ 0.20 & 0.15 & 0.20 & 0.70 \end{bmatrix} \times \begin{bmatrix} 123 \\ 176 \\ 207 \\ 264 \end{bmatrix} + \begin{bmatrix} 20 \\ 25 \\ 25 \\ 40 \end{bmatrix} = \begin{bmatrix} 120 \\ 191.75 \\ 251.05 \\ 317.2 \end{bmatrix}$$

317 litres of Yellow paint will be required in 2018 A1

Module 2 – Networks and decision mathematics

Q1

(a) TGHSNRT, TRNSHGT (other answers possible) A1

(b) Hamiltonian Cycle A1

(c) He needs an Eulerian circuit or path and neither of these is possible in a network with more than 2 odd degree vertices. This network has four intersections of odd degree, H, R, N and T. A1

Q2

(a) The zeros in this matrix can be covered with only three lines, it will only be ready for allocation when the minimum number of lines to cover the zeros is equal to the number of people and tasks i.e. four. A1

(b)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} R & L & P & W \end{array} \\ \begin{bmatrix} 0 & 3 & 2 & 1 \\ 2 & 4 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \text{ OR } \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} R & L & P & W \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 \\ 3 & 4 & 2 & 0 \\ 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A1

(c) Alf registrations, Barbara roadworthiness certificates, Clarice licences and Donald learner's permits. A1

Q3

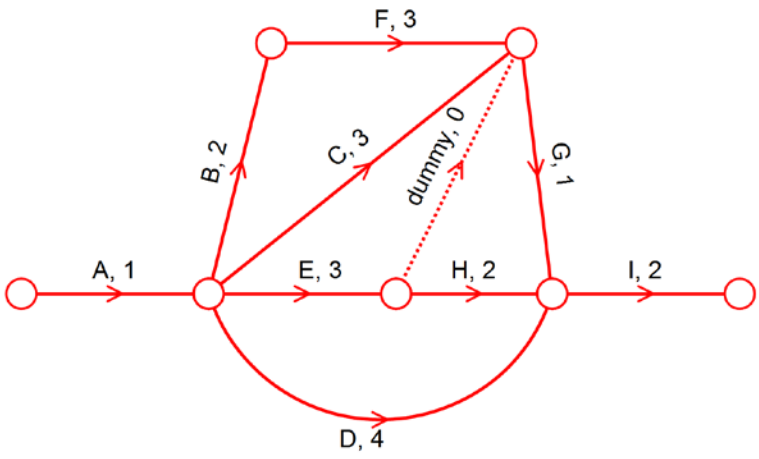
(a) C and F. A1

(b) 9 hours A1

(c) 2 hours A1

(d) 8 hours. New critical path is AEHI with a length of 8 hours. A1

(e)



A1

(f) C, G and D. A1

Module 3 – Geometry & Measurement

Q1

(a) Radius sphere = $120 \div 2$ = 60 mm = 6.0 cm

Volume = $\frac{4}{3} \times \pi \times 6.0^3$ = 904.778… ≈ 905 cm^3 A1

(b) Density = $\frac{7260}{905}$ = 8.0220… ≈ 8.02 g/cm^3 A1

(c) 8.00 = $\frac{4000}{\text{volume}}$, giving volume = $\frac{4000}{8}$ = 500 cm^3 A1

500 = $\frac{4}{3} \times \pi \times r^3$, giving r = $\sqrt[3]{\frac{3 \times 5000}{4 \times \pi}}$ = 4.923…

Diameter = 2×4.923 = 9.846 ≈ 9.8 cm A1

Q2

(a) Distance = $6400 \times 2 \times \pi \times \frac{(55 + 2)}{360}$ = 6366.9611… ≈ 6367 km A1

Note that the angles are added since one place is north of the equator and the other is south.

(b) Since Melbourne is to the east of Nairobi, its time will be AHEAD.
So, ADD seven hours to 8.30 pm to give 3.30 am Saturday morning. A1

(c) Radius small circle = 6400 cos 34° = 5305.84 ≈ 5306 km
Difference in longitude = 151° – 18° = 133° M1

Arc distance = $\frac{5306 \times \pi}{180} \times 133$ = 12 316.75… ≈ 12 320 km A1

Q3

(a) Michael, the official and the competitor form a right-angled triangle.
From the competitor, the angle between the official and Michael can be found from

$\cos\left(\frac{23.0}{30.0}\right)$ = 39.9445…° A1

The bearing of Michael is therefore 90° – 39.9445 = 50.0555 ≈ 050° A1

(b) Radius of circle = $2.135 \div 2$ = 1.0675 m
Use the cosine rule to find the angle at the centre, with the sides being
1.0675 m, 1.0675 m and 1.20 m (opposite the angle) M1

Angle *POQ* = $\cos^{-1}\left(\frac{1.0675^2 + 1.0675^2 - 1.20^2}{2 \times 1.0675 \times 1.0675}\right)$ = 68.396884… ≈ 68.40° A1

Module 4 – Graphs and Relations

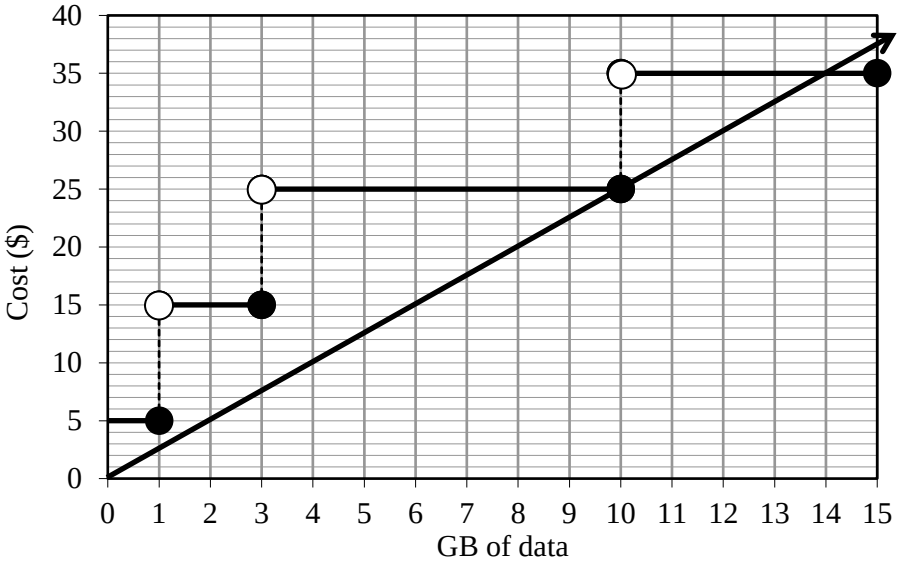
Q1

(a) \$25 A1

(b) $e = 25,\ f = 3,\ g = 10$ A1

(c) $C = 2.5d,\ d \geq 0$ A1

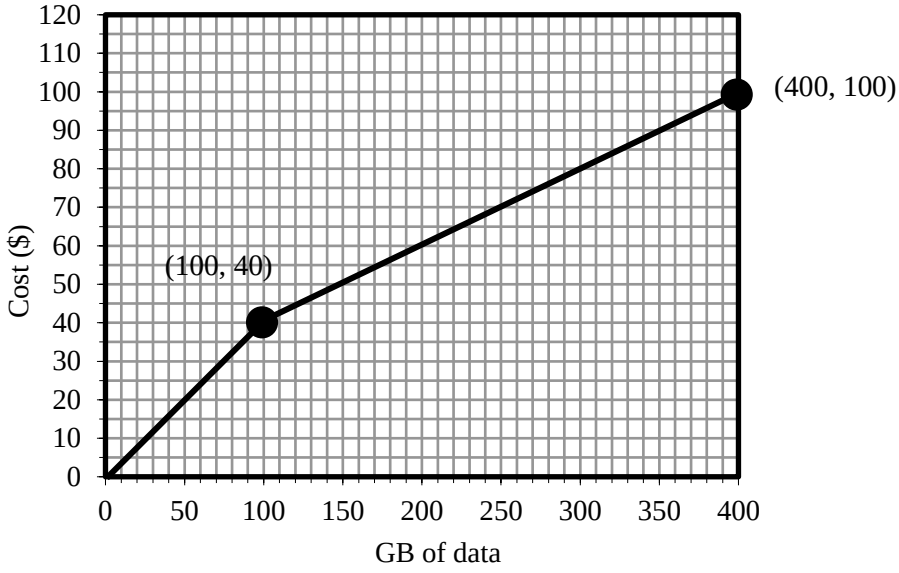
(d) Line passes through (0, 0) and (10, 25) A1



(e) It would be less expensive at any time unless Bill used exactly 10 GB or 14 GB of data at which times it would be the same or if Bill used more than 14 GB of data, at which time the step plan is cheaper. A1

Q2 Line to point (100, 40) A1

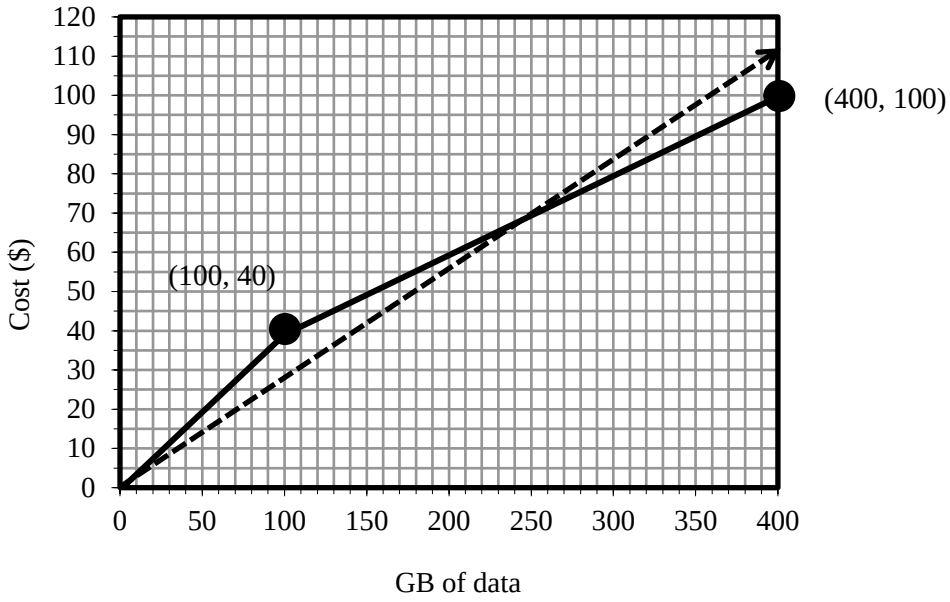
Line to point (400, 100) A1



(b) $j = 0.2,\ k = 20$ A1

(c) $P = \begin{cases} -0.12d, & 0 < d \leq 100 \\ 0.08d - 20 & 100 < d \leq 400 \end{cases}$ A1

(d) More than 250 GB. (Line for $R = 0.28d$ is shown dashed) A1



OR (Algebraically)
Revenue = Cost
$0.28d = 0.2d + 20$
$0.08d = 20$
$d = 20/0.08 = 250$

Q3 $-1 < \text{gradient} < -\frac{1}{2}$ lower limit (– 1) A1

upper limit $\left(-\frac{1}{2}\right)$ A1

END OF SOLUTIONS