Neap:

Trial Examination 2017

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 (8 marks)

a. **i.** 6 A1

ii. no mode A1

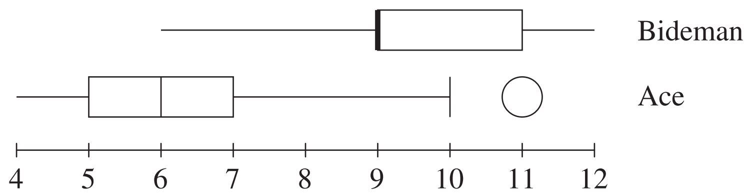
b. 6, 9, 9, 11, 12 A2

Note: 1 mark for a partially correct solution where at least two figures are correct.

c. There are 33 sales.

$Q_1 = 5$, $Q_3 = 7$, so the $IQR = 7 - 5 = 2$. A1

d.



accurate graph A1
correct scale A1

e. *For example:*

- All of the data for the Bideman sales is greater than or equal to the median of the Ace sales.
- Since Q_1, Q_2 and Q_3 are all greater for Bideman than Q_3 for Ace shoe sales, we can say that Bideman shoes are bought by more people requiring larger sizes.
- The IQR for both sets of data is 2.

A1

Note: 1 mark for an accurate comment using mathematical terminology.

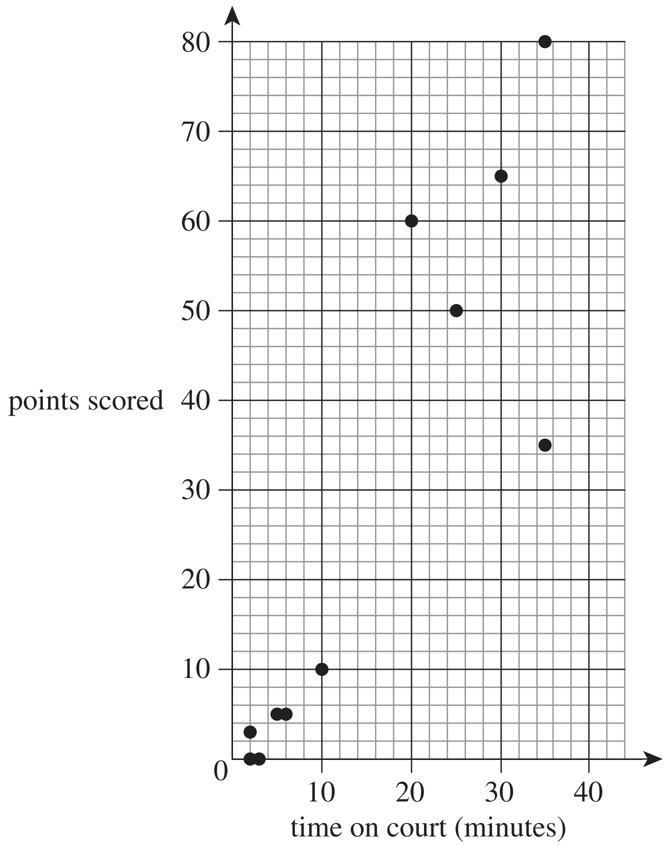
Question 2 (7 marks)

a. When considering creating a scatterplot for *time on court* and *total points*, the variable *total points* is the **response** variable because A1
the longer a player spends on court, the greater chance they have to score.
The *total points* depends upon the explanatory variable, *time on court*. A1

b. Students should enter the values for *total points* and *time on court* into their calculator and find the value of $r = 0.93$. A1

c. *time on court* = 0.43 × *total points* + 4.33 A1

d.

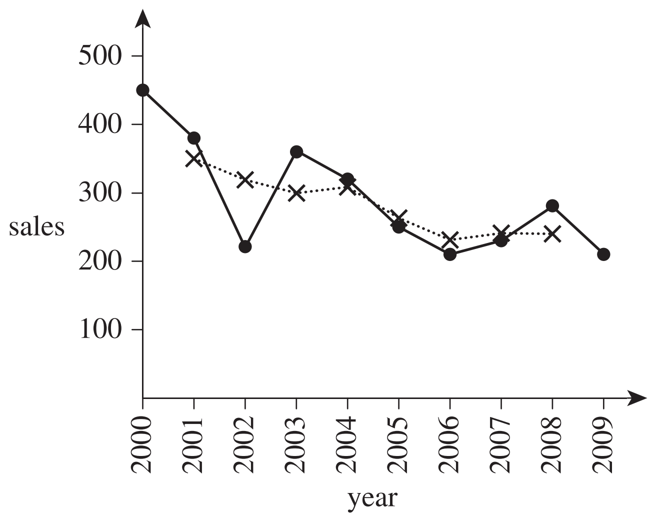


accuracy of plotting the points A1
correct scaling and labelling A1

e. Both the total points and personal fouls depend upon the time spent on court, so neither is the response variable. A1

Question 3 (9 marks)

a.



Year	01	02	03	04	05	06	07	08
Sales	350	320	300	310	260	230	240	240

correct calculation of points A1
accurate plotting A1

b. *For example:*

There is a negative linear trend. A1

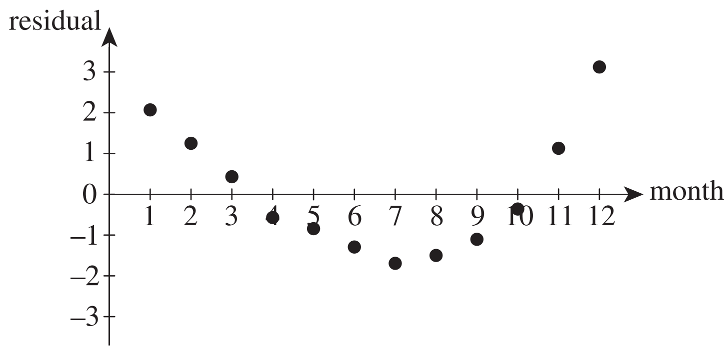
c. value = 1.24 × month – 2.02, $r = 0.95$ A1

Note: Both solutions are required for the mark.

d. Use your calculator to create a table of residuals and plot month versus residual.

OR

Calculate the residuals using your regression equation and plot the points. Note as we are only interested in the shape of the residual plot. it is only necessary to find the residuals approximately.



correct format of plot A1

accuracy of plot A1

e. There is a distinct pattern in the residuals so the assumption of linearity is not correct. A1

f. After applying an x^2 transformation, the value of r increases to 0.99, A1
which is an improved fit over the linear model. A1

Recursion and financial modelling

Question 4 (2 marks)

a. The value of every term is 2. A1

b. 3, 4, 7, 11, 18 A1

Question 5 (8 marks)

a. $P_{n+1} = P_n \times 1.1, P_1 = 9000$ A1

b. \$10 890 A1

c. Solving $9000 \times 1.1^n = 50\ 000$ gives $n = 17.99$, or 18 months. A1

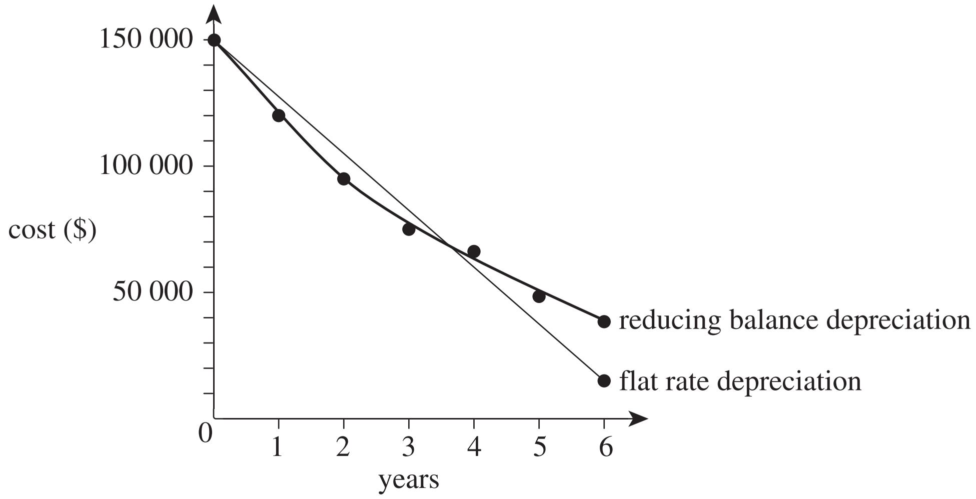
d. Method 1:

$P_v = 150\ 000 - 22\ 500n$ $(0.15 \times 150\ 000 = 22\ 500)$ A1

Method 2:

$P_v = 150\ 000 \times 0.8^n$ A1

e.



accurate graph A1
correct axes A1

f. The intersection of the graphs is at about 3.8 years. A1

Note: Accept solutions between 3.5 and 4.

Question 6 (2 marks)

A = $1250.00

B = $247 738.73

The interest rate per month is $\left(1+\frac{\frac{6}{12}}{100}\right) = 1.005$.

The new balance is found by *current balance* × 1.005 – 2000.

Date	Deposit	Withdrawal	Interest	Balance
January 1	$250 000			$250 000
January 31		$2000	**$1250**	**$249 250**
February 28		$2000	**$1246.25**	**$248 496.25**
April 31		$2000	**$1242.48**	**$247 738.73**

correct value A A1
correct value B A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (5 marks)

a. $$T = \begin{array}{c} \begin{matrix} B & I & A & D \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0.40 & 0 \\ 0.80 & 0 & 0.00 & 0 \\ 0 & 0.90 & 0.85 & 0 \\ 0.20 & 0.10 & 0.15 & 1.00 \end{bmatrix} \end{array} \begin{matrix} B \\ I \\ A \\ D \end{matrix} \quad S_0 = \begin{bmatrix} 2000 \\ 1500 \\ 3000 \\ 0 \end{bmatrix}$$ A1

If 20% of baby koalas die each year, 80% remain to become immature. This needs to appear in the baby koalas column as this was what they **were** before the transition and in the immature koalas row as this is what they **became**. $T_{33} = 0.85$ for the same reason. 15% of adult koalas dying means that 85% remain as adults. A1

No immature koalas become baby koalas and so $T_{12} = 0$.

Clearly all dead koalas will remain dead and so $T_{34} = 0$.

b. Students just need to multiply.

$$S_1 = \begin{bmatrix} 0 & 0 & 0.45 & 0 \\ 0.75 & 0 & 0.0 & 0 \\ 0 & 0.85 & 0.80 & 0 \\ 0.25 & 0.15 & 0.20 & 1.00 \end{bmatrix} \begin{bmatrix} 2000 \\ 1500 \\ 3000 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1350 \\ 1500 \\ 3675 \\ 1325 \end{bmatrix}$$

Thus we expect 1350 baby koalas, 1500 immature koalas and 3675 adult koalas. We are not asked about dead koalas. A1

c. $$T = \begin{bmatrix} 0 & 0 & 0.45 & 0 \\ 0.75 & 0 & 0.0 & 0 \\ 0 & 0.85 & 0.80 & 0 \\ 0.25 & 0.15 & 0.20 & 1.00 \end{bmatrix}^{10} \begin{bmatrix} 2000 \\ 1500 \\ 3000 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2664.49 \\ 1890 \\ 6255.17 \\ 16\,463 \end{bmatrix}$$

Thus there are 2664 baby koalas, 1890 immature koalas and 6255 adult koalas now. A1

Thus the koalas have not died out but have become unsustainable in that time as total population is now 10 807 koalas. A1

Question 2 (5 marks)

a. $$B = \begin{array}{c} \begin{matrix} S & I & C & P & H \end{matrix} \\ \begin{bmatrix} 0 & 2 & 0 & 1 & 2 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} \\ S \\ I \\ C \\ P \\ H \end{matrix}$$ A2

b. The matrix B lists the number of direct connections between each of the five islands.

The matrix B^2 thus lists the indirect connections that exist between corresponding islands using exactly two boats. A1

c. Students need to find matrix B^2.

$$B^2 = \begin{bmatrix} 0 & 2 & 0 & 1 & 2 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \end{bmatrix}^2$$

$$= \begin{bmatrix} 9 & 1 & 3 & 4 & 1 \\ 1 & 5 & 1 & 2 & 5 \\ 3 & 1 & 2 & 1 & 1 \\ 4 & 2 & 1 & 4 & 3 \\ 1 & 5 & 1 & 3 & 6 \end{bmatrix}$$ A1

We are concerned with column 1 (corresponding to Spanish Island) and row 4 (Portuguese Island).

The value in this position is 4, meaning there are four ways to travel between these islands using two boat rides. We already know that there is one way to travel between these islands by a single boat ride. Thus there is a total of five ways. A1

Question 3 (2 marks)

There are possibly two issues with the information as presented. The table is presented as a row whereas it would normally be a column; also, the matrix is presented in a format inconsistent with multiplying by a column vector. Thus we can either rearrange both and multiply, or premultiply the existing matrix by the row vector.

a. $$\begin{bmatrix} 2.0 & 1.6 & 1.2 & 1.5 & 2.0 \end{bmatrix} \begin{bmatrix} 112 & 96 & 102 & 118 & 145 \\ 43 & 41 & 45 & 60 & 72 \\ 25 & 27 & 29 & 41 & 50 \\ 61 & 60 & 58 & 68 & 83 \\ 58 & 50 & 51 & 78 & 109 \end{bmatrix} = \begin{bmatrix} 530.3 & 480.0 & 499.8 & 639.2 & 807.7 \end{bmatrix}$$ A1

b.

Monday	\$530.30
Tuesday	\$480
Wednesday	\$499.80
Thursday	\$639.20
Friday	\$807.70

Alternatively, the calculation could be:

$$\begin{bmatrix} 112 & 43 & 25 & 61 & 58 \\ 96 & 41 & 27 & 60 & 50 \\ 102 & 45 & 29 & 58 & 51 \\ 118 & 60 & 41 & 68 & 78 \\ 145 & 72 & 50 & 83 & 109 \end{bmatrix} \begin{bmatrix} 2.0 \\ 1.6 \\ 1.2 \\ 1.5 \\ 2.0 \end{bmatrix} = \begin{bmatrix} 530.3 \\ 480.0 \\ 499.8 \\ 639.2 \\ 807.7 \end{bmatrix}$$

total takings = 530.3 + 480 + 499.8 + 639.20 + 807.70

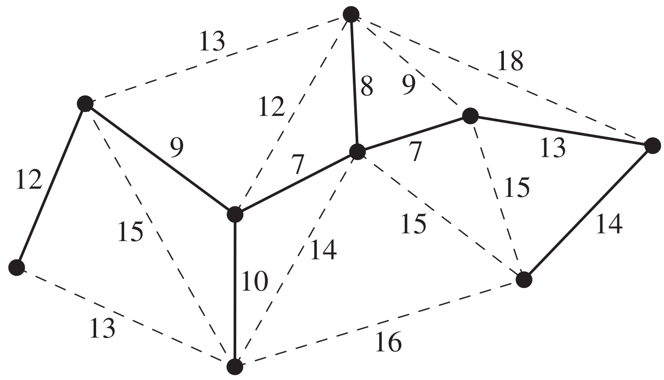
= \$2956 A1

 FMU34EX2_SS_2017.FM

Module 2 – Networks and decision mathematics

Question 1 (5 marks)

a.

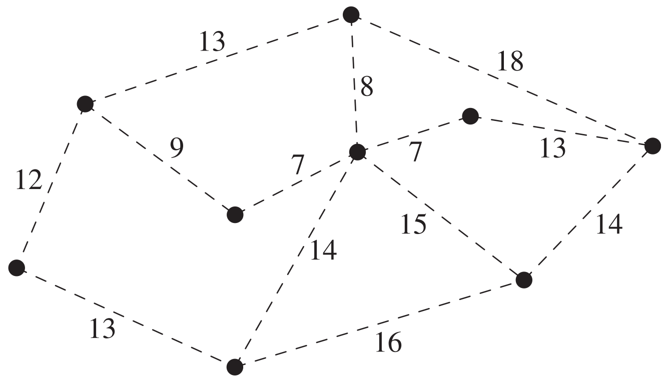


A1

b. The network shown on the diagram in **part a.** can be referred to as the **minimal spanning tree** of the original network. A1

c. 12 + 9 + 10 + 7 + 8 + 7 + 13 + 14 = 80 km A1

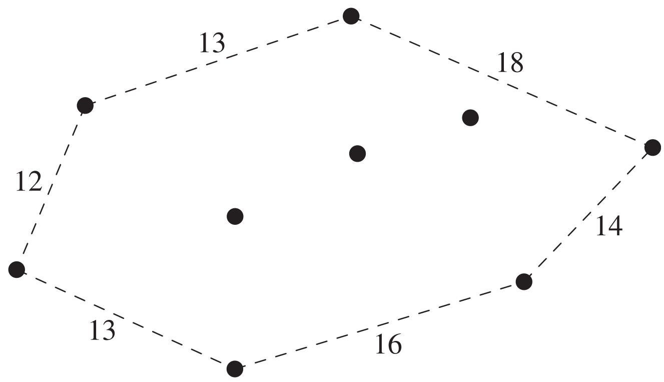
d. After a few edges have been eliminated using Dijkstra's algorithm, we have the diagram shown below.



M1

Note: Students must show method. An interim diagram is sufficient.

From this point, it is easy to remove any other unnecessary edges to obtain the final solution.



There are, in fact, two equally good routes that may be taken. In both cases, the total distance is 43 km. A1

Question 2 (4 marks)

a. If this subtraction step was omitted, the process would minimise the total energy instead of maximising it. By subtracting the figures from 10, the order is reversed and this is exactly what is required.

A1

Note: The first sentence would suffice for the 1 mark.

b. After row operations:

$$\begin{bmatrix} 0.0 & 0.5 & 1.2 & 2.3 \\ 0.0 & 0.2 & 1.9 & 2.7 \\ 0.0 & 0.4 & 1.4 & 2.6 \\ 0.0 & 1.1 & 1.6 & 3.1 \end{bmatrix}$$

After columns:

$$\begin{bmatrix} 0.0 & 0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.4 \\ 0.0 & 0.2 & 0.2 & 0.3 \\ 0.0 & 1.8 & 0.4 & 0.8 \end{bmatrix}$$

A1

c. Place lines to cover the zeros.

$$\begin{bmatrix} 0.0 & 0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.4 \\ 0.0 & 0.2 & 0.2 & 0.3 \\ 0.0 & 1.8 & 0.4 & 0.8 \end{bmatrix}$$

We add the lowest uncovered value to all covered values. There are two values that are covered twice and these will have the lowest value added twice.

$$\begin{bmatrix} 0.4 & 0.5 & 0.2 & 0.2 \\ 0.4 & 0.2 & 0.9 & 0.6 \\ 0.2 & 0.2 & 0.2 & 0.3 \\ 0.2 & 1.8 & 0.4 & 0.8 \end{bmatrix}$$

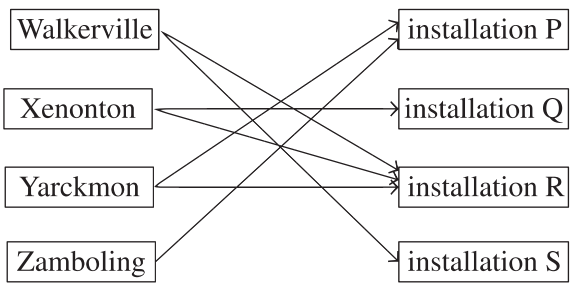
After row and column subtractions:

$$\begin{bmatrix} 0.2 & 0.3 & 0.0 & 0.0 \\ 0.2 & 0.0 & 0.7 & 0.4 \\ 0.0 & 0.0 & 0.0 & 0.1 \\ 0.0 & 0.6 & 0.2 & 0.6 \end{bmatrix}$$ M1

This requires four lines to cover all zeros.

$$\begin{bmatrix} 0.2 & 0.3 & 0.0 & 0.0 \\ 0.2 & 0.0 & 0.7 & 0.4 \\ 0.0 & 0.0 & 0.0 & 0.1 \\ 0.0 & 0.6 & 0.2 & 0.6 \end{bmatrix}$$

Thus we can allocate.



Installation Q must go to Xenonton as nowhere else matches it.

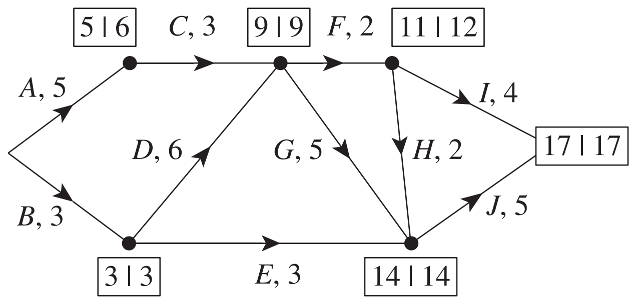
Walkerville must get installation S as nowhere else matches it above.

Zamboling gets installation P as it is the only installation that it is matched with.

That leaves Yarckmon getting installation R. A1

Question 3 (3 marks)

The earliest and latest completion times have been calculated and appear below.



a. As can be seen above, the earliest completion time is 17 days. A1

b. The critical path links the activities with earliest and latest times. Thus it is *BDFHJ*. A1

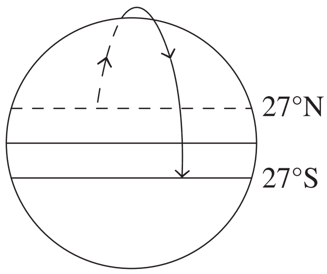
c. Look at non-critical activities. *A* can extend one day, as can *C* (but not both).
G can extend one day also. *I* can extend two days.
E can actually extend seven days, however. It is activity *E* that can extend the most. A1

Module 3 – Geometry and measurement

Question 1 (2 marks)

The key point is that Easter Island and the Thar Desert are directly opposite each other on the Earth. We can tell this from the fact that their latitudes are equally distant from the equator (27° south and north respectively for Easter Island and the Thar Desert and they are 180° of longitude apart.

Thus the Thar Desert and Easter Island are on the same longitude line if it is continued over the North Pole, as shown in the diagram below.



Thus the distance is half the circumference of the Earth. A1

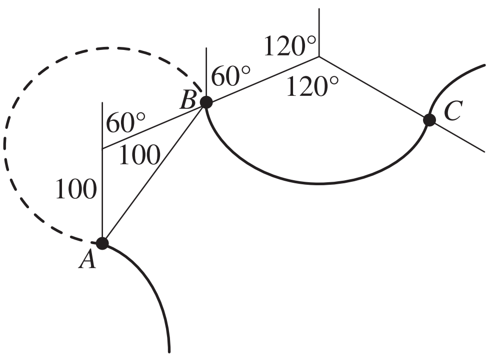
$\frac{1}{2} \times 2\pi \times 6400 = 20\ 106$ km A1

Question 2 (10 marks)

a. $\text{length} = \frac{240}{360} \times 2\pi \times 100$

$= 419$ m A1

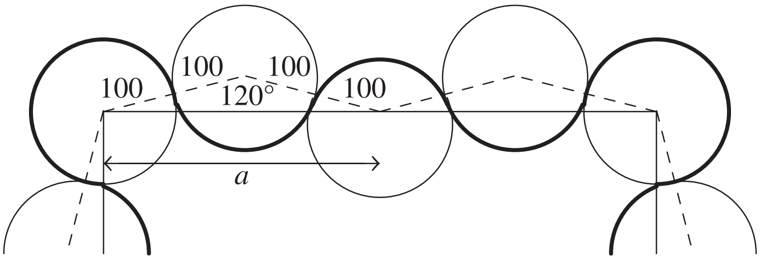
b. To find the bearings we will need to draw a new diagram with the relevant information present.



The angle at the centre of the circle with A on its circumference must be (180 – 60 =) 120°. Since the sum of all triangle angles must be 180°, the angle in the triangle at A must be 30° by symmetry.

That gives is a bearing of 30°T (or N30°E) from A to B. A1

c. We will need to determine the side length of the square.



M1

Using cosine rule:

$$a^2 = 200^2 + 200^2 - 2 \times 200 \times 200\cos(120°)$$
$$= 120\,000$$
$$\therefore a = 346.4 \text{ km}$$ A1

The side length of the square is thus 2 × 346.4 = 692.8 km.

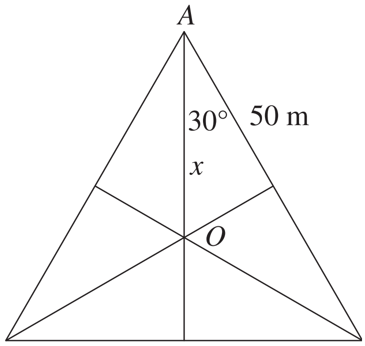
Therefore area must be $692.8^2 = 480\,000 \text{ m}^2$. A1

d. $$\text{area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2} \times 100 \times 100\sin(60°)$$
$$= 4330 \text{ m}^2$$ A1

e. $\cos(30°) = \frac{50}{x}$

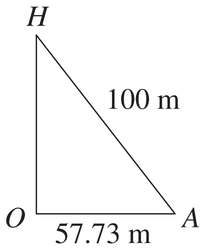
$$x = \frac{50}{\cos(30°)}$$
$$= 57.73 \text{ m}$$

Now construct a vertical axis triangle (shown below).



Let H be the top of the tetrahedron.

We know that AH is 100 as it is the side of one of the equilateral triangles.



The height OH can now be found using Pythagoras' theorem.

$$OH = \sqrt{100^2 - 57.73^2}$$
$$= 81.65$$ A1

We now have the height, so we also need to use the area of the base to find the volume.

$$V = \frac{1}{3} \times 4330 \times 81.65$$
$$= 117\,847 \text{ m}^3$$ A1

f. Surface area ratio will be 1 : 1000^2 = 1 : 10^6.

Volume ratio will be 1 : 1000^3 = 1 : 10^9. M1

$$\text{thus surface area} = 4 \times \frac{4330}{1\,000\,000}$$
$$= 1.732 \times 10^{-2} \text{ m}^2$$
$$= 173.2 \text{ cm}^2$$

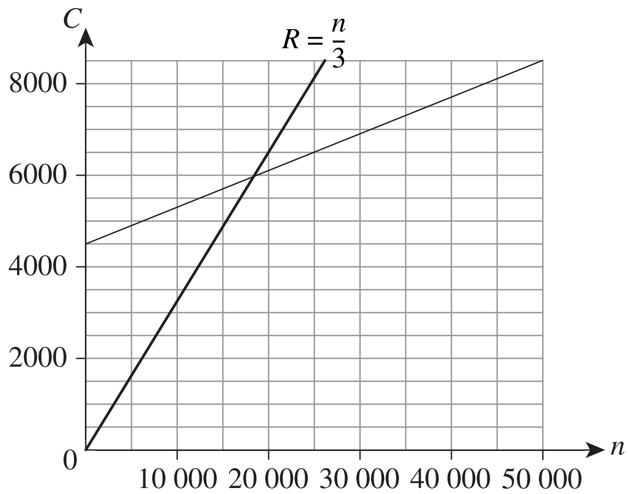
$$\text{volume} = 117\,847 \times 10^{-9}$$
$$= 1.17847 \times 10^{-4} \text{ m}^3$$
$$= 117.847 \text{ cm}^3$$ A1

Module 4 – Graphs and relations

Question 1 (4 marks)

a. $C = 4500 + 0.08n$ A1

b. **i. ii.**



correct graph A1
correct equation A1

c. From the graph, the lines meet at approximately (18 000, 60 000).

Thus the company breaks even when 18 000 mini-pizzas are made and sold. A1

Question 2 (8 marks)

a. $2x + 5y \leq 30$ A1

b. $x + y = 10$ meets $2x + 5y = 30$.

Multiply first equation by 2: $2x + 2y = 20$

Subtract: $3y = 10$

$B(6.67, 3.33)$ to two decimal places A1

Note: Students can obtain this point from the equations and thus two decimal place accuracy is required.

c. The equation consists of two terms. The first must give the total profit from standard parties ($120x$) and the second term, the total profit from deluxe parties ($235y$). We must remember that each of these total profits are the individual profits per party multiplied by the number of such parties. Thus, from the equation, we see that each standard party must make a profit of \$120 and that each deluxe party must make a profit of \$235. A1

d. We have seen that point B does not have integer coordinates. Thus we must find the nearest points within the feasible region that do have integer coordinates.

(0, 6), (10, 0), (6, 3) A1

(7, 3), (5, 4), (2, 5) A1

 FMU34EX2_SS_2017.FM

e. At (7, 3), $P = 120 \times 7 + 235 \times 3$

$= 1545$

At (5, 4), $P = 120 \times 5 + 235 \times 4$

$= 1540$

At (0, 6), $P = 1410$

At (10, 0), $P = 1200$

At (6, 3), $P = 1425$

At (2, 5), $P = 1415$ M1

The maximum profit is \$1545 using 7 standard and 3 deluxe packages. A1

f. At B, $P = 120 \times 6.67 + 235 \times 3.33$

$= 1582.95$

Thus they overestimated 1582.95 – 1545 = \$37.95 per week from this error. A1

