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FURTHER MATHEMATICS TRIAL EXAMINATION 1 SOLUTIONS 2018

SECTION A (answers)

Core

1. E	**13.** A
2. D	**14.** C
3. E	**15.** C
4. D	**16.** E
5. E	**17.** D
6. A	**18.** B
7. B	**19.** C
8. C	**20.** C
9. C	**21.** C
10. A	**22.** B
11. C	**23.** B
12. B	**24.** A

SECTION B (answers)

Module 1 Matrices	**Module 2 Networks & decision maths**	**Module 3 Geometry & measurement**	**Module 4 Graphs & relations**
1. D	**1.** E	**1.** A	**1.** A
2. C	**2.** C	**2.** C	**2.** E
3. A	**3.** A	**3.** B	**3.** D
4. B	**4.** B	**4.** B	**4.** D
5. E	**5.** D	**5.** A	**5.** C
6. D	**6.** D	**6.** D	**6.** D
7. B	**7.** E	**7.** C	**7.** B
8. E	**8.** E	**8.** A	**8.** C

SECTION A – Core - solutions

Data analysis

Question 1

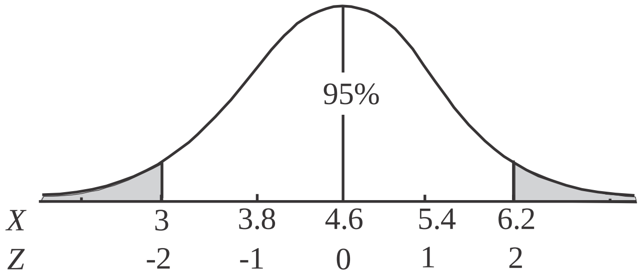
$$\text{range} = 690 - 330$$
$$= 360$$

The answer is E.

Question 2

Enter the data into your CAS and using $1-Var$ stats we see that $\bar{x} = 1.6$ and $s_x = 0.46291\ldots$
The closest answer is $s_x = 0.46$.
Note that $\sigma_x = 0.4330...$ is the population standard deviation not the sample standard deviation.
The answer is D.

Question 3



The mean is 4.6 and the standard deviation is 0.8 so $3 = 4.6 - 2 \times 0.8$ and $6.2 = 4.6 + 2 \times 0.8$.
We are looking at the proportion of patrons whose waiting times lie between 2 standard deviations either side of the mean. Since 95% will lie between 2 standard deviations either side of the mean then 95% of 28 000 equals 26 600.
The answer is E.

Question 4

The middle value or median, is the 12th value from either end which will lie between $\log_{10}(2)$ and $\log_{10}(3)$.

i.e. $2 < \log_{10}(\textit{grape production}) < 3$

so $10^2 < \textit{grape production} < 10^3$

i.e. $100 < \textit{grape production} < 1000$

The answer is D.

Question 5

The variable *destination* is a categorical variable. The variable *weight* is not a numerical variable in this context, i.e. it gives categories of weights rather than just ordinary weights. We can reject option A.
The variable *destination* is a nominal variable because it names the destination where the package must be sent but there is no order in these destinations. We can reject options B and C.
The variable *weight* is an ordinal variable because there is an order (smaller to larger) of the categories of weights given. We can reject option D.
The answer is E.

Question 6

In order to suggest that there is an association between the two variables *weight* and *destination* it needs to be shown that there is a difference between two different destinations in the same category of weight.
For example, 30% of packages with a CBD destination weighed $10 - 19.9$ kg and 50% of packages with a regional destination weighed $10 - 19.9$ kg.
The answer is A.

Question 7

A person's weight, measured in kilograms, is a numerical variable.
A person's place of residence (city or regional) is a categorical variable.
A scatterplot is used to display two numerical variables so reject option A.
A parallel box plot is used to display an association between a numerical and a categorical variable so it would be appropriate.
Note that an ordered stem plot, a dot plot and a histogram are all used to display just one variable which is numerical.
The answer is B.

Question 8
We focus on the middle column of the table which shows the female siblings who took two attempts to obtain their licence.
There are $8+10+5=23$ of these female siblings.
Of these, 5 had a male sibling who took three attempts.
So $\left(\frac{5}{23}\times\frac{100}{1}\right)\%=21.73...\%$.
The closest answer is 22%.
The answer is C.

Question 9
We need to find the equation of the least squares line.
Choose any two points on the line given on the scatterplot, for example
$(20,18.5)$ and $(35,29)$.

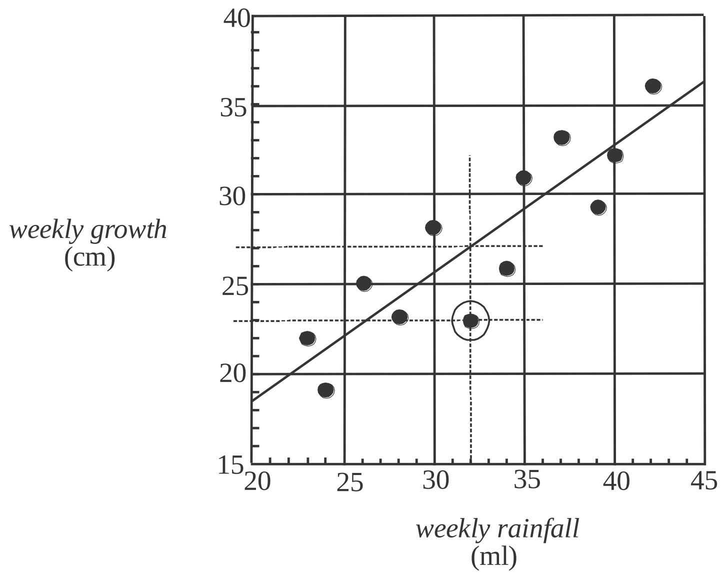
$$\begin{aligned}\text{slope} &= \frac{29-18.5}{35-20}\\ &= 0.7\end{aligned}$$

Equation of a straight line given by $y-y_1=m(x-x_1)$.
Using $m=0.7$ and $(35,29)$ we have

$$\begin{aligned}y-29 &= 0.7(x-35)\\ y &= 4.5+0.7x\end{aligned}$$

The intercept is closest to 4.5 so when weekly rainfall (the x –variable) is zero, the weekly growth (the y-variable), will be 4.5cm.
The answer is C.

Question 10
The data point representing the crop with a location which has weekly rainfall of 32 ml, is circled. The weekly growth for that crop is approximately 23cm.



The least squares line predicts that when weekly rainfall is 32ml, weekly growth will be approximately 27cm.

$$\begin{aligned}\text{residual value} &= \text{actual value} - \text{predicted value} \qquad \text{(formula sheet)}\\ &= 23-27\\ &= -4\end{aligned}$$

The answer is A.

Question 11

The correct conclusion is that office workers who tend to spend more time at a stand up desk each day tend to weigh less.
The answer is C.

Question 12

We are told that for a 90 year old patient,

$$\frac{1}{weight} = 0.016$$
$$1 = 0.016 \times weight$$
$$weight = \frac{1}{0.016}$$
$$= 62.5$$

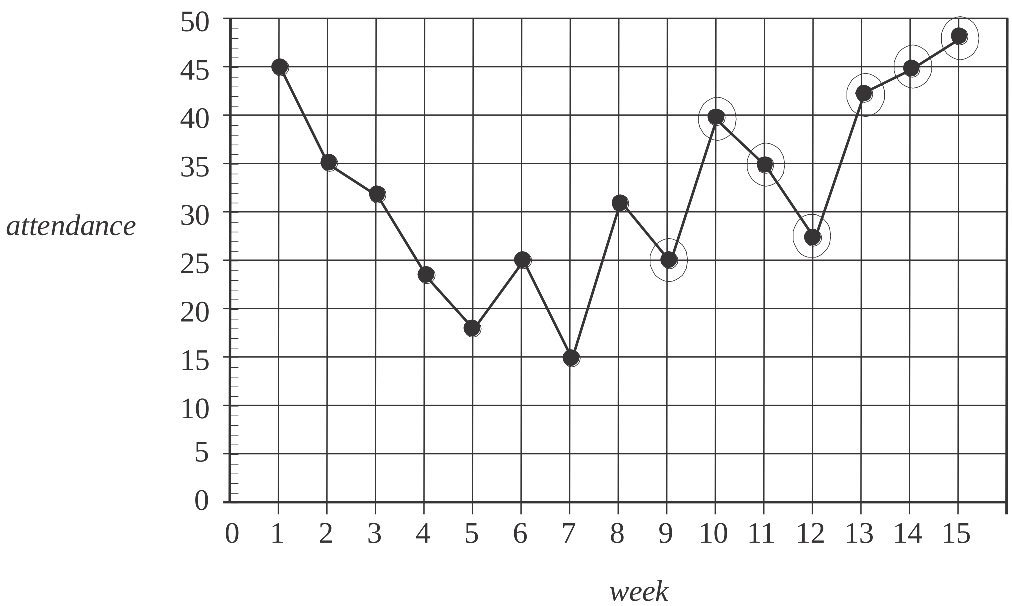
So the weight of a 90 year old patient is expected to be 62.5 kg.
The answer is B.

Question 13

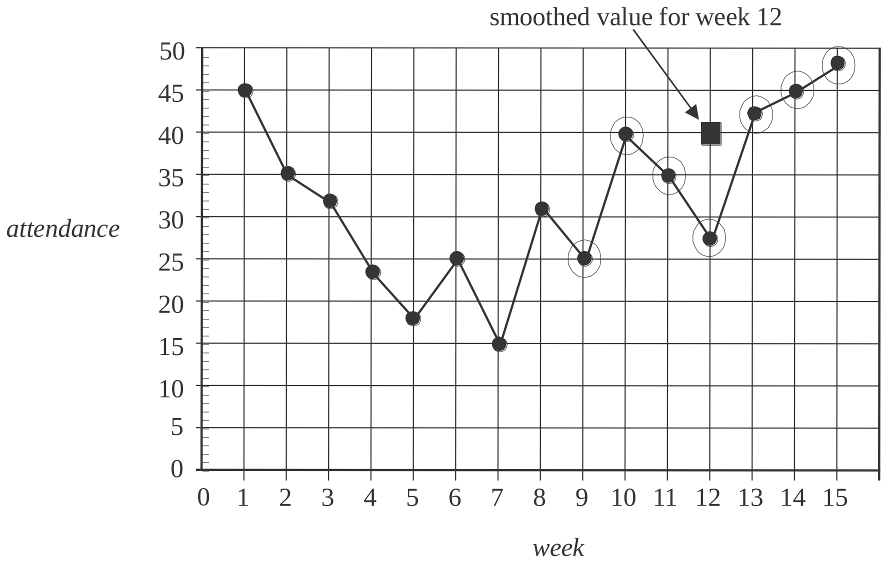
Enter the variables *year* and *number of staff* into your CAS. Now add an extra column which gives $\log_{10}(year)$. This variable, $\log_{10}(year)$ is the explanatory variable (or *x*-variable) and *number of staff* is the response variable (or *y*-variable).
We know this because we are told in the question that the *number of staff* can be predicted from $\log_{10}(year)$.
So $y = a + bx$ becomes $number\ of\ staff = 5.204... + 36.181... \times \log_{10}(year)$.
The closest equation to this is $number\ of\ staff = 5.2 + 36 \times \log_{10}(year)$.
The answer is A.

Question 14

Looking at the graph, note the position of the data points for weeks 9, 10, 11 (i.e. the three points to the left of the week 12 data point), and the data points for weeks 13, 14 and 15 (i.e. the three points to the right of the week 12 data point).



Now forget about where the 7 data points are positioned horizontally. Just focus on where they are positioned vertically. Three are below 40 (the data value for week 10) and three are above 40. So the smoothed attendance for week 12 is 40 as shown on the graph below.



The answer is C.

Question 15

$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}} \qquad \text{(formula sheet)}$$

$$\begin{aligned}\text{So deseasonalised figure} &= \frac{\text{actual figure}}{\text{seasonal index}}\\ &= \frac{\text{actual figure}}{0.8}\\ &= 1.25 \times \text{actual figure}\\ &\text{(i.e. } 1 \div 0.8 = 1.25)\end{aligned}$$

So to obtain the deseasonalised sales for winter, the actual sales figure needs to be increased by 25%,
The answer is C.

Question 16

The daily seasonal index for Thursday is equal to
$7-(0.95+0.87+0.73+1.24+1.17+1.21)=0.83$.

$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}} \quad \text{(formula sheet)}$$

$$0.83 = \frac{3\,820}{\text{deseasonalised figure}}$$

$$\begin{aligned}\text{So deseasonalised figure} &= \frac{3\,820}{0.83}\\ &= 4\,602.41\end{aligned}$$

The closest answer is $4 602.
The answer is E.

Recursion and financial modelling

Question 17

Use trial and error.
For option A, the sequence is $-1, 0, 1, ...$
For option B, the sequence is $-1, 0, 2, ...$
For option C, the sequence is $-1, 0, 3, ...$
For option D, the sequence is $-1, 0, 4, 20, 84, ...$
The answer is D.

Question 18

The depreciation of the coffee machine in the 3 years is $\$42\,500 - \$9\,600 = \$32\,900$.
Over this 3 year period it produces $3 \times 90\,000 = 270\,000$ servings of coffee.
So $\frac{32\,900}{270\,000} = 0.121851...$
The closest answer is 12 cents.
The answer is B.

Question 19

Annual interest is 4.6% so quarterly interest is

$$\begin{aligned} 4.6\% \div 4 &= 1.15\% \\ &= \frac{1.15}{100} \\ &= 0.0115 \end{aligned}$$

The recurrence relation is $V_0 = 12\,000, \quad V_{n+1} = 1.0115 V_n$.
The answer is C.

Question 20

Because this is an interest only loan, none of the principal borrowed will be paid back during the five years, only interest will be paid. So the amount owing is $120 000.
The answer is C.

Question 21

$$\begin{aligned} r_{\text{effective}} &= \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\% \qquad \text{(formula sheet)} \\ &= \left[\left(1 + \frac{7.8}{100 \times 2}\right)^2 - 1\right] \times 100\% \\ &= 7.9521 \end{aligned}$$

The closest answer is 7.95%.
The answer is C.

Question 22

We need to find the interest rate per period being used.
For payment number 8, interest is \$348.64 so,

$$\begin{aligned} 348.64 &= \frac{x}{100} \times 6\,972.79 \\ x &= \frac{348.64 \times 100}{6\,972.79} \\ &= 5.0000... \end{aligned}$$

So interest rate per period is 5%.
(Double check this for payment number 9,

i.e. $\frac{5}{100} \times 4\,721.43$

$= 236.07$ which is correct).

For the final payment

$$\begin{aligned} \text{interest} &= \frac{5}{100} \times 2\,357.50 \\ &= 117.88 \end{aligned}$$

So the final payment needs to be $2\,357.50 + 117.88 = 2\,475.38$.
The closest answer is \$2 475.
The answer is B.

Question 23

For option A, the truck would depreciate by 8% of \$12 000 i.e. \$960 each year.
Its value for year 1 would be $\$12\,000 - \$960 = \$11\,040$. Reject option A.

For option B, use finance solver.

$\text{N}: 1$

$\text{I(\%)}: 8$

$\text{PV}: -12\,000$

$\text{Pmt}: 2\,000$

$\text{FV}: ?$

$\text{PpY}: 1$

$\text{CpY}: 1$

$\text{FV} = 10\,960$

Check when $\text{N} = 1$, $\text{FV} = 10\,960$

Check when $\text{N} = 2$, $\text{FV} = 9836$

Check when $\text{N} = 3$, $\text{FV} = 8\,623$

Check when $\text{N} = 4$, $\text{FV} = 7\,313$

Check when $\text{N} = 5$, $\text{FV} = 5\,898$

The answer is B.

Question 24

We work backwards in this question.
Starting with the final 8 years of the loan and using finance solver, we have

$\text{N}: 96$

$\text{I(\%)}: 5.4$

$\text{PV}: ?$

$\text{Pmt}: -2\,900$

$\text{FV}: 0$

$\text{PpY}: 12$

$\text{CpY}: 12$

$\text{PV} = 225\,659.42$

The amount still owing on the loan at the start of the final 8 years (i.e. 96 months) is $225 659.42.

For the first 7 years, we have

$N: 84$

$\text{I(\%)}: 5.15$

$\text{PV}: 360\,000$

$\text{Pmt}: ?$

$\text{FV}: -225\,659.42$

$\text{PpY}: 12$

$\text{CpY}: 12$

$\text{Pmt}: -2\,876.70$

Note that FV is negative here because after 7 years, Julian still owes the bank $225 659.42
During the first 7 years of the loan, Julian's repayments were $ 2876.70.
The closest answer is $2 877.
The answer is A.

SECTION B - Modules

Module 1 – Matrices

Question 1

A permutation matrix is a square, binary matrix so we can eliminate option A which is not square and option E which contains a 2. Note that a binary matrix contains only zeros and ones.
A permutation matrix has only one ‘1’ in each row and column so we can eliminate option B (row 3 and column 1 each contain two 1’s) and option C (row 2 contains two 1’s).
Option D shows a permutation matrix.
The answer is D.

Question 2

$$M=\begin{array}{c} \begin{matrix} van\,1 & van\,2 \end{matrix} \\ \begin{bmatrix} 870 & 940 \\ 1\,580 & 1\,790 \\ 2\,340 & 2\,560 \\ 910 & 870 \end{bmatrix} \end{array} \begin{matrix} Thurs \\ Fri \\ Sat \\ Sun \end{matrix}$$

Matrix M is a 4×2 matrix.
Therefore if the matrix product $H\times M$ is to exist,

$$(1\times 4)\times(4\times 2)$$

then matrix H must have 4 columns, since matrix M has 4 rows.
Also, if the resulting matrix is to show 2 values then matrix H must have 1 row, so the resulting matrix will be a 1×2 matrix. So matrix H is of order 1×4.
We can eliminate options A, B and E.

For option C, $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\times\begin{bmatrix} 870 & 940 \\ 1\,580 & 1\,790 \\ 2\,340 & 2\,560 \\ 910 & 870 \end{bmatrix}$ will produce a 1×2 matrix containing the sum of each of the columns. Option D contains zeros so not all the values in the columns will be added.
The answer is C.

Question 3

If the determinant of the 2×2 matrices in these equations **does not** equal zero then they will have a unique solution.

$$\det\begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} = 4\times 4 - 8\times 2 = 0$$

$$\det\begin{bmatrix} 6 & -3 \\ 4 & -2 \end{bmatrix} = 6\times -2 - 4\times -3 = 0$$

$$\det\begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} = 0\times 1 - -1\times 0 = 0$$

$$\det\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} = -2\times 1 - 2\times -1 = 0$$

Since they all have a determinant of zero, then none of them will have a unique solution.
The answer is A.

Question 4

For this matrix equation, the order of the matrices are $(5\times5)\times(5\times1)=(5\times1)$ ie matrix R must be a column matrix with 5 rows.
The 5×5 matrix is a permutation matrix.
The first row of this matrix is multiplied by the column of letters in matrix R to produce the letter T.

$$\text{So } R=\begin{bmatrix} - \\ - \\ T \\ - \\ - \end{bmatrix}$$

So we can eliminate options A and E.
The second row of the permutation matrix is multiplied by the column of letters in matrix R to produce the letter E.

$$\text{So } R=\begin{bmatrix} - \\ - \\ T \\ E \\ - \end{bmatrix}$$

So we can eliminate option D.
The third row of the permutation matrix is multiplied by the column of letters in matrix R to produce the letter A.

$$\text{So } R=\begin{bmatrix} - \\ A \\ T \\ E \\ - \end{bmatrix}$$

So eliminate option C.
The answer is B.

Question 5

Since $A-B=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then matrix A and matrix B are both of order 2×3.

$$\text{So } A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \text{ where } a_{ij}=3i-j$$

$$=\begin{bmatrix} 3\times1-1 & 3\times1-2 & 3\times1-3 \\ 3\times2-1 & 3\times2-2 & 3\times2-3 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 1 & 0 \\ 5 & 4 & 3 \end{bmatrix}$$

$$\text{Since } A-B=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{then } B=\begin{bmatrix} 1 & 0 & -1 \\ 4 & 3 & 2 \end{bmatrix}$$

The answer is E.

Question 6

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array} + \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 0 \end{bmatrix} \quad \begin{array}{c} \text{total} \\ 4 \\ 2 \\ 3 \\ 5 \end{array}$$

The one-step dominance scores are given by summing the rows of the given matrix.
The two-step dominance scores are given by summing the rows of the given matrix squared.
Ranking the students using the sum of the one-step and two-step dominance scores gives

$$\begin{array}{l} D\ (5) \\ A\ (4) \\ C\ (3) \\ B\ (2) \end{array}$$

So the correct ranking is Danica, Andrew, Chris, Brent.
The answer is D.

Question 7

T is a regular transition matrix whose columns add to give 1 so the sum of the elements in each of the state matrices must be the same.

That is, $a+5+20=16+b+c$

i.e. $a-b-c=-9 \qquad -(1)$

Also $S_1 = TS_0$

$$\begin{bmatrix} 16 \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.1 & 0.5 & 0.6 \\ 0.4 & 0.2 & 0.3 \\ 0.5 & 0.3 & 0.1 \end{bmatrix} \begin{bmatrix} a \\ 5 \\ 20 \end{bmatrix}$$

So
$$\begin{aligned} 16 &= 0.1a + 0.5\times 5 + 0.6\times 20 \\ 16 &= 0.1a + 14.5 \\ 1.5 &= 0.1a \\ a &= 15 \end{aligned}$$

In (1) $15-b-c=-9$

$b+c=24 \qquad -(2)$

Also
$$\begin{aligned} b &= 0.4\times a + 0.2\times 5 + 0.3\times 20 \\ b &= 6+1+6 \\ &= 13 \end{aligned}$$

In (2) $c=11$

So $a=15,\ b=13,\ c=11$

So options A, C, D and E are all false and option B is true.
The answer is B.

Question 8

Using the rule,

$$E_1 = \begin{bmatrix} 0.75 & 0 & 0 & 0 \\ 0.20 & 0.80 & 0 & 0 \\ 0 & 0.01 & 0.90 & 0 \\ 0.05 & 0.19 & 0.10 & 1 \end{bmatrix} \begin{bmatrix} 160 \\ 700 \\ 40 \\ 0 \end{bmatrix} + F$$

$$= \begin{bmatrix} 120 \\ 592 \\ 43 \\ 145 \end{bmatrix} + F$$

Since the organisation wants to keep employee numbers constant.

then $\begin{bmatrix} 120 \\ 592 \\ 43 \\ 145 \end{bmatrix} + F = \begin{bmatrix} 160 \\ 700 \\ 40 \\ 0 \end{bmatrix} \begin{matrix} P \\ G \\ M \\ L \end{matrix}$

So $F = \begin{bmatrix} 40 \\ 108 \\ -3 \\ -145 \end{bmatrix} \begin{matrix} P \\ G \\ M \\ L \end{matrix}$

This means that each month 40 probationary staff will need to be added, 108 general staff will need to be added, 3 managers will be asked to leave and 145 employees will decide themselves to leave the organisation.
Only option E reflects this.
The answer is E.

Module 2 – Networks and decision mathematics

Question 1

The sum of the degrees of the four vertices is $2+3+5+4=14$. Note that a loop, which connects the top right hand vertex to itself, counts for two degrees of the vertex, so its total number of degrees is 5.
The answer is E.

Question 2

For an Eulerian trail we need exactly two vertices that have an odd degree. This graph has vertices with degrees of 2, 3, 3, 5 and 1.
Reject option A.
For an Eulerian circuit we need all vertices to have an even degree.
Reject option B.
For a Hamiltonian path we need every vertex on the graph to be visited once. This is possible.
Option C is correct.
Note that for option E, since the graph is not a weighted graph (i.e. no numbers on the edges) we cannot find a minimum spanning tree.
The answer is C.

Question 3

An adjacency matrix that could be used to represent the graph is

$$\begin{array}{c} \\ R \\ S \\ T \\ U \end{array}\begin{array}{c} \begin{array}{cccc} R & S & T & U \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

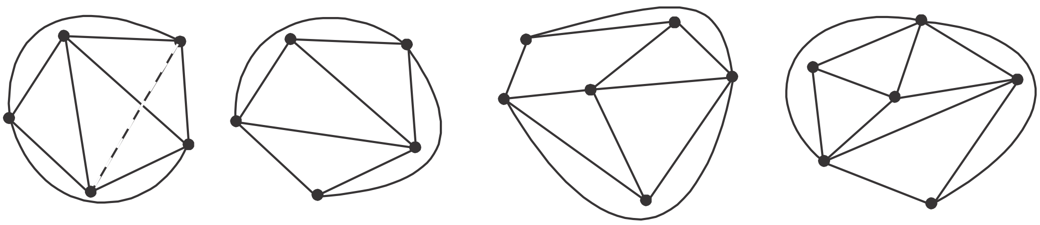
The answer is A.

Question 4

In these questions, you must make sure that a cut separates the source from the sink, that is, the source should be on one side of the cut and the sink should be on the other side.
Cut 1 has the source and the sink both on its right hand side.
Cut 4 has the source and the sink both on its left hand side.
Therefore neither of these cuts can be used to determine the minimum capacity and hence the maximum flow.
The other three cuts can be used.
Cut 2 had a capacity of $60+20+90=170$.
Cut 3 had a capacity of $60+80+60+80=280$.
Cut 5 had a capacity of $100+80=180$.
The minimum capacity is 170 hence the maximum flow is 170 litres per hour.
The answer is B.

Question 5

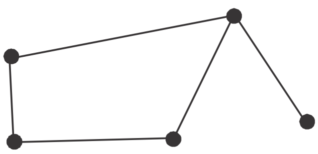
A graph is planar if it can be drawn so that no two edges intersect (except of course at the vertices).
The graphs have been redrawn and are shown below.



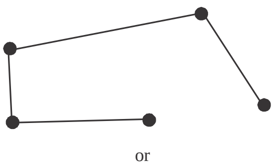
The first graph cannot be redrawn so that no edges intersect.
It therefore is not planar.
All the other graphs can be redrawn so that no edges intersect and therefore they are planar.
We have 3 planar graphs.
The answer is D.

Question 6

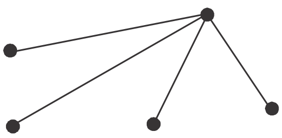
Option A is true, since if the graph has a bridge then it must be a connected graph (if the bridge is removed, the graph becomes disconnected).
Option B is true. For example



If the graph has 4 edges, then it will have 4 bridges.
For example,



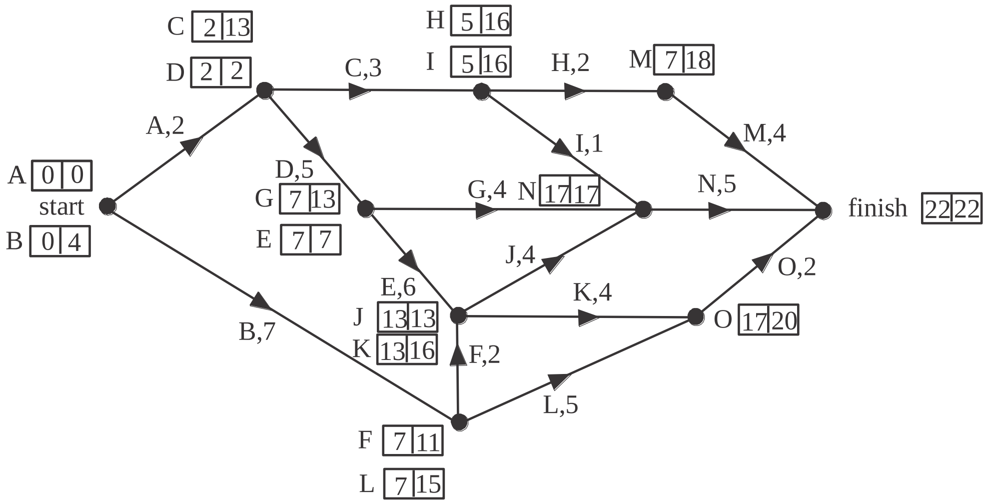
or



etc.
etc
Option C is true, if it did not contain a cycle, there would be more than one bridge.
Since a tree cannot contain a cycle, then option E must be true.
Option D is not true, since if the graph is complete, every vertex is connected to every other vertex via an edge so the graph cannot have a bridge.
The answer is D.

Question 7

Do a forward and backward scan to find the earliest and latest start time for each activity.



The minimum time, in days, required to complete the project is 22 days.
The answer is E.

Question 8

Using the forward and backward scan from question 7 solutions, we see that activity $C\,(13-2=11)$, activity $H\,(16-5=11)$, activity $I\,(16-5=11)$ and activity $M\,(18-7=11)$ all have float times of 11 days.
So four activities have a float time of 11 days.
The answer is E.

Module 3 – Geometry and measurement

Question 1

Since Δ's ABE and ACD are similar,

then $\dfrac{BE}{CD} = \dfrac{AB}{AC}$

so $\dfrac{6}{12} = \dfrac{5}{AC}$

$$AC = \frac{5 \times 12}{6}$$
$$= 10$$

So $BC = 10 - 5$

$$= 5 \text{ cm}$$

The answer is A.

Question 2

$$\text{total volume} = \text{volume of cylinder} + \text{volume of hemisphere}$$
$$= \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3 \qquad \text{(formula sheet)}$$
$$= \pi \times 25^2 \times 40 + \frac{2}{3} \times \pi \times 25^3$$
$$= 111\,264.7398...$$

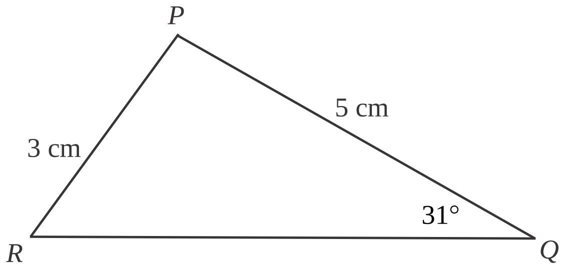
The closest answer is 111 265 cm^3.
The answer is C.

Question 3

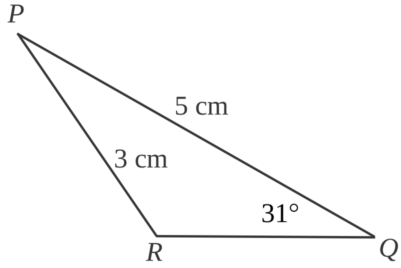
We look at the longitude of the cities (i.e. the second number and direction given in the brackets) to decide where the sun will set first (and rise first).
The city furthest to the east is Hobart. Moving west the next city is Vladivostok, the next is San Sebastian and the city furthest to the west is Trelew.
The order is Hobart, Vladivostok, San Sebastian, Trelew.
The answer is B.

Question 4

Do a quick sketch. Note that because we are given 2 sides of the triangle and an angle that is not included, we can have what is known as 'the ambiguous case' ie two possible triangles exist.



OR



Using the sine rule,

$$\frac{\sin(\angle PRQ)}{5} = \frac{\sin(31°)}{3}$$

$$\sin(\angle PRQ) = 0.8583...$$

$$\angle PRQ = \sin^{-1}(0.8583...)$$

$$= 59.137...°$$

$$= 59° \text{ (to the nearest degree)}$$

This gives us $\angle PRQ$ in the diagram on the left.
So in the same triangle,

$$\angle QPR = 180° - 31° - 59°$$

$$= 90° \text{ (to the nearest degree)}$$

In the triangle on the right,

since $\sin(180 - \theta) = \sin(\theta)$

then $180° - 59° = 121°$

So $\angle PRQ = 121°$

and so in this triangle,

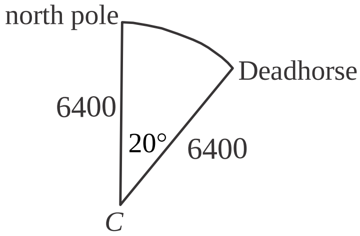
$$\angle QPR = 180° - 31° - 121°$$

$$= 28°$$

The only one of these four angles i.e. $59°, 90°, 121°$ and $28°$ that is offered is $28°$.
The answer is B.

Question 5

We are looking for the length of an arc (from Deadhorse to the north pole) with angle $20°$ and radius 6 400 km.



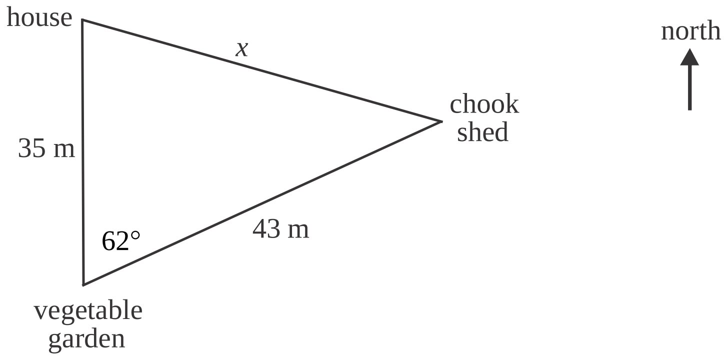
$$l = r \times \frac{\pi}{180} \times \theta° \qquad \text{(formula sheet)}$$

$$= 6\,400 \times \frac{\pi}{180} \times 20$$

$$= 2\,234.021...$$

The closest answer is 2 234 km.
The answer is A.

Question 6



We have two sides of a triangle and the included angle.
Let the distance between the chook shed and the house be *x* metres.

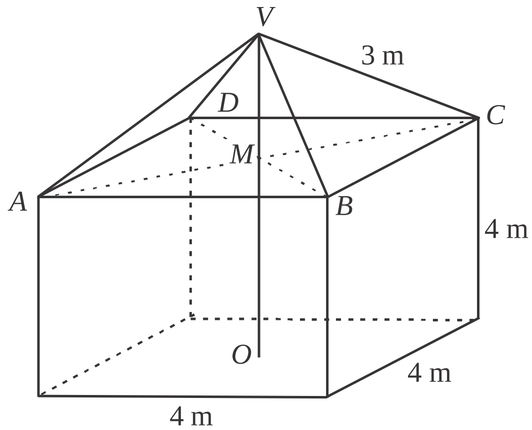
Using the cosine rule,

$$x^2 = 35^2 + 43^2 - 2 \times 35 \times 43 \times \cos(62°)$$
$$= 1\,660.8905...$$
$$x = \sqrt{1\,660.8905...}$$
$$= 40.7540...$$

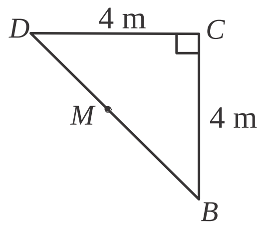
The closest answer is 41 metres.
The answer is D.

Question 7

Let *M* be the centre point of the square *ABCD*. The pole runs through *M*.



In the right-angled triangle *BCD*,

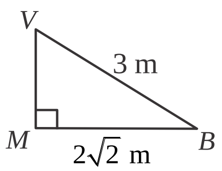


$$(BD)^2 = 4^2 + 4^2 \quad \text{(Pythagoras)}$$
$$BD = \sqrt{32}$$
$$= 4\sqrt{2}$$

So $BM = \dfrac{4\sqrt{2}}{2}$

$$= 2\sqrt{2} \qquad (\approx 2.8...)$$

In the right-angled triangle *BMV*,

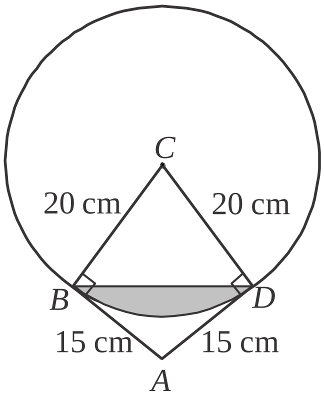


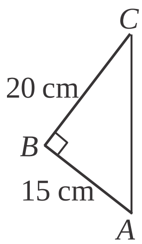
$$(MV)^2 = 3^2 - \left(2\sqrt{2}\right)^2 \qquad \text{(Pythagoras)}$$
$$MV = \sqrt{1}$$
$$= 1 \text{ metre}$$

So the length of the pole is $4 + 1 = 5$ metres.
The answer is C.

Question 8

Δ's ABC and ACD are congruent triangles (i.e. the same). They are both right angled triangles.





In ΔABC, $\tan(\angle ACB) = \dfrac{15}{20}$

$$\angle ACB = \tan^{-1}\left(\frac{15}{20}\right)$$
$$= 36.8698...^\circ$$

So $\angle ACD = 36.8698...^\circ$ as well.
So the segment is formed by an angle of $2 \times 36.8698...^\circ = 73.7397...^\circ$ in a circle of radius 20 cm.

Area of segment = area of sector – area of triangle

$$= \pi \times 20^2 \times \frac{73.7397...^\circ}{360} - \frac{1}{2} \times 20 \times 20 \times \sin(73.7397...^\circ)$$
$$= 65.400...\text{cm}^2$$

The closest answer is 65.
The answer is A.

Module 4 – Graphs and relations

Question 1

The points $(3,-1)$ and $(3,3)$ both have *x*-coordinates of 3.
The line that passes through them has the equation $x=3$.
The answer is A.

Question 2

Option A is not true because if Jason leaves home at 8am it will take him 80 minutes to get to work which is the slowest travel time.
Option B is not true because it will take 60 minutes to get to work if he leaves home at 6am or at 8.30am i.e. leaving at 6am is not quicker.
Option C is not true because if Jason leaves home at 10am it will take 30 minutes to get to work which is the quickest trip possible.
Option D is not true because if he leaves home at 6am it will take 60 minutes which is faster than if he leaves at 8am when it will take 80 minutes.
Option E is correct.
The answer is E.

Question 3

For a member who plays *n* games in a season, total payment $=180+15n$.
So, $\$255=180+15\times5$ where $n=5$

$\$300=180+15\times8$ where $n=8$

$\$360=180+15\times12$ where $n=12$

For \$380, after subtracting the registration fee of \$180, \$200 remains and this amount is not divisible by 15 so \$380 could not represent a member's total payment for the season.
The answer is D.

Question 4

Option A, is incorrect because the inequality should be $x\le5$.
Option B is incorrect because the inequality should be $1\le y\le4$.
Option C is incorrect because the point $(3,3)$, for example, does not satisfy the inequality
$2x-y\ge7$
i.e. $6-3\le7$
Option D is correct because the point $(3,3)$, for example, satisfies the inequality
$5x+4y\ge20$
i.e. $15+12\ge20$
To confirm this, for option E, the point $(3,3)$, for example, does not satisfy the inequality.
$x+y\ge20$
$3+3\le20$
The answer is D.

Question 5

For option A, we would have the number of residents being less than or equal to the number of registered nurses divided by 8. This means there would be very few residents!
Reject option A.
For option B, we would have the number of residents being greater than or equal to eight times the number of registered nurses. This means there could be way too many residents.
Reject option B.
For option C, we would have the number of residents being less than or equal to 8 times the number of registered nurses. This is what is required.
The answer is C.

Question 6

In March the cost was $5 \times \$7 = \$35$.
In June the cost was $25 \times \$6 = \$150$.
In September the cost was $70 \times \$4.50 = \$315$.
The total cost was $500.
The answer is D.

Question 7

Let $x =$ the number of shorts boots purchased.
Let $y =$ the number of long boots purchased .

$$x + y = 11 \quad -(1)$$
$$60x + 90y = 750 \quad -(2)$$

Method 1 – using CAS
Solve the two equations for *x* and *y*.
$x = 8, y = 3$
So 3 pairs of long boots are purchased.
The answer is B.

Method 2 – by hand

$(1) \times 60 \qquad 60x + 60y = 660 \quad -(3)$

$(2) - (3) \qquad 30y = 90$

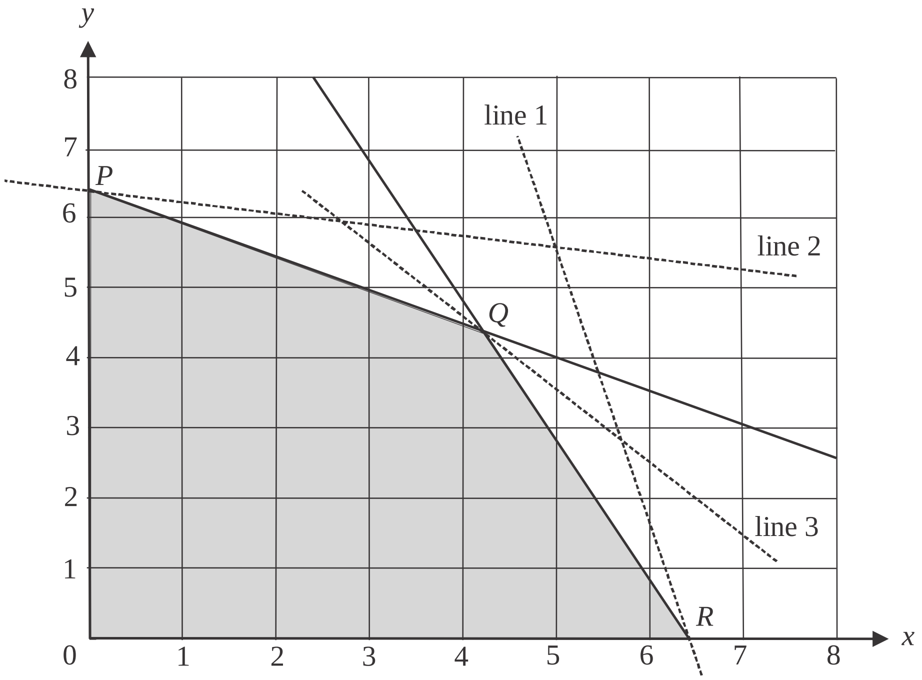
$y = 3$

In (1) $\qquad x = 8$

So 3 pairs of long boots are purchased.
The answer is B.

Question 8

For options A and B the objective function is $Z = 4x + y$.
Rearranging this gives $y = -4x + Z$.
So this line has a gradient of – 4. When you slide a line with a gradient of – 4 from left to right across the feasible region; the last point it touches in the feasible region is point *R*. This is indicated by the dotted line 1 in the graph below.



This means that the maximum value of this objective function occurs at point *R*.
This means that neither option A nor B is correct.

In options C and D, the objective function is $Z = x + 6y$.

Rearranging this gives $y = -\frac{1}{6}x + \frac{Z}{6}$.

This line has a gradient of $-\frac{1}{6}$.

When we slide this line from left to right (or more from bottom to top) the last point in the feasible region that it touches is point *P*. This is shown in line 2.
So the maximum value of this objective function occurs at point *P*.
So option C is correct and option D isn't.

Line 3 in the diagram shows the sliding line for option E which has a maximum value of *Z* occurring at point *Q* not point *R* so option E is incorrect.
The answer is C.