THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025
info@theheffernangroup.com.au
www.theheffernangroup.com.au

FURTHER MATHEMATICS TRIAL EXAMINATION 2 SOLUTIONS 2018

SECTION A - Core

Data analysis

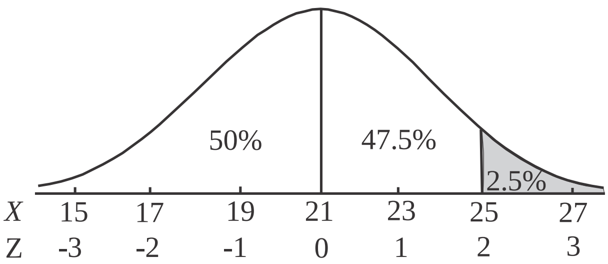
Question 1 (5 marks)

a. **i.** $\text{median} = 20$ **(1 mark)**

ii. $Q_3 = 24$ **(1 mark)**

b. $\left(\frac{4}{16} \times \frac{100}{1}\right)\% = 25\%$ **(1 mark)**

c. 95% of hens laid between 17 and 25 eggs during January (i.e. between two standard deviations above and below the mean).
So 5% lay less than 17 or more than 25. Because the normal curve is symmetrical we know that 2.5% must have laid more then 25 eggs. The answer is 2.5%



(1 mark)

d. $z = \frac{x - \bar{x}}{s_x}$ (formula sheet)

$$-1.5 = \frac{x - 21}{2}$$

$$x = 18$$

This hen lay 18 eggs. **(1 mark)**

Question 2 (7 marks)

a. The modal weight (the most popular or frequently occurring weight) is 67.7 g. **(1 mark)**

b. **i.** The maximum is 72.5 so the $\text{range} = 72.5 - 65.0$.
$= 7.5$ **(1 mark)**

ii. $\text{interquartile range} = Q_3 - Q_1$
$= 70.5 - 68.5$
$= 2.0$ **(1 mark)**

c. $\text{lower fence} = Q_1 - 1.5 \times IQR$ (formula sheet)
$= 68.5 - 1.5 \times 2.0$
$= 65.5$ **(1 mark)**

Since $65.0 < 65.5$, the egg with a weight of 65.0 g is an outlier. **(1 mark)**

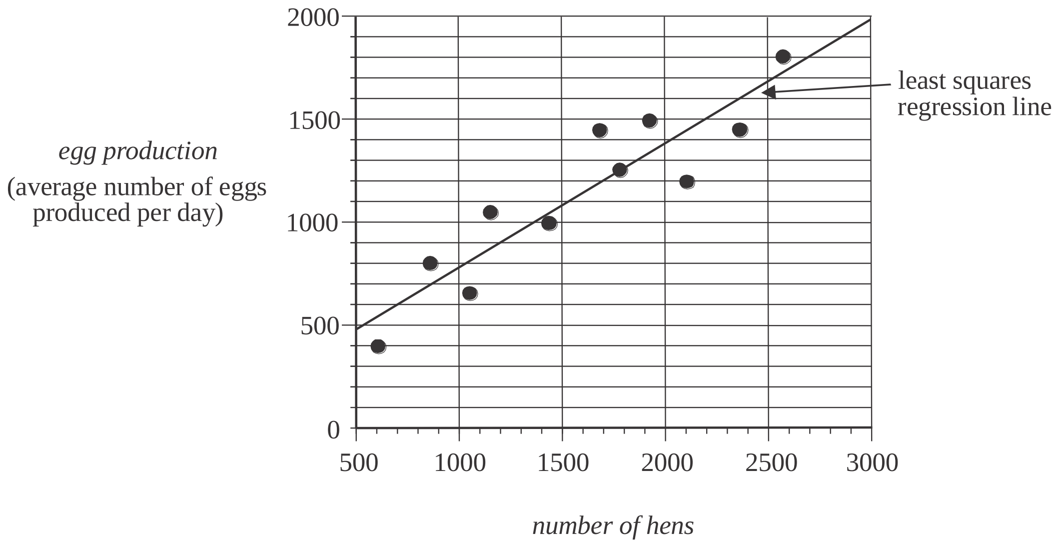
d. The medians for the eggs collected in the nesting box and outside differ. **(1 mark)**
For those eggs collected from the nesting box the median weight was 69.5 g and for those collected outside it was 68.0 g. **(1 mark)**
Alternatively the *IQR*'s differ. For eggs collected from the nesting box the *IQR* was 2.0 and for those collected outside it was $69.0 - 67.5 = 1.5$.

Question 3 (8 marks)

a. The response variable (given on the vertical axis) is *egg production*.

(1 mark)

b.



(1 mark)

c. For every increase of one in the *number of hens* kept on a farm there is an increase of 0.6 in the *egg production* per day at that farm.

(1 mark)

d. The least squares regression line equation predicts that at this farm the egg production will be:

$$\text{egg production} = 194 + 0.6 \times 2\,100$$
$$= 1\,454$$

$$\text{residual value} = \text{actual value} - \text{predicted value} \qquad \text{(formula sheet)}$$
$$= 1\,200 - 1\,454$$
$$= -254$$

The least squares line overestimates the egg production by 254 eggs.

(1 mark)

e. Finding the *egg production* for a farm where the *number of hens* is 4 500 requires us to extrapolate; that is, to go beyond the data that we have used to formulate the least squares line equation. Doing this can be unreliable.

(1 mark)

f. Since $r = 0.9182$

then $r^2 = 0.843091...$

So 84.31% of the variation in *egg production* can be explained by the variation in the *number of hens* kept at these 11 farms.

(1 mark)

g. Enter the data in the table into your CAS and calculate the least squares (regression) line.

Enter *number of hens* as the explanatory variable (*x* variable) and *egg production* as the response variable (*y* variable).

$\textit{egg production} = 79.75 + 0.74 \times \textit{number of hens}$ where the values have been expressed correct to two decimal places.

(1 mark) – 79.75

(1 mark) – 0.74

Question 4 (4 marks)

a. The pattern shows seasonality with an increasing trend.

(1 mark)

b. The five data values centred around Summer 2017 are

2 400 5 420 5 680 4 840 3 810

$$\text{mean } 1 = \frac{2\,400 + 5\,420 + 5\,680 + 4\,840}{4}$$
$$= 4\,585$$

$$\text{mean } 2 = \frac{5\,420 + 5\,680 + 4\,840 + 3\,810}{4}$$
$$= 4\,937.5$$

(1 mark)

$$\text{Average of mean 1 and mean 2} = \frac{4\,585 + 4\,937.5}{2}$$
$$= 4\,761.25$$

The four-mean smoothed egg production centred on Summer 2017 is 4 761.25.

(1 mark)

c. The average of the long-term average

$$= \frac{5\,100 + 4\,380 + 3\,260 + 5\,850}{4}$$
$$= 4\,647.5$$

$$\text{Seasonal index for winter} = \frac{3\,260}{4\,647.5}$$
$$= 0.701452...$$
$$= 0.70 \quad \text{(correct to 2 significant figures)}$$

(1 mark)

Note that after a decimal point, all zeros to the right of any non-zero digits are significant.

Recursion and financial modelling

Question 5 (4 marks)

a. $4 590

(1 mark)

b. $V_0 = 54\,000$
$V_1 = 54\,000 - 4\,590$
$= 49\,410$
$V_2 = 49\,410 - 4\,590$
$= 44\,820$ as required

(1 mark)

c. annual flat rate of depreciation
$= \left(\frac{4\,590}{54\,000} \times \frac{100}{1}\right)\%$
$= 8.5\%$

(1 mark)

d. $V_0 = 54\,000, \quad V_{n+1} = 0.925 \times V_n$

(1 mark)

Note that $7.5\% = \frac{7.5}{100}$
$= 0.075$
and $1 - 0.075 = 0.925$

Question 6 (4 marks)

a. $\$5\,600 + 1.8\% \text{ of } \$5\,600 + \$15$
$= \$5\,600 + 0.018 \times \$5\,600 + \$15$
$= \$5\,715.80$

(1 mark)

b. $D_0 = 5\,600, \quad D_{n+1} = 1.018 \times D_n + 15$
(1 mark) **(1 mark)**

c. Generate on your CAS, the sequence defined by the recurrence relation found in part **b**.
5 600, 5 715.80, 5 833.68, 5 953.69,…6 856.45, 6 994.86, 7 135.77,…
Note that the initial debt was $5 600, after 1 month the debt was $5 715.80, after 2 months it was $5 833.68 and so on. Therefore, after 12 months, the debt first exceeded $7 000 (i.e. it was $7 135.77).

(1 mark)

Question 7 (4 marks)

a. Using finance solver.

$$\begin{aligned} N &: 20 \\ I(\%) &: 4.2 \\ PV &: -80\,000 \\ PmT &: -5\,000 \\ FV &: ? \\ PpY &: 4 \\ CpY &: 4 \end{aligned}$$

$FV = 209\,218.6852...$

After 5 years the value of the investment is \$209 218.69.

(1 mark)

b. Over the five years, Jacinta invested

$$\begin{aligned} &\$80\,000 + 5 \times 4 \times \$5\,000 \\ &= \$180\,000 \end{aligned}$$

$$\begin{aligned} \text{Interest earned} &= \$209\,218.69 - \$180\,000 \\ &= \$29\,218.69 \end{aligned}$$

(1 mark)

c. Using finance solver,

$$\begin{aligned} N &: 8 \\ I(\%) &: 4.2 \\ PV &: -209\,218.69 \\ Pmt &: ? \\ FV &: 300\,000 \\ PpY &: 4 \\ CpY &: 4 \end{aligned}$$

$\text{Pmt}: -8740.3740...$

Jacinta's new quarterly payment needs to be \$8 740.37.

(1 mark)

d.

$$\begin{aligned} \text{weekly payment } &= \frac{3.9/52}{100} \times \$300\,000 \\ &= \$225 \end{aligned}$$

(1 mark)

SECTION B - Modules

Module 1 - Matrices

Question 1 (4 marks)

a. There were $720+708+690=2118$ accessories sold over the three-month period. **(1 mark)**

b. **i.** \$65 750 **(1 mark)**

ii. Since $\begin{bmatrix} 720 & 405 & 980 \\ 708 & 420 & 950 \\ 690 & 390 & 970 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 67\,375 \\ 66\,970 \\ 65\,750 \end{bmatrix}$

then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 720 & 405 & 980 \\ 708 & 420 & 950 \\ 690 & 390 & 970 \end{bmatrix}^{-1}\begin{bmatrix} 67\,375 \\ 66\,970 \\ 65\,750 \end{bmatrix}$

$$=\begin{bmatrix} 15 \\ 55 \\ 35 \end{bmatrix}$$

The profit made on the sale of a pair of boots is \$55. **(1 mark)**

iii. $G=\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ **(1 mark)**

Question 2 (3 marks)

a. 10%. **(1 mark)**

b.

$$T_1=\begin{array}{c} \text{this week} \\ \begin{array}{ccc} \text{level 1} & \text{level 2} & \text{level 3} \end{array} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & \mathbf{0.6} & 0.1 \\ \mathbf{0.1} & 0.1 & 0.7 \end{bmatrix} \end{array}\begin{array}{l} \text{level 1} \\ \text{level 2 next week} \\ \text{level 3} \end{array}$$

(1 mark)

c. <u>Method 1</u> – using the transition diagram

From the transition diagram we see that from one week to the next,

- 10% on level 1 change to level 3
- 10% on level 2 change to level 3
- 70% on level 3 stay on level 3

The number of staff who are expected to park on level 3 in week 2 is given by

$10\%\text{ of }65+10\%\text{ of }91+70\%\text{ of }82$

$=0.1\times 65+0.1\times 91+0.7\times 82$

$=73$ **(1 mark)**

<u>Method 2</u> – using the transition matrix

$N_2=T_1\,N_1$

$$=\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.6 & 0.1 \\ 0.1 & 0.1 & 0.7 \end{bmatrix}\begin{bmatrix} 65 \\ 91 \\ 82 \end{bmatrix}$$

$$=\begin{bmatrix} 76.2 \\ 88.8 \\ 73 \end{bmatrix}$$

So 73 staff are expected to park on level 3 in week 2. **(1 mark)**

Question 3 (5 marks)

a. We are interested in the elements of the leading diagonal of the transition matrix. These four numbers tell us that

- 60% of staff who choose accounting in Q1 also choose it in Q2.
- 30% of staff who choose customer service in Q1 also choose it in Q2.
- 20% of staff who choose design in Q1 also choose it in Q2.
- 50% of staff who choose health and safety in Q1 also choose it in Q2.

In total, $0.6\times30+0.3\times10+0.2\times20+0.5\times40=45$ staff choose the same training session for quarters 1 and 2. **(1 mark)**

b. Using the matrix equation

$$S_{n+1}=T_2\,S_n$$

we have $S_2=T_2\,S_1$

$$=\begin{bmatrix}28\\22\\17\\33\end{bmatrix}\begin{matrix}A\\C\\D\\H\end{matrix}$$

(Using CAS, and having defined T_2 and S_1 on your CAS).
So in quarter 2, 33 staff are expected to choose health and safety *(H)* as their training session.

(1 mark)

c. Start by finding S_3 in order to find how many staff did accounting in that quarter.

$$S_3=T_2\,S_2$$

$$=\begin{bmatrix}30.6\\22.8\\17.2\\29.4\end{bmatrix}\begin{matrix}A\\C\\D\\H\end{matrix}$$

So 30.6 were expected to choose accounting in Q3. **(1 mark)**
From part **b.**, 17 staff chose design in Q2.
From the transition matrix T_2, 0.1 or 10% of staff are expected to change from design to accounting each quarter. So 10% of $17=1.7$.
So the percentage of staff expected to choose accounting in Q3 after having chosen design in Q2 is

$$\left(\frac{1.7}{30.6}\times\frac{100}{1}\right)\%$$

$$=5.555...\%$$

Re-read the question. We need to round this percentage to the nearest whole number.
So 6% is the required answer. **(1 mark)**

d.

$$S_1=\begin{bmatrix}30\\10\\20\\40\end{bmatrix},\quad S_2=\begin{bmatrix}28\\22\\17\\33\end{bmatrix},\quad S_3=\begin{bmatrix}30.6\\22.8\\17.2\\29.4\end{bmatrix}\text{ and }S_4=\begin{bmatrix}32.14\\22.66\\16.94\\28.26\end{bmatrix}$$

The minimum number of staff expected in the accounting training sessions is 28.

(1 mark)

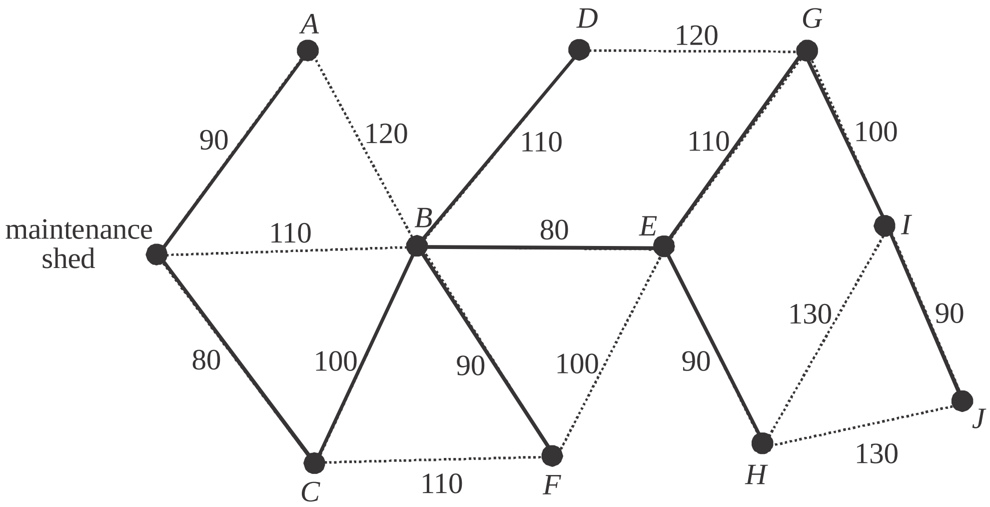
Module 2 - Networks and decision mathematics

Question 1 (3 marks)

a. Using trial and error the shortest distance is 410 metres (shed, *B, E, H, J*).

(1 mark)

b. i.



(1 mark)

ii. The minimum length of vehicle tracks required is
$90+80+100+110+80+90+110+90+100+90=940$ metres.

(1 mark)

Question 2 (2 marks)

a. $\boxed{8}_{v} + \boxed{8}_{f} = \boxed{14}_{e} + \boxed{2}$

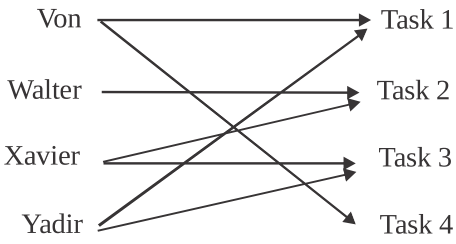
(1 mark)

b. The value of *f*, which is 8, tells us that there are 7 paddocks within the wind farm and 1 area (i.e. the outside world!) which is outside the wind farm.

(1 mark)

Question 3 (2 marks)

a. Using the table given, draw a bipartite graph.



The optimal assignment is

Technician	Task
Von	4
Walter	2
Xavier	3
Yadir	1

(1 mark)

b. Using the first table given in the question, the minimum time is
$6+8+18+10=42$ hours. **(1 mark)**

Question 4 (5 marks)

a. The three immediate predecessors of activity *L* are *F*, *G* and *H*. **(1 mark)**

b. Activity *G* has activity *C* as an immediate predecessor and activity *C* has activity *A* as its immediate predecessor. So activity *G* has to wait $2+2=4$ weeks for activity *C* to be completed.
Activity *G* also has activity *E* as an immediate predecessor which has activity *D* as its immediate predecessor which has activity *A* as its immediate predecessor. So activity *G* has to wait $2+3+3=8$ weeks for activity *E* to be completed.
The earliest start time for activity *G* is therefore 8 weeks. **(1 mark)**

c. Because we're told that the minimum completion time is 18 weeks, we can just look for a path that takes 18 weeks rather than doing a forward and backward scan.
The critical path is therefore *A, D, E, G, L, M, N.* **(1 mark)**

d. The latest start time for activity *J* is $18-1-4=13$ (doing a backward scan through *N* and *J*). **(1 mark)**

e. The possible paths and their completion times for this project are shown below.

A	*B*	*F*	*J*			*N*	16 weeks
A	*B*	*F*	*K*			*N*	15 weeks
A	*B*	*F*	*L*		*M*	*N*	17 weeks
A	*C*	*G*	*K*			*N*	12 weeks
A	*C*	*G*	*L*		*M*	*N*	14 weeks
A	*D*	*E*	*G*	*K*		*N*	16 weeks
A	*D*	*E*	*G*	*L*	*M*	*N*	18 weeks
A	*D*	*H*	*K*			*N*	14 weeks
A	*D*	*H*	*L*		*M*	*N*	16 weeks
A	*D*	*I*			*M*	*N*	15 weeks

The two longest paths take 18 weeks (i.e. the critical path) and 17 weeks. All other paths take 16 weeks or less. These two longest paths are:

A *B* *F* *L* *M* *N* which takes 17 weeks

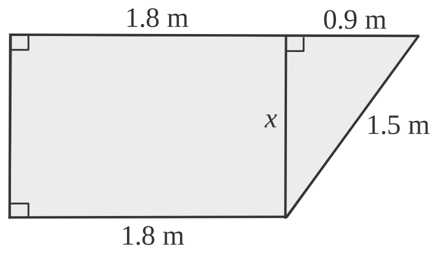
AND

A *D* *E* *G* *L* *M* *N* which takes 18 weeks

We will need to reduce this latter path by 2 weeks and the former path by 1 week.
We can't reduce activities *A* or *N*. The only other activities that these two paths share are *L* and *M*. So the two activities that should be crashed are *L* and *M*.
(1 mark)
(Note that if we don't choose these activities, then we have to choose from *B* or *F*, which won't reduce the critical path, or *D*, *E* or G which won't reduce the 17 week path.)

Module 3 - Geometry and measurement

Question 1 (3 marks)



a. In the right-angled triangle, we have

$1.5^2 = x^2 + 0.9^2$ (Pythagoras theorem)

$x^2 = 1.5^2 - 0.9^2$

$= 1.44$

$x = \sqrt{1.44}$

$= 1.2$

The height of the desk is 1.2 metres.

(1 mark)

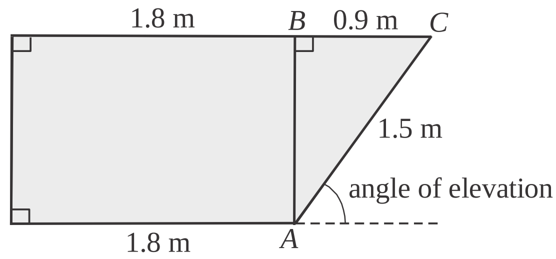
b. surface area

$= 2\times\left(1.8\times1.2+\frac{1}{2}\times0.9\times1.2\right)+1.3\times1.2+1.3\times1.5+2.7\times1.3+1.8\times1.3$

$= 14.76$ square metres

(1 mark)

c. There are many ways to do this question.



In ΔABC, $\sin(\angle BAC) = \frac{0.9}{1.5}$

$\angle BAC = \sin^{-1}\left(\frac{0.9}{1.5}\right)$

$= 0.6435...$

$= 36.8698...°$

So the angle of elevation is $90° - 36.8698...°$

$= 53.1301...°$

$= 53°$ (to nearest degree)

(1 mark)

Question 2 (3 marks)

a. Since the longitude of Durban is 31°E and the longitude of Melbourne is 145°E, then Durban time is 9 hours behind Melbourne time.
Kristy left Melbourne at 1.15pm on 6 March or 13.15 on the 24 hour clock.
Method 1
In Durban time, Kristy's departure time would be 4.15 Monday on the 24 hour clock.
Travel time of 22 hours and 30 minutes, means it would be 2.45am the next day.
Kristy arrives on Tuesday 7 March at 2.45am Durban time.

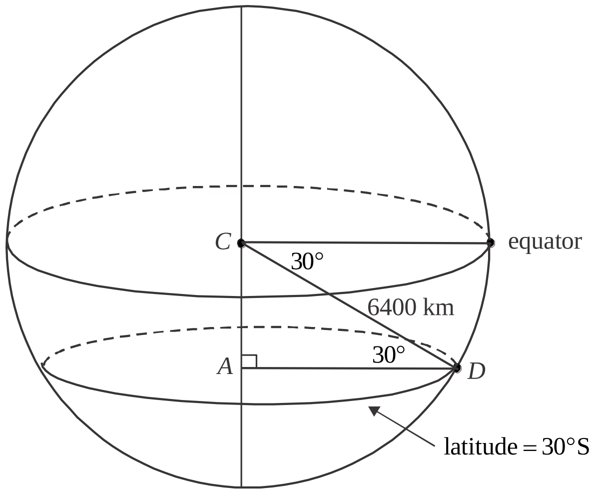
Method 2
Kristy leaves Melbourne at 13.15 on the 24 hour clock on Monday.
She travels for 22 hours and 30 minutes which takes it to 11.45am Melbourne time on Tuesday 7 March. In Durban time this would be 2.45am on Tuesday 7 March.

(1 mark)

b. In the diagram, *AD* represents the radius of the small circle of earth at latitude 30°S.
In ΔACD,

$$\cos(30°) = \frac{AD}{6\,400}$$

$$AD = 6\,400 \times \cos(30°)$$
$$= 5\,542.5625...$$

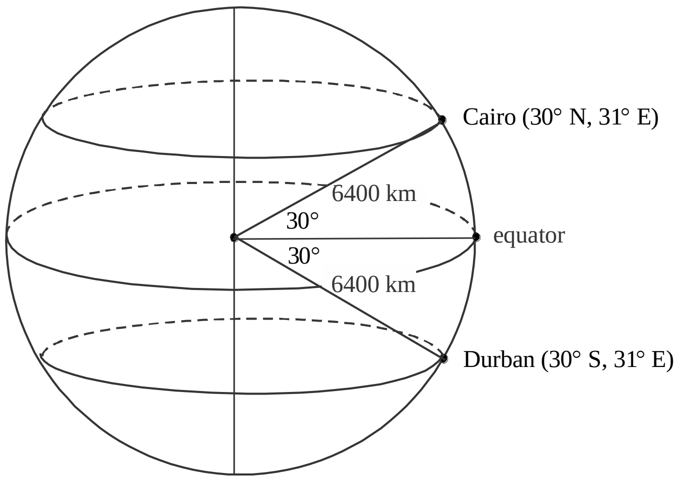


The radius of the small circle of Earth at 30°S is 5 543km (correct to the nearest km).

(1 mark)

c. The great circle distance between Cairo and Durban is represented by the length of the curved edge of the sector with radius of 6 400 and angle at the centre of 60°.

$$l = r \times \frac{\pi}{180} \times \theta° \text{(formula sheet)}$$
$$= 6\,400 \times \frac{\pi}{180} \times 60$$
$$= 6\,702.0643... \text{ km}$$



The required distance is 6702 km (correct to the nearest kilometre)

(1 mark)

Question 3 (3 marks)

a. The ratio of the sidelengths of the smaller pyramid to the larger pyramid is $1:2$. Therefore the ratio of the volume of the smaller pyramid to the larger pyramid will be

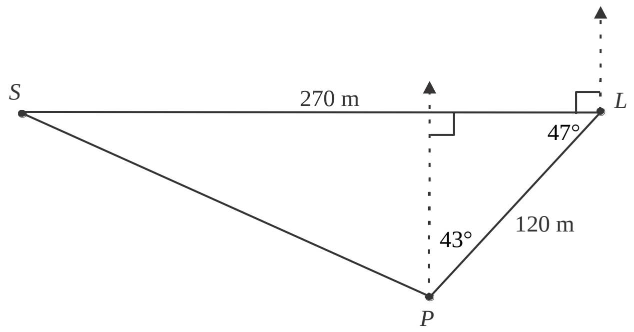
$1:2^3$

ie. $1:8$

Since the smaller pyramid has a volume of 200 000 m^3, the larger pyramid will have a volume of $8 \times 200\,000 = 1\,600\,000\,\text{m}^3$.

(1 mark)

b. **i.**



Using the diagram, we see that the bearing of the carpark at P from the large pyramid at L is equal to $360° - 90° - 47° = 223°$.

(1 mark)

ii. In ΔLPS, we have

$$(PS)^2 = 270^2 + 120^2 - 2 \times 270 \times 120 \times \cos(47^\circ)$$
$$= 43106.5062...$$
$$PS = \sqrt{43106.5062...}$$
$$= 207.6210...$$

Distance from S to P is 208 metres (to the nearest metre).

(1 mark)

Question 4 (3 marks)

a. In the smaller sector,

$$AB = r \times \frac{\pi}{180} \times \theta° \qquad \text{(formula sheet - length of an arc)}$$

i.e. $1.537 = r \times \dfrac{\pi}{180} \times 110$

Solve for r.

i.e. $r = \dfrac{1.537 \times 180}{(\pi \times 110)}$

$= 0.800578...$

$= 0.8$ (correct to 1 decimal place)

as required.

(1 mark)

b. area of shaded section of baggage carousel

$=$ area of larger sector $-$ area of smaller sector

$= \pi \times 2.6^2 \times \dfrac{110}{360} - \pi \times 0.8^2 \times \dfrac{110}{360}$ (formula sheet - area of a sector) **(1 mark)**

$= 5.8747...$

$= 6$ square metres (correct to the nearest square metre)

(1 mark)

Module 4 - Graphs and relations

Question 1 (3 marks)

a. 8 km

(1 mark)

b. Ben has ridden $8\text{km}+3\text{km}=11\text{km}$ after 35 minutes.

(1 mark)

c. The dead tree is 4 km from Ben's home. He is 4 km from home when $t=10$ and then again (on the trip back home) when $t=40$.
So it is $40-10=30$ minutes later that he passes this dead tree again.

(1 mark)

Question 2 (2 marks)

a. Choose any point on the graph.
For example (1, 4).
Since $\text{speed}=kt^2$.
When $t=1$ and speed = 4,

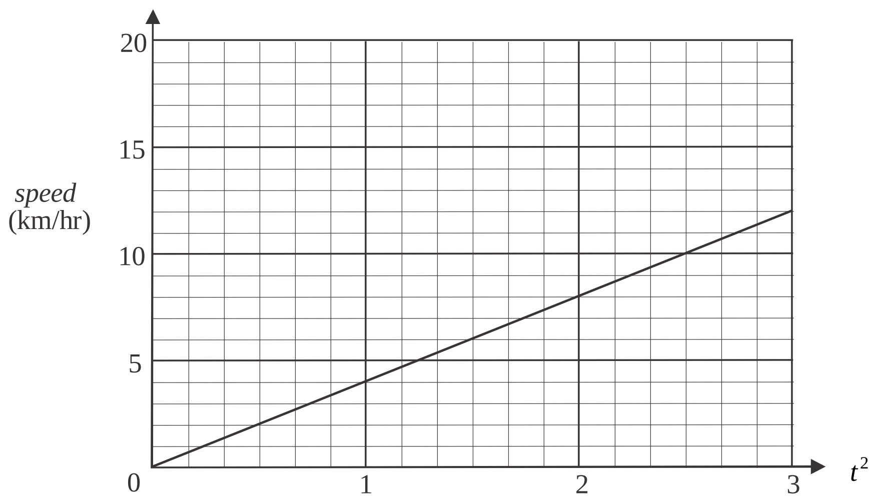
$$4=k\times 1$$
$$k=4$$

(1 mark)

You can double-check this with any other point on the graph.
For example $(3,36)$.
When $t=3$, $\text{speed}=36$

$$36=k\times 3^2$$
$$k=4$$

b.



(1 mark)

Question 3 (3 marks)

a. Using the equation $revenue = 35n$, we see that Rick charges $35 for a service.

(1 mark)

b. **i.** Break even occurs when

$$\text{revenue} = \text{cost}$$
$$35n = 10n + 500$$
$$25n = 500$$
$$n = 20$$

Rick needs to service 20 bikes to break even.

(1 mark)

ii.

$$\text{Profit} = \text{revenue} - \text{cost}$$
$$500 = 35n - (10n + 500)$$
$$= 35n - 10n - 500$$
$$1\,000 = 25n$$
$$n = 40$$

Rick needs to service 40 bikes to make a profit of $500.

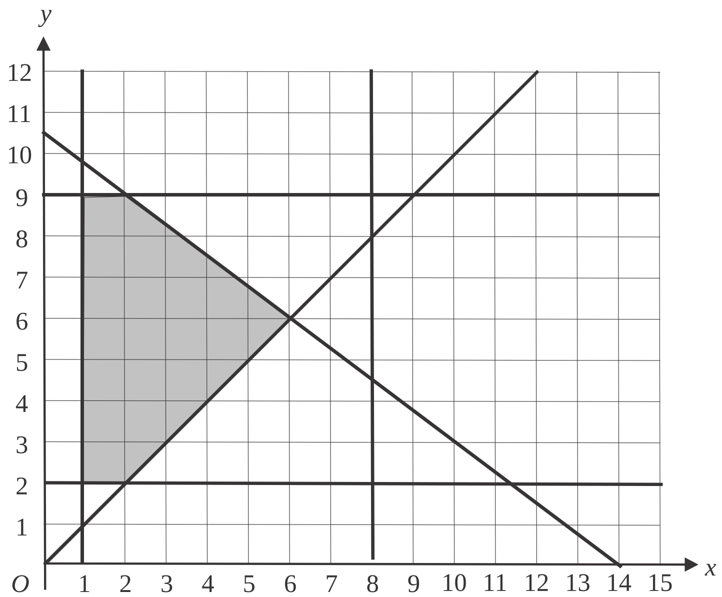
(1 mark)

Question 4 (4 marks)

a. The inequality $3x + 4y \le 42$ relates to the time Rick has available for checking and cleaning. He has a maximum of 42 minutes available each Sunday.

(1 mark)

b.



(1 mark)

c. $P = 15x + 20y$

Maximum profit will occur at one of the corner points of the shaded region.
Corner points are $(1,2),(1,9),(2,2),(2,9)$ and $(6,6)$.
We can ignore $(1,2)$ because $(1,9)$ will give a bigger profit.
Similarly we can ignore $(2,2)$ because $(2,9)$ will give a bigger profit.

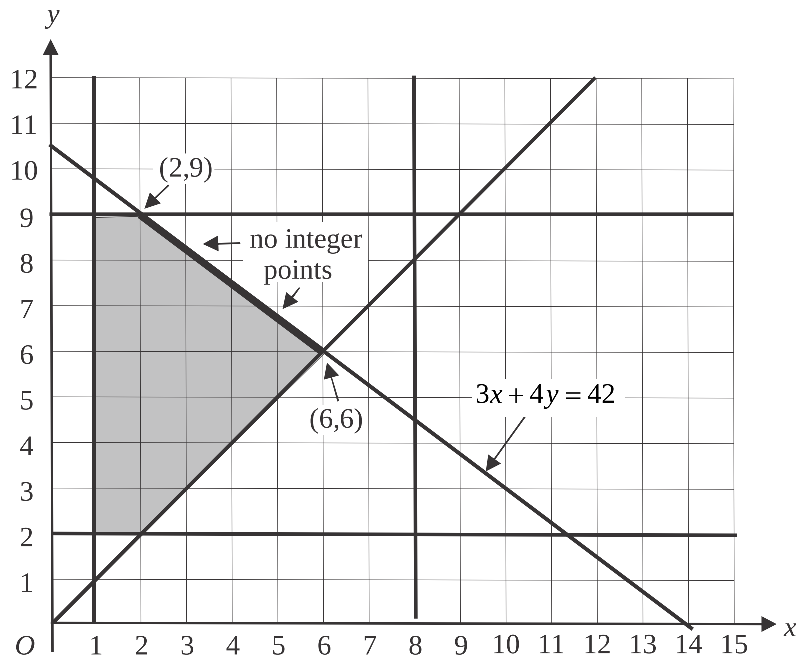
At $(2,9)$ $\quad P = 15\times 2 + 20\times 9$
$\quad = 210$

At $(6,6)$ $\quad P = 15\times 6 + 20\times 6$
$\quad = 210$

At $(1,9)$ $\quad P = 15\times 1 + 20\times 9$
$\quad = 195$

Maximum profit is \$210 and it occurs at $(2,9)$ and $(6,6)$. **(1 mark)**

Since two corner points give the same maximum profit then using the corner point principle, all the points along the line joining these two points will also give the same maximum profit. Note that this line is the boundary of Inequality 4, $3x + 4y \le 42$.
Looking at the graph we see that along this boundary line between the points $(2,9)$ and $(6,6)$ there are no points with integer (whole number) coordinates.



So the maximum profit occurs when $2 + 9 = 11$ or $6 + 6 = 12$ bikes and helmets are hired.
The smallest total number of bikes and helmets that need to be hired to produce a maximum profit is therefore 11.

(1 mark)