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Year 12 *Trial Exam Paper*

2018

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students must write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 31 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All responses must be written in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

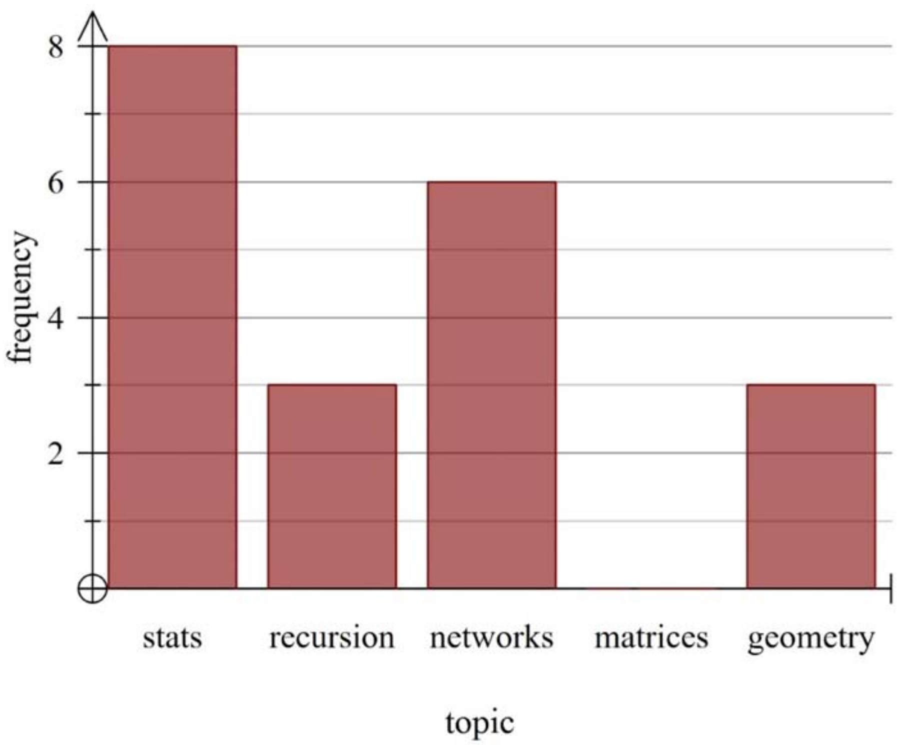
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (2 marks)

A mathematics teacher surveyed the 25 students in her Year 12 mathematics class to investigate their favourite topic studied in mathematics.

The graph below displays the information she gathered, with the data for matrices missing.



a. What percentage of the Year 12 students selected geometry as their favourite topic?

1 mark

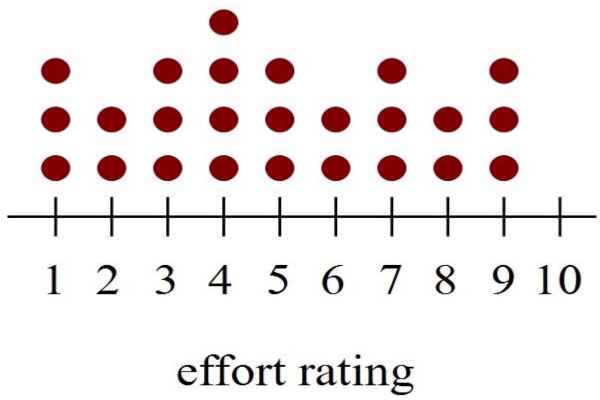
b. Complete the graph above by drawing the bar for the number of students who selected matrices as their favourite topic.

1 mark

Question 2 (3 marks)

The mathematics teacher continued her survey by asking the students to rate their effort in mathematics in previous years of study on a scale from 1 to 10.

The information obtained from the students is displayed in the dot plot below.



a. Find the median *effort rating* given by the students.

1 mark

___

b. Complete the following statement.

1 mark

☐ per cent of students gave an *effort rating* between 3 and 7.

c. Is the variable *effort rating* numerical discrete or numerical continuous?

1 mark

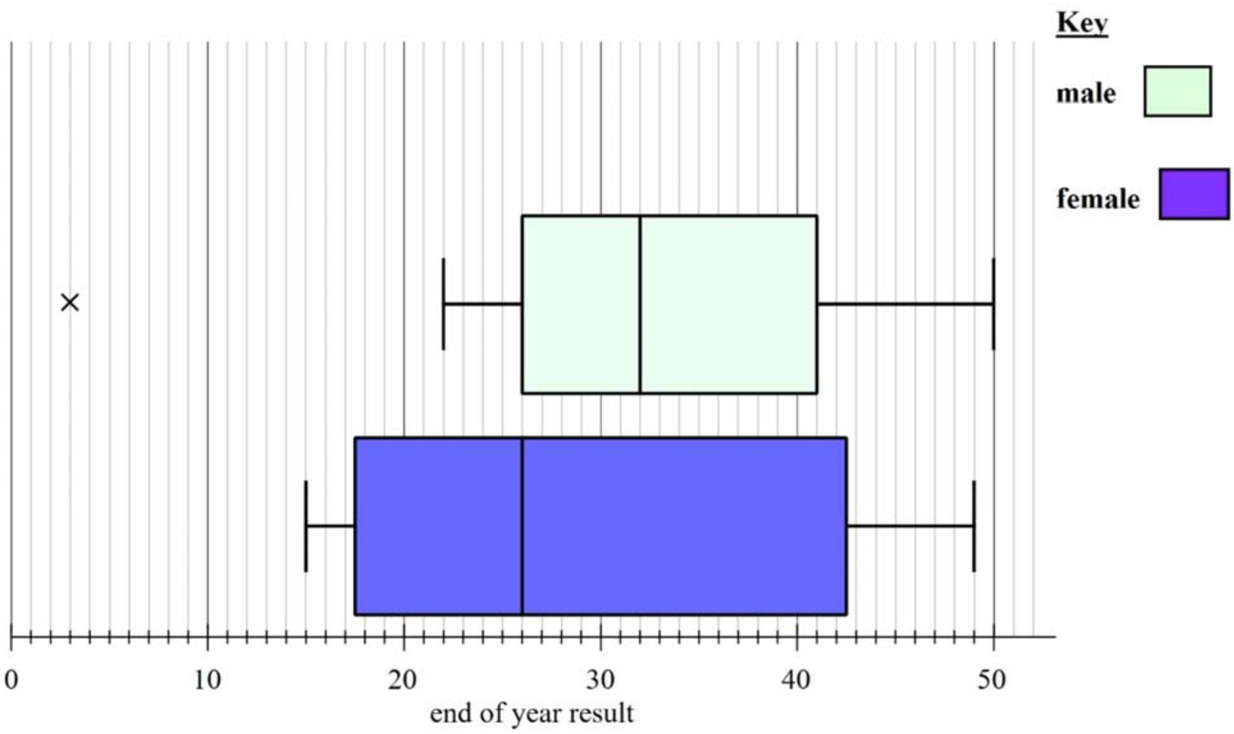
___

SECTION A – continued
TURN OVER

Question 3 (7 marks)

End of year results for the Year 11 mathematics exam were recorded by the teacher as a score out of 50. The teacher was interested in determining whether *gender* is related to *end of year result*.

The students' Year 11 mathematics results are shown in the graph below.



a. Describe the shape of the boxplot for **female** *end of year result*.

1 mark

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b. Complete the following five-number summary for **male** *end of year result*.

2 marks

	End of year result				
Gender	minimum	Q_1	median (M)	Q_3	maximum
Male		26	32		50
Female	15	17.5	26	42.5	49

c. Show that the male student who received a result of 3 out of 50 for his end-of-year examination is an outlier.

2 marks

d. The data shown in the boxplots above suggest that *end of year result* is associated with *gender*.

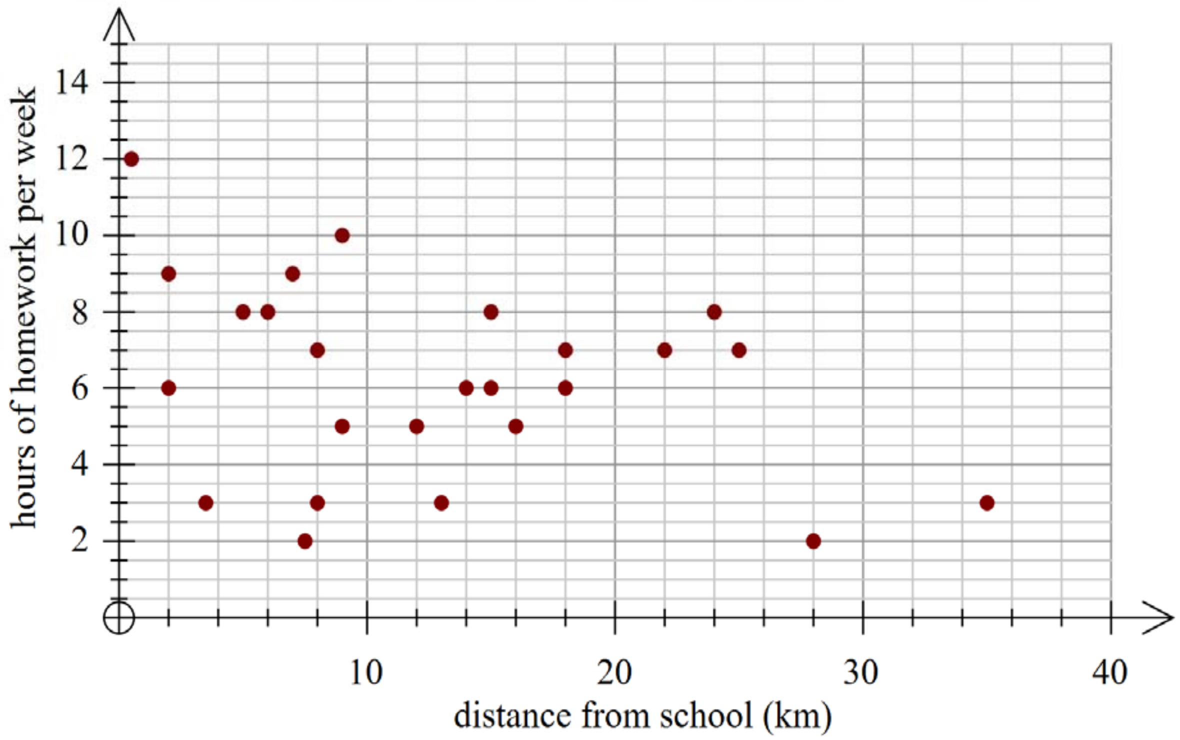
Explain why, quoting the values of an appropriate statistic.

2 marks

Question 4 (8 marks)

The teacher became concerned about the amount of homework her students were completing, and so began an investigation into the association between *hours of homework per week* and the *distance from school* for her students.

A scatterplot of the data that the teacher gathered is shown below.



The equation of the least squares regression line is

$$hours\ of\ homework\ per\ week = 7.46 - 0.0977 \times distance\ from\ school$$

a. On the scatterplot above, draw the least squares regression line.

2 marks

b. Describe the scatterplot in relation to its strength, direction and form.

1 mark

c. Interpret the slope of the regression line in terms of the variables *hours of homework per week* and *distance from school*.

1 mark

d. The value of the coefficient of determination for the scatterplot is 0.1117.
Find the value of Pearson's correlation coefficient, correct to three decimal places.

1 mark

e. The *hours of homework per week* is 3 when the *distance from school* is 8 km.

Determine the residual value if the least squares line is used to predict the *hours of homework per week* for this distance. Round your answer to two decimal places.

1 mark

f. The teacher has a new student in her class who lives 11 kilometres away from school.

i. Predict the number of *hours of homework per week* that this student can complete, correct to one decimal place.

1 mark

ii. Explain why this is an appropriate prediction.

1 mark

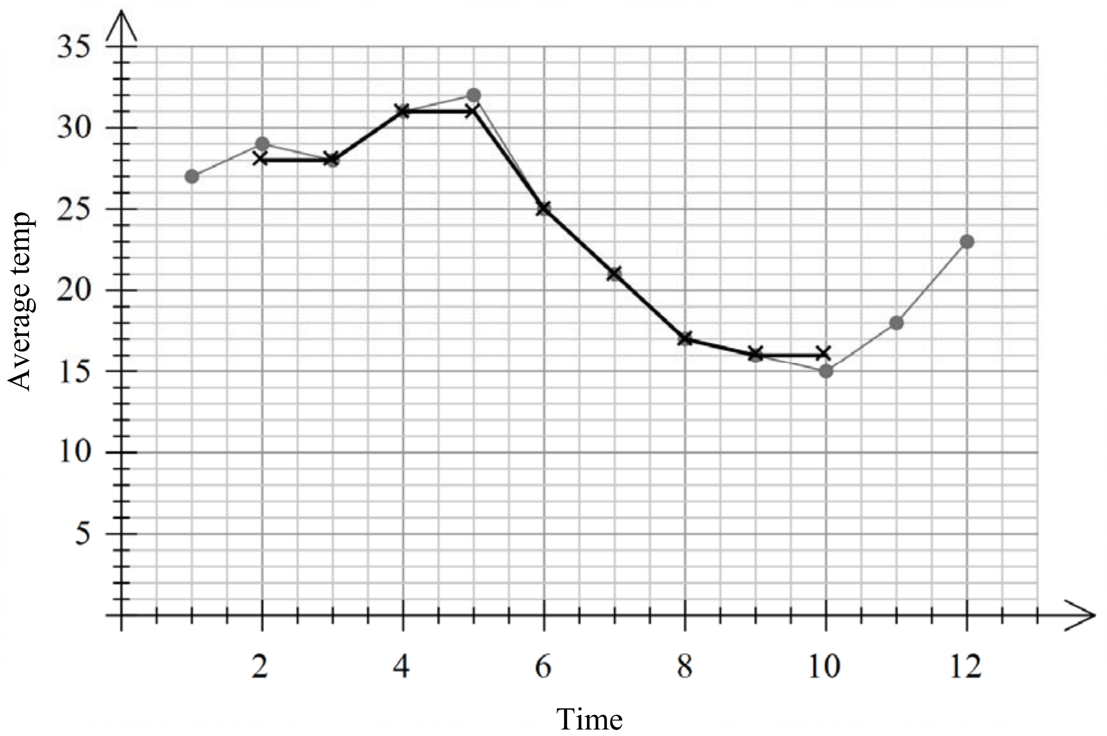
Question 5 (4 marks)

The following table shows the average temperature (°C) of the mathematics classroom, as recorded by the teacher, in the 12 months leading up to the students' final mathematics SAC.

The teacher was concerned that the room may be too hot for the students.

Time	Oct 2017	Nov 2017	Dec 2017	Jan 2018	Feb 2018	Mar 2018	Apr 2018	May 2018	Jun 2018	Jul 2018	Aug 2018	Sept 2018
Month no.	1	2	3	4	5	6	7	8	9	10	11	12
Temp (°C)	27	29	28	31	32	25	21	17	16	15	18	23

The data is also displayed graphically below.



a. The graph above shows both the original data for *average temperature* and the three-median smoothed data. The three-median smoothed value for August 2018 is missing.

Complete the graph for three-median smoothing above.

1 mark

The table below shows the seasonal indices (SI) for each month:

Time	Oct 2017	Nov 2017	Dec 2017	Jan 2018	Feb 2018	Mar 2018	Apr 2018	May 2018	Jun 2018	Jul 2018	Aug 2018	Sept 2018
SI	1.13	1.23	1.19	1.33	1.35	1.08	0.89	0.74	0.68	0.64	0.79	0.98

b. Calculate the deseasonalised value for June 2018, correct to one decimal place.

1 mark

The equation for the deseasonalised data is

$$deseasonalised\ \ temperature = 23.71 + 0.457 \times month$$

c. Use this equation to predict the temperature in the mathematics classroom on the day of the students' last mathematics SAC, which will occur in October 2018. Round your answer to the nearest whole degree.

2 marks

Recursion and financial modelling

Question 6 (6 marks)

Jonathan purchased a car for \$24 990.

The value of Jonathan's car was depreciated using the unit cost method of depreciation.

The value of the car, in dollars, after n kilometres, V_n, can be modelled by the rule shown below.

$$V_0 = 24\,990, \qquad V_n = 24\,990 - 0.0015n$$

a. Show that the value of the car after 20 000 kilometres is \$24 960.

2 marks

b. What is the cost, in cents per kilometre, that Jonathan's car is being depreciated?

1 mark

c. If Jonathan were to use the flat rate method of depreciation, it would take him 3 years to depreciate the car to \$23 191.

Find the annual flat rate of depreciation. Give your answer as a percentage. Round your answer to one decimal place.

1 mark

It was suggested to Jonathan that a reducing balance method of depreciation may be more appropriate to depreciate the value of his car.

d. If the car is depreciated at a reducing balance rate of 5.8% per annum, write down a recurrence relation, in terms of A_0, A_{n+1} and A_n, for the value of Jonathan's car, in dollars, after n years.

2 marks

Question 7 (6 marks)

In order to purchase his new car, Jonathan had to borrow \$15 000.

He was presented with the opportunity to borrow the money from his parents.

The arrangement with his parents can be modelled by the recurrence relation below, which shows the value, in dollars, of his loan, L_n, after n months,

$$L_0 = 15\ 000, \qquad L_{n+1} = 1.0022L_n - 200$$

a. Find the value, in dollars, of Jonathan's loan after 6 months.

1 mark

As an alternative, Jonathan can borrow the money from a bank.

The bank will offer Jonathan a \$15 000 loan with interest calculated monthly at a rate of 4.5% per annum. Jonathan will be required to make monthly payments to his loan and will repay his loan in 7 years.

b. What monthly payment will Jonathan make on his loan?

1 mark

c. How much interest, to the nearest cent, will Jonathan pay in the first 2 years of his loan?

2 marks

SECTION A – Question 7 – continued
TURN OVER

After 3 years, Jonathan feels that he is able to increase his monthly repayments to $325.

The interest rate on his loan will remain the same.

d. How much time, to the nearest month, will Jonathan save on his loan under these new conditions?

2 marks

END OF SECTION A

CONTINUES OVER PAGE

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (4 marks)

A local gymnasium offers members an everyday (E), frequent (F) and occasional (O) membership, based on how regularly they attend the gym. Membership is renewed on a monthly basis.

The number of members who chose each option for the first three months of 2018 is shown in the matrix G.

$$\begin{array}{cc} & \begin{array}{ccc} E & F & O \end{array} \\ G = & \begin{bmatrix} 125 & 76 & 35 \\ 107 & 95 & 52 \\ 95 & 120 & 43 \end{bmatrix} \begin{array}{l} \text{January} \\ \text{February} \\ \text{March} \end{array} \end{array}$$

a. In total, how many memberships were taken at the gym in February 2018?

1 mark

b. What does the element in row 2, column 3 of matrix G indicate?

1 mark

Consider the matrix equation

$$\begin{bmatrix} 125 & 76 & 35 \\ 107 & 95 & 52 \\ 95 & 120 & 43 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 19\,045 \\ 19\,474 \\ 19\,831 \end{bmatrix}$$

where a = cost of everyday membership, b = cost of frequent membership and c = cost of occasional membership.

c. Write the equation that represents the total membership revenue for February 2018.

1 mark

d. What is the cost, per month, of an occasional membership?

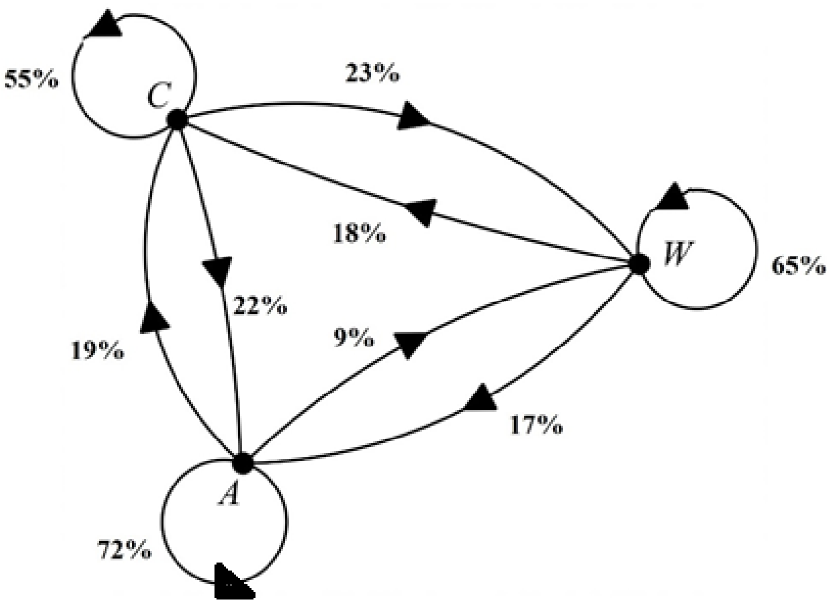
1 mark

Question 2 (3 marks)

Members at the gym are offered a selection of three group classes, which they can attend once a week during their monthly membership.

Members can choose from cycle (C), weights (W) and aerobics (A) classes.

The transition diagram below shows the way in which the gym members are expected to change their choice of group class from week to week.



a. Of the members who chose aerobics (A) in one week, what percentage are not expected to choose aerobics (A) the following week?

1 mark

___

Matrix S_1 lists the number of gym members who chose each exercise class in the first week of their month membership.

$$S_1 = \begin{bmatrix} 51 \\ 101 \\ 73 \end{bmatrix} \begin{matrix} C \\ W \\ A \end{matrix}$$

b. How many gym members are expected to choose the weights class (W) in their second week of membership?

1 mark

___

c. Of the members who chose the cycle class (C) in the first week, how many will remain in the cycle class in the third week of their membership?

1 mark

___

Question 3 (5 marks)

There are three competing fitness centres in the area, Fit Gym (*F*), Healthy Life (*H*) and Smash It (*S*).

Local citizens will choose between the three centres at the beginning of each year, although some people who were gym members choose to leave (*L*) and not sign up to a new centre.

The transition matrix *T* shows the way in which the local citizens are expected to change their choice of fitness centre from year to year.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{array}{cccc} F & H & S & L \end{array} \\ T = & \begin{bmatrix} 0.44 & 0.34 & 0.12 & 0 \\ 0.18 & 0.35 & 0.11 & 0 \\ 0.32 & 0.22 & 0.66 & 0 \\ 0.06 & 0.09 & 0.11 & 1 \end{bmatrix} \begin{array}{c} F \\ H \\ S \\ L \end{array} \quad \textit{next year} \end{array}$$

Let M_1 be the state matrix for the number of citizens expected to choose each fitness centre in 2018.

$$M_1 = \begin{bmatrix} 120 \\ 120 \\ 120 \\ 0 \end{bmatrix} \begin{array}{c} F \\ H \\ S \\ L \end{array}$$

a. How many people will change to a different fitness centre at the beginning of 2019?

1 mark

___

___

b. Complete M_3, the state matrix for 2020, below. Round element values to the nearest whole numbers.

1 mark

$$M_3 = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix} \begin{array}{c} F \\ H \\ S \\ L \end{array}$$

c. How many people will choose Smash It (S) in 2022?

1 mark

___

d. The manager at Healthy Life (*H*) is concerned that the membership numbers are decreasing rapidly. If the number of members falls below 25% of the enrolments in 2018, the fitness centre will be forced to close.

The manager believes that he will have to close his fitness centre by 2025.

Is the manager correct in his assumption? Use evidence to support your answer.

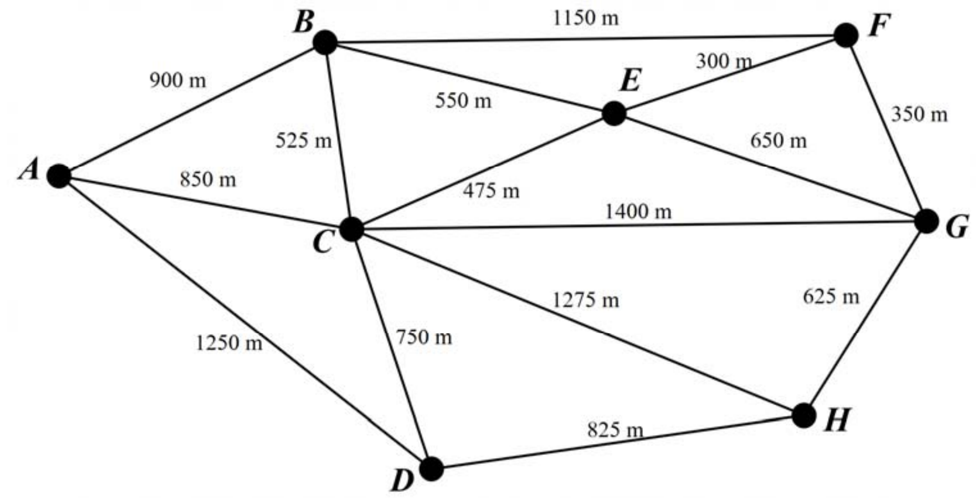
2 marks

Module 2 – Networks and decision mathematics

Question 1 (6 marks)

Michelle is planning a fun run to raise money for charity.

The graph below shows a series of roads that Michelle is considering using for her running event. Included on the graph are a series of landmarks, *A* to *H*, and the distances between these.



a. The checkpoints on the graph represent significant buildings or commemorative parks within the town. It is Michelle's intention to ensure that each of these is included in the running path.

Write down one possible circuit, starting and ending at *C*, that the runners could follow to ensure that they pass each checkpoint. Give the length of this circuit in metres.

2 marks

b. The roads must be checked for potholes or other hazards prior to the fun run being given approval by the local council. It is not possible for a council inspector to plot his way through the town without passing over a number of roads more than once.

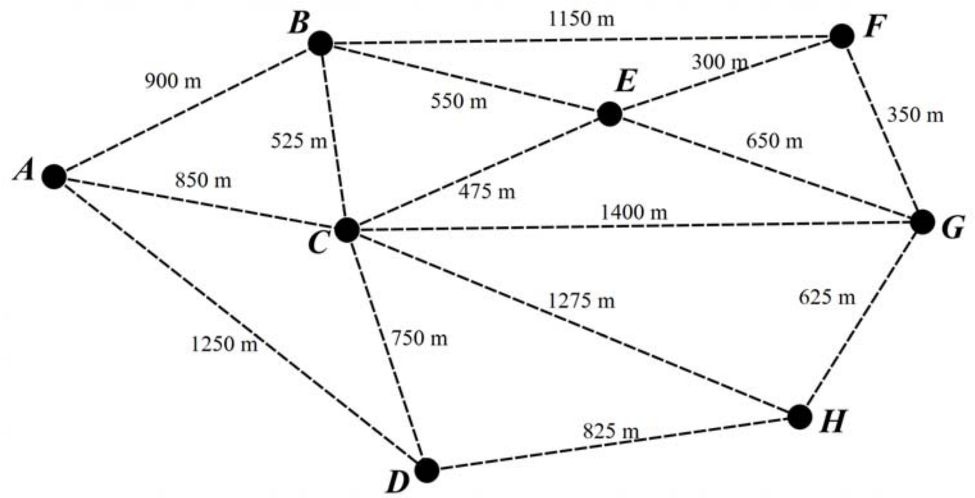
Explain why this is the case, with reference to the degree of the vertices in the network.

1 mark

c. For the safety of all participants, emergency services must be provided with a path to each checkpoint should an injury occur. Emergency services will position themselves at point A during the event.

i. On the diagram below, sketch the minimum spanning tree that allows emergency services to connect to each of the checkpoints.

1 mark



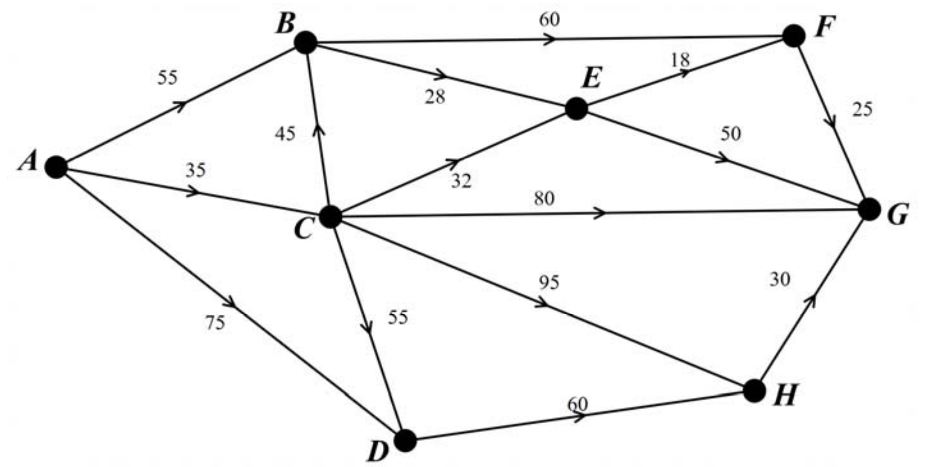
ii. What is the length of this minimum spanning tree?

1 mark

___

The graph below shows another version of the running course that documents the number of people who can safely run along each road in an hour and the direction they can run in.

Michelle is considering an under 12s event before the fun run, the purpose of which would be to get as many children as possible to run through the town wearing coloured T-shirts and costumes.



d. If the children must start at A and finish at G, what is the maximum number of children who can run through the town in one hour?

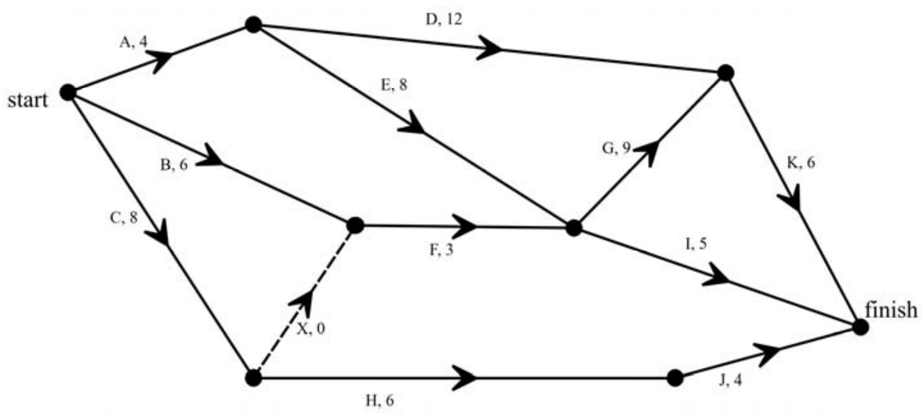
1 mark

___

Question 2 (6 marks)

It was decided that the roads must be re-surfaced prior to the running event. The project involves activities *A* to *K*.

The directed graph below shows these activities and their completion time in days.



a. Write down the activities that must be completed before activity *G* can begin. 1 mark

____

b. Find the minimum completion time for this project and write down the critical path. 2 marks

____

____

____

c. Determine the float time, in days, for activity *H*. 1 mark

____

d. The project can be completed earlier if some of the activities are crashed.

Activities *E*, *F*, *G* and *I* can be crashed by a maximum of 2 days.

The cost of crashing is $2000 per activity per day.

i. What is the minimum number of days in which the project can now be completed? 1 mark

____

____

ii. What is the minimum cost of completing the project in this time? 1 mark

____

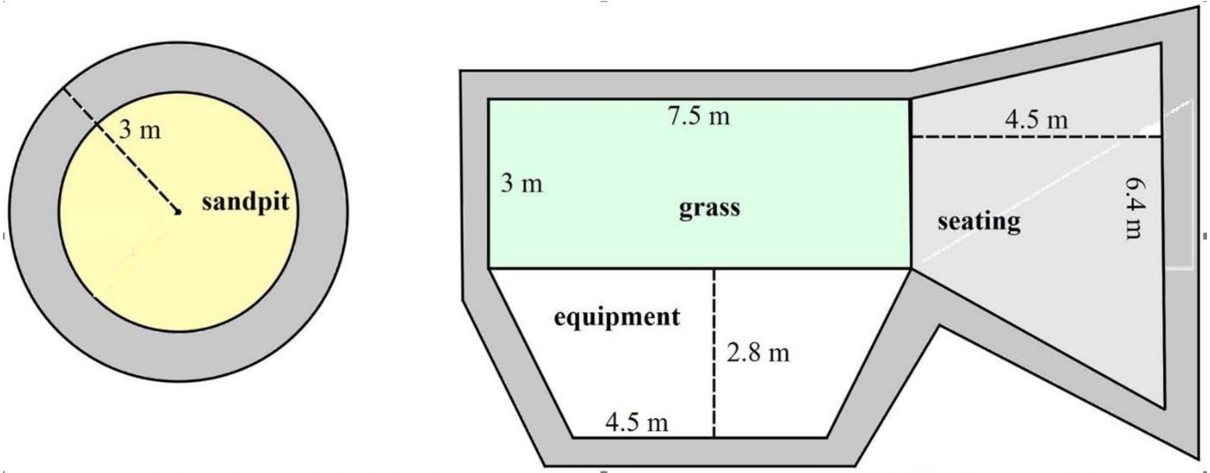
____

Module 3 – Geometry and measurement

Question 1 (4 marks)

Belinda is designing the outdoor play area for a new early childhood centre to be opened in her local area.

Below is her initial sketch, not to scale, of what this area might look like.



Belinda has chosen to include a 1.5 metre wide concrete path around all play areas. This is also shown in her sketch.

a. A fence will be installed around the grass area and the sandpit, with the path placed outside this fence. Calculate the length of fencing, to the nearest metre, required for these two areas.

1 mark

b. The equipment and seating areas will be fitted with a non-slip material. Calculate the area that must be covered by this non-slip material, to the nearest square metre.

1 mark

c. If the sandpit is to be 1.3 metres deep, calculate the volume, to the nearest whole cubic metre, of sand required to fill the pit.

1 mark

d. The centre will be a two-storey building, with all staff areas having a clear view of the outdoor play area from the second storey, which is 3.5 metres above the ground.

If Belinda is standing in her office, she has a clear view into the sandpit along an angle of depression of $38°$.

Find the horizontal distance from Belinda's office to the sand pit, correct to one decimal place.

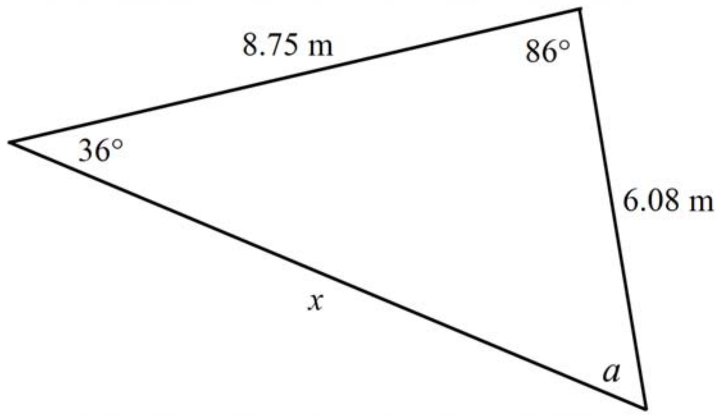
1 mark

Question 2 (5 marks)

A second outdoor passive area will be created within the early childhood centre for older children who attend an after-school care program. The area will contain seating and down ball courts, all of which will be covered by shade sails.

The company that supplies the shade sails offers two sizes.

A diagram of the small sail is shown below.



a. What is the value of the unknown angle, a, in this triangle?

1 mark

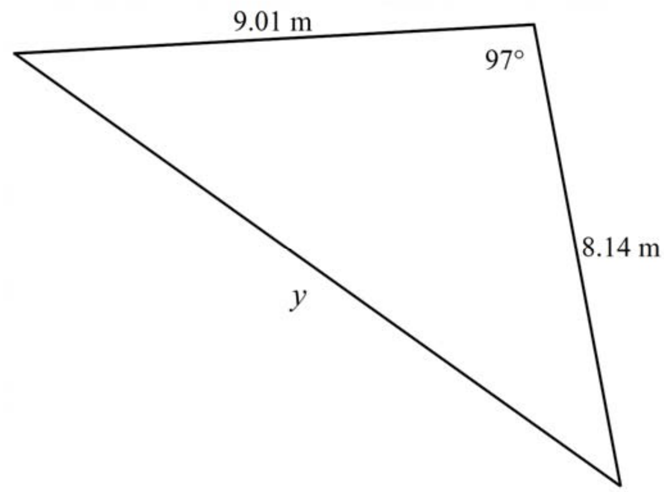
b. Find the length of x, to the nearest metre.

1 mark

c. What is the total area that this shade sail will provide shade for? Give your answer to the nearest square metre.

1 mark

A diagram of the larger sail is shown below.



d. Use the cosine rule to show that the length of y is 12.9 metres.

2 marks

Question 3 (3 marks)

Belinda lives in Mildura $(34° \text{ S}, 142° \text{ E})$ and has found a childcare consultant in Beijing $(40° \text{ N}, 116° \text{ E})$ who will help to refine her design for the play area. She organises to speak to the consultant on the phone at 9:30 am on Tuesday morning.

The time difference between Mildura and Beijing is 3 hours at that time of year.

a. On what day and at what time will Belinda speak to the consultant in Beijing?

1 mark

b. Assume that the radius of the Earth is 6400 km.

Find the shortest great circle distance to the equator from Beijing. Round your answer to the nearest kilometre.

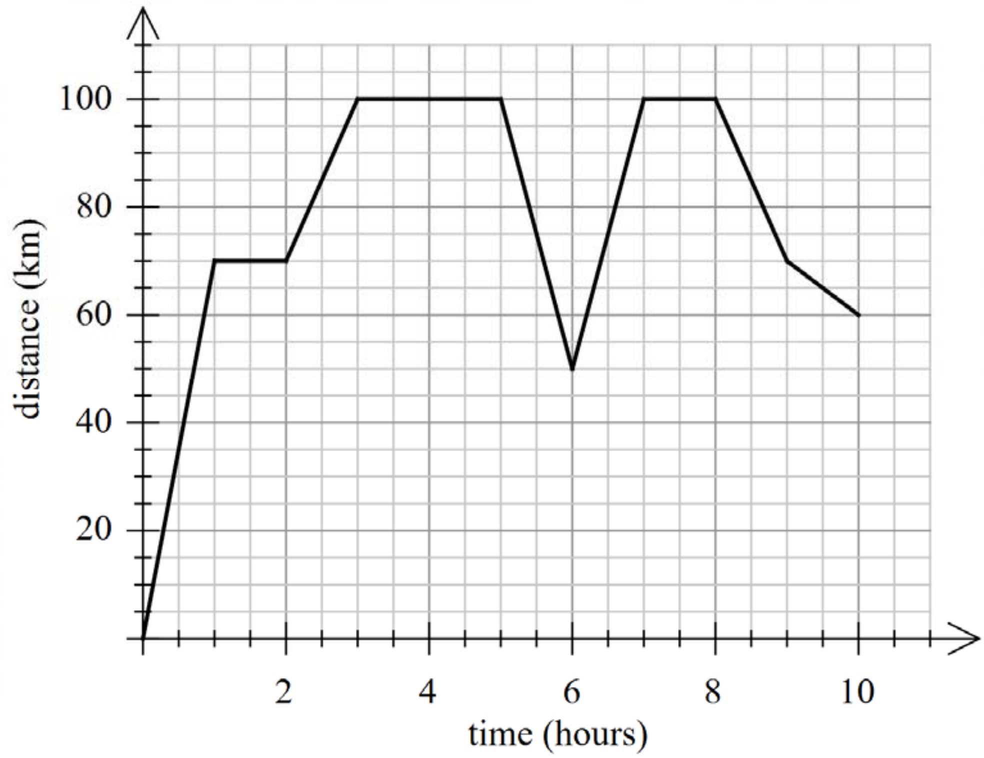
2 marks

Module 4 – Graphs and relations

Question 1 (5 marks)

A family is driving from their home in Geelong to a family member's house on Christmas Eve.

The graph below shows the distance–time graph for their journey.



a. How far has the family travelled in the first 3 hours of their trip?

1 mark

____

b. What does the horizontal line between 3 and 5 hours represent?

1 mark

____

____

c. Find the equation for the line between 5 and 6 hours in the form $y = mx + c$.

2 marks

d. Show that the slope for the line between 8 and 9 hours is -30.

1 mark

Question 2 (7 marks)

To ensure the safety of the family during their car trip, constraints must be placed on the amount of driving that the mother and father can do in one day.

Let x be the number of hours that the mother can drive.

Let y be the number of hours that the father can drive.

Constraint 1 $x \leq \dfrac{y}{3}$

a. Interpret Constraint 1 in terms of the number of hours that the mother and father can drive in one day.

1 mark

b. If the father drives 8 hours in one day, how many hours can the mother drive? Give your answer in hours and minutes.

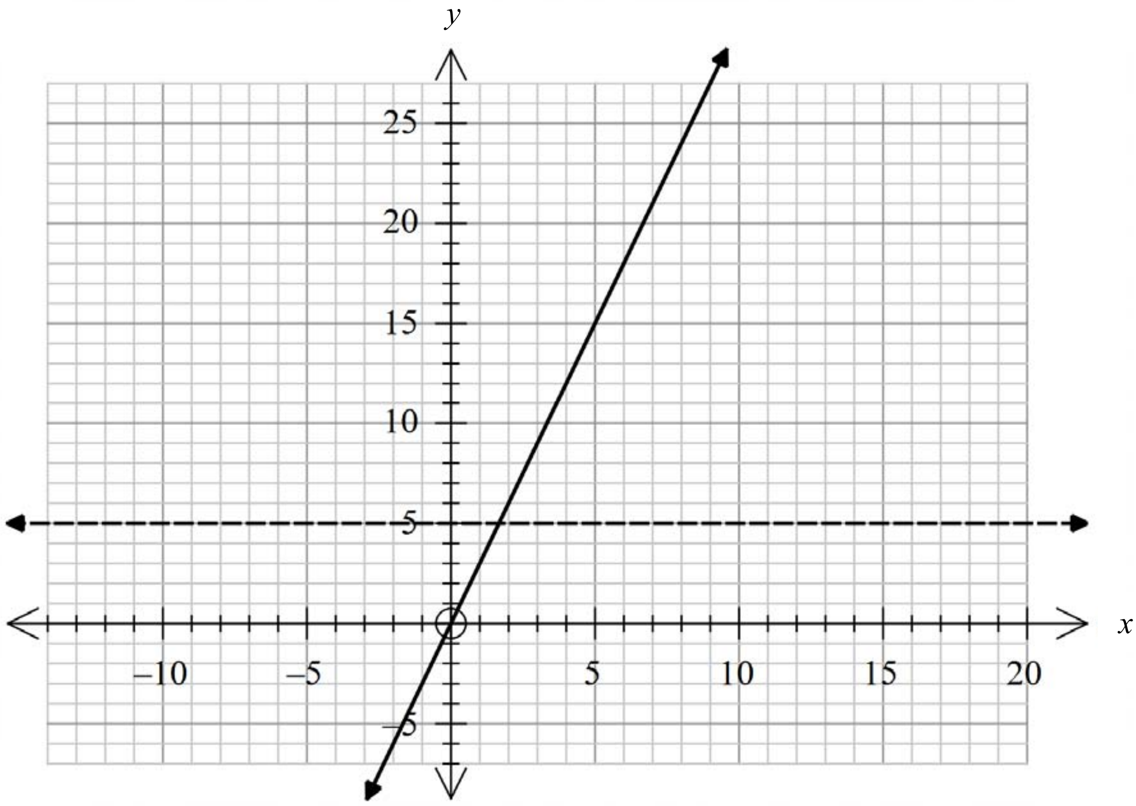
1 mark

There are two other constraints

Constraint 2 $x + y \leq 15$

Constraint 3 $y > 5$

The graph below shows the lines representing Constraints 1 and 3.



c. Sketch the line for Constraint 2 on the graph above.

2 marks

It is decided that the family's 18-year-old son can also contribute to the driving.

Let x be the total number of hours that the parents can drive per day.

Let y be the number of hours that the son can drive per day.

The following constraints exist:

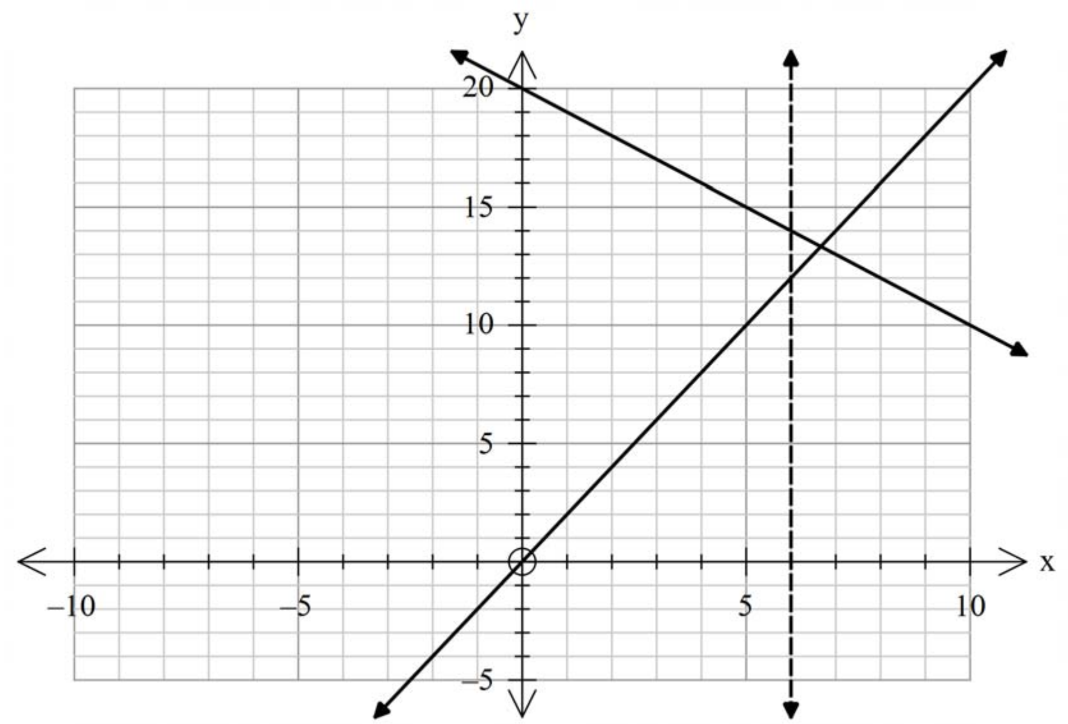
Constraint 1 $y \geq 2x$

Constraint 2 $x < 6$

Constraint 3 $x + y \leq 20$

Constraint 4 $x > 0$

These are shown on the graph below.



d. On the graph above, shade the region that satisfies Constraints 1 to 4.

1 mark

e. What is the maximum amount of driving that can be done by the family in one day?

2 marks

END OF QUESTION AND ANSWER BOOK