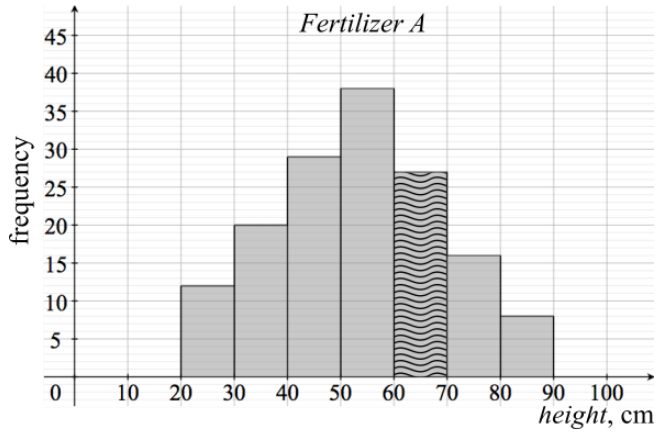
FURTHER MATHEMATICS – UNITS 3&4
2018
Written examination 2 Solutions

SECTION A – Core

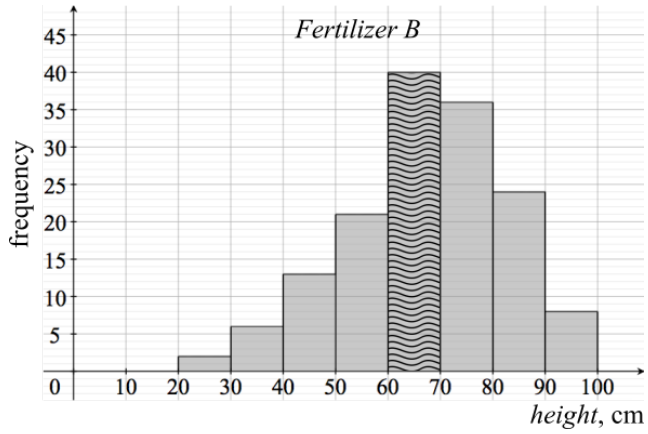
Data analysis

Question 1 (9 marks)

a.



1A



1A

b. Modal class = 50 –< 60 cm **1A**

c. Negatively skewed. **1A**

d. mean $= \dfrac{\Sigma(xf)}{n}$ **1M**

$$= \frac{2\times25+6\times35+13\times45+21\times55+40\times65+36\times75+24\times85+8\times95}{150}$$

$= 67.33$

≈ 67 cm **1A**

e. i. 53.5 + 15.8 = 69.3 cm ⇒ above one standard deviation to the right of the mean: $\dfrac{100-68}{2} = 16\%$ **1A**

ii. $53.5 - 2 \times 15.8 = 21.9$ cm ⇒ below two standard deviations to the left of the mean: $\frac{100-95}{2} = 2.5\%$ **1A**

f. z – score $= \frac{x-\mu}{\sigma}$

$= \frac{26.6-53.5}{15.8}$

$= -1.70$

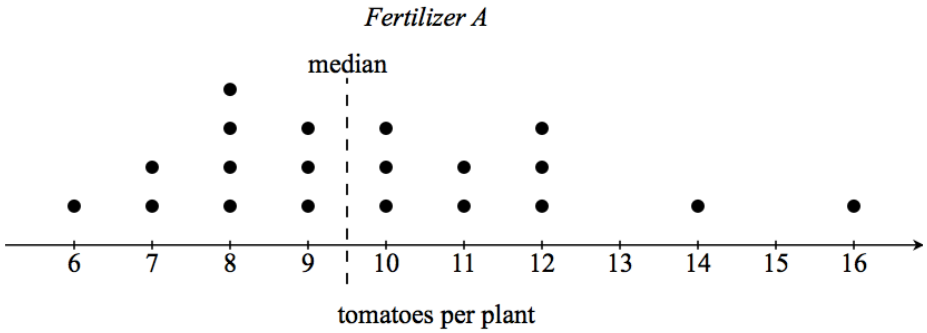
A height of 26.6 cm is 1.7 standard deviations to the left of the mean. **1A**

Question 2 (5 marks)

a. The variable *Type of fertilizer* (*Fertilizer A* and *Fertilizer B*) is categorical nominal. **1A**

The variable *Number of tomatoes per plant* is numerical discrete. **1A**

b.



median $= \frac{9+10}{2} = 9.5$ **1A**

c. IQR $= Q_3 - Q_1$

$= 14 - 11$

$= 3$ **1A**

Lower fence $= 11 - 1.5 \times 3$

$= 11 - 4.5$

$= 6.5 \Rightarrow 6 < 6.5$, hence it is an outlier … as required **1A**

Question 3 (5 marks)

a. Using available CAS technology, use the *diameter* as the explanatory variable and *weight* as the response variable.

weight $= 36.9 + 12.3 \times$ *diameter* **2A**

b. 86.6% of the variation in the *weight* of tomatoes can be explained by the variation in the *diameter* of tomatoes. **1A**

c. Substitute *diameter* = 5.4 into the equation *weight* $= 42.6 + 11.1 \times$ *diameter*.

weight $= 42.6 + 11.1 \times 5.4$

$= 102.54$ grams **1A**

d. Residual = actual – predicted

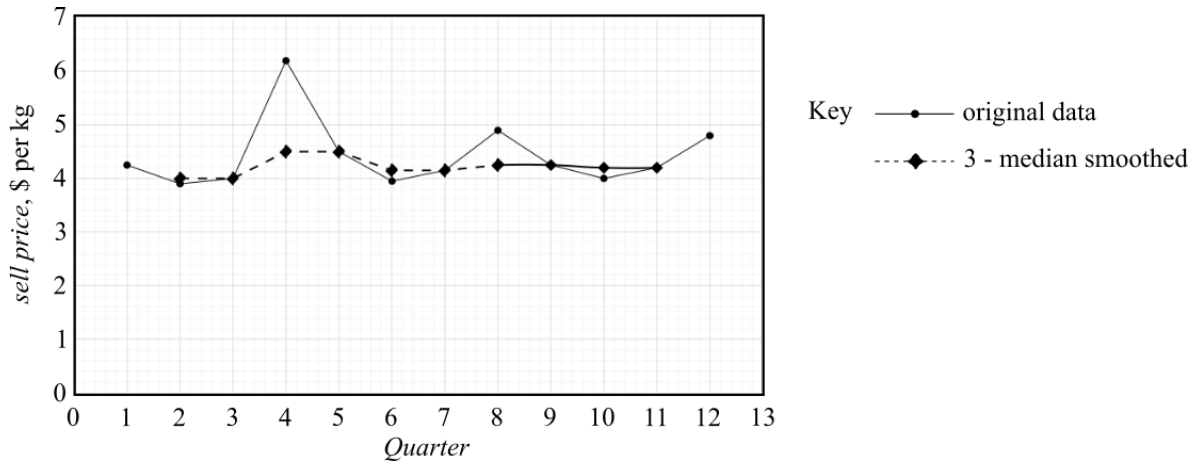
$= 98 - 102.54$

$= -4.54$ **1A**

Question 4 (5 marks)

a. \$4.25 **1A**

b.



1A

c.

Year	*Season*	*Quarter*	*Price*, \$	*4-mean smoothing*	*Centring*
2016	Spring	1	4.25		
	Summer	2	3.90	4.5875	
	Autumn	3	4.00	4.65	4.61875
	Winter	4	6.20		
2017	Spring	5	4.50		
	Summer	6	3.95		
	Autumn	7	4.15		
	Winter	8	4.90		
2018	Spring	9	4.25		
	Summer	10	4.00		
	Autumn	11	4.20		
	Winter	12	4.80		

\$4.62 **1A**

d. $0.98 + S + 0.93 + 1.20 = 4 \Rightarrow S = 0.89$ **1A**

e. Re-seasonalised data = predicted (de-seasonalised) × seasonal index

$= 5.10 \times 0.98$

= \$4.998

≈ \$5 **1A**

Recursion and financial modelling

Question 5 (6 marks)

a. $d = \dfrac{h_4 - h_0}{4} = \dfrac{470 - 350}{4}$

$d = 30$ mm **1A**

b. $h_{n+1} = h_n + 30$ **1A**

c. Substitute h_0 and h_1 into $h_1 = ah_0 + b \Rightarrow 440 = 500a + b$ … [1] **1M**

Substitute h_1 and h_2 into $h_2 = ah_1 + b \Rightarrow 392 = 440a + b$ … [2]

Use available CAS technology to solve the system of simultaneous equations [1] and [2].

$a = 0.8$ and $b = 40$

$h_{n+1} = 0.8h_n + 40$ … as required **1A**

Altenative 'by hand' method

Subtract [1] – [2]: $48 = 60a \Rightarrow a = \dfrac{48}{60} = 0.8$

Substitute $a = 0.8$ into [1]: $440 = 500 \times 0.8 + b \Rightarrow b = 440 - 400 = 40$

$h_{n+1} = 0.8h_n + 40$ … as required

d. The 4th picket has height $h_3 = 0.8h_2 + 40$.

$h_3 = 0.8 \times 392 + 40$

$= 353.6$ mm **1A**

The 5th picket has height $h_4 = 0.8h_3 + 40$.

$h_3 = 0.8 \times 353.6 + 40$

$= 322.88$

≈ 323 mm **1A**

Question 6 (3 marks)

a. 26000 – 16250 = \$9750 **1A**

b. 16250 ÷ 5 = \$3250 per year $\Rightarrow$ Depreciation per unit $= \dfrac{3250}{15000}$

$= \$0.217$

$\approx \$0.22$ **1A**

c. 26000 ÷ 3250 = 8 years

8 – 5 = 3 years **1A**

Question 7 (3 marks)

a. Using finance solver on available CAS technology

N: 36

I(%): **6.2** **1A**

PV: -14000

Pmt: -100

FV: 20800

PpY: 12

CpY: 12

b. account balance = $20800\left(1+\frac{8.5}{400}\right)+300$ or account balance = $20800 \times 1.02125 + 300$ **1A**

c. Using finance solver on available CAS technology
N: 1
I(%): 8.5 **1A**
PV: -20800
Pmt: -300
FV: 21542
PpY: 4
CpY: 4

SECTION B – Applications

Module 1 – Matrices

Question 1 (5 marks)

a.

$$W_1 = \begin{bmatrix} T & U & V \\ 12.50 & \underline{13.75} & \underline{17.20} \end{bmatrix}$$ **1A**

$$W_2 = \begin{bmatrix} T & U & V \\ \underline{18.00} & 21.50 & \underline{25.60} \end{bmatrix}$$ **1A**

b. Order = 1×5 **1A**

c. $W_2 R = \begin{bmatrix} 188.05 & 251.40 \end{bmatrix}$ **1A**

d. \$188.5 = total amount of money earned by the three shop assistants on Saturday
\$251.40 = total amount of money earned by the three shop assistants on Sunday **1A**

Question 2 (4 marks)

a. 22 + 106 + 30 = 158 items **1A**

b. $\det(M) = 2622 \neq 0 \Rightarrow$ the matrix is invertible **1A**

c. $\begin{bmatrix} c \\ s \\ j \end{bmatrix} = \begin{bmatrix} \underline{17} & \underline{-10} & \underline{-3} \\ \underline{-13} & \underline{8} & \underline{2} \\ \underline{21} & \underline{-13} & \underline{-3} \end{bmatrix} \times \begin{bmatrix} \underline{1127} \\ \underline{1517} \\ \underline{1287} \end{bmatrix}$ **1A**

d. Using available CAS technology solve the matrix equation from **part c.** for c, s and j.
$c = 128$, $s = 59$ and $j = 85$ **1A**

Question 3 (3 marks)

a. $\begin{bmatrix} 0.32 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.10 \end{bmatrix} \times \begin{bmatrix} 200 \\ 380 \\ 160 \end{bmatrix} = \begin{bmatrix} 64 \\ 19 \\ 16 \end{bmatrix} \Rightarrow$ 19 shirts **1A**

b. $B = \begin{bmatrix} 200 \\ 380 \\ 160 \end{bmatrix} - \begin{bmatrix} 64 \\ 19 \\ 16 \end{bmatrix} = \begin{bmatrix} 136 \\ 361 \\ 144 \end{bmatrix}$ **1A**

c. $S_1 = TS_0 + B$

$S_2 = TS_1 + B$

$= T(TS_0 + B) + B$

$= T^2S_0 + TB + B$ **1A**

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

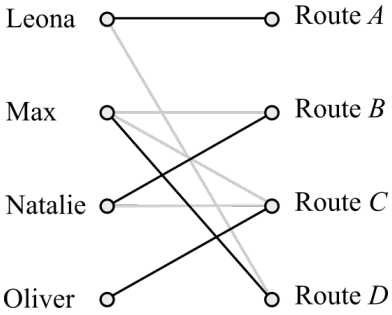
a.

	Leona	Max	Natalie	Oliver
Route *A*	0	1	3	5
Route *B*	4	0	0	1
Route *C*	2	0	0	0
Route *D*	0	0	1	4

The minimum number of lines that cover the 0s is four, which is equal to the number of rows. **1A**

b. Oliver can be assigned a route first because there is only one 0 in his column. **1A**
If the lines are crossed horizontally, then Leona can be assigned a route first because there is only one 0 in her row.

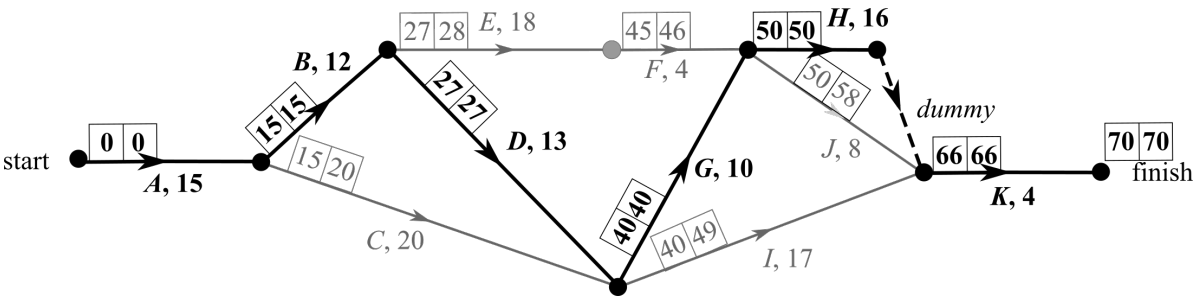
c. Leona (*A*; 10 min) + Oliver (*C*; 9 min) + Natalie (*B*; 12 min) + Max (*D*; 12 min) = 43 min **1A**



Question 2 (5 marks)

a. The *dummy* activity is introduced to clearly show the precedence of activity *H* to activity *K*. **1A**

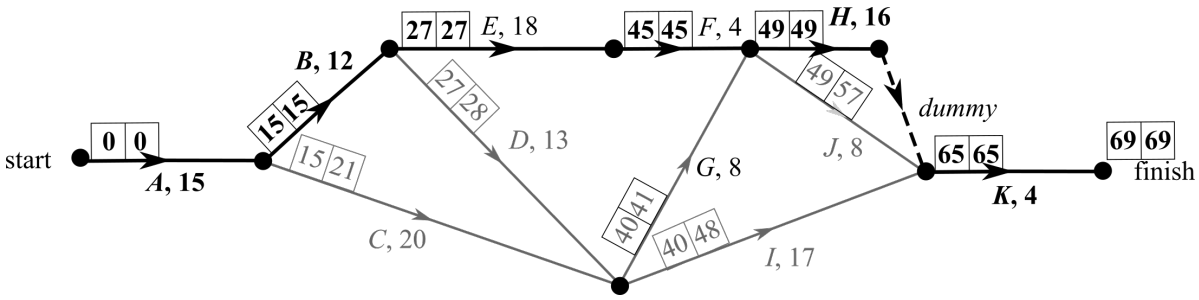
b. The critical path includes activities *A*, *B*, *D*, *G*, *H*, *dummy* and *K*. **1A**



The completion time is 70 minutes or 1 hour and 10 minutes. Therefore the organisers will be ready to start the race as planned since they have 1 hour and 20 minutes available. **1A**

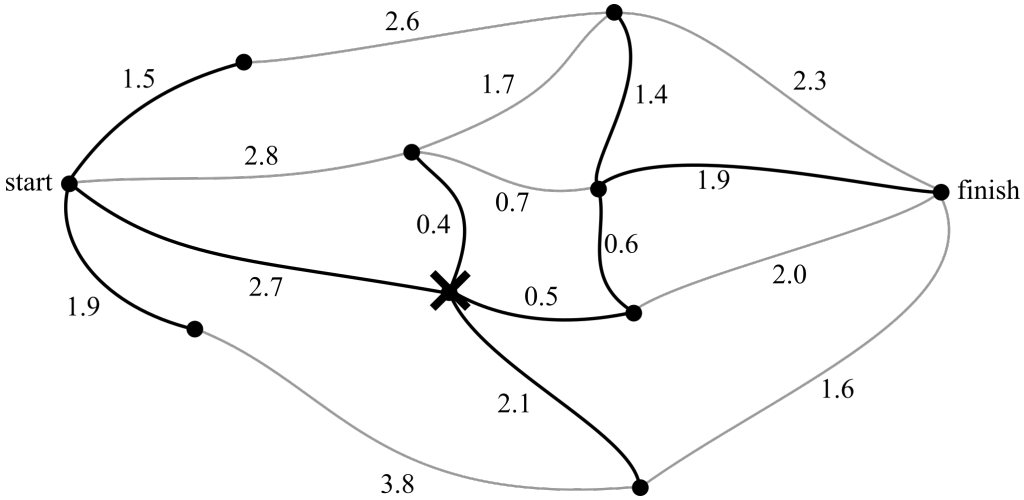
c. Activities *E* and *F* have a float time of 1 minute. **1A**

d. Reduces the completion time by 1 minute and the critical path to *A*, *B*, *E*, *F*, *H*, *dummy* and *K*. **1A**



Question 3 (4 marks)

a.



2A

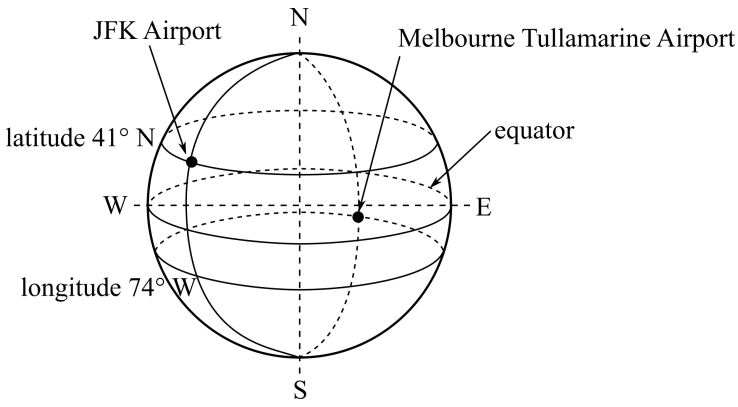
b. No. The minimum spanning tree should have edge 1.6 instead of edge 2.1. **1A**

c. Hamiltonian circuit. **1A**

Module 3 – Geometry and trigonometry

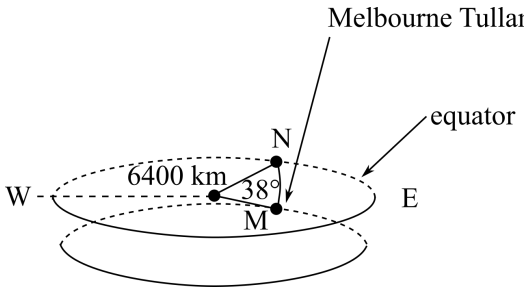
Question 1 (5 marks)

a.



2A

b.



$$\text{Arc length } MN = 2\pi r\frac{\theta°}{360°}$$
$$= 2\pi \times 6400 \times \frac{38°}{360°}$$
$$= 4244.64 \text{ km} \quad \textbf{1A}$$

c. Difference in longitude = 145° – (–74°)
= 219°

1 hour ≈ 15° $\Rightarrow \frac{219°}{15°}$ = 14.6 ≈ 15 hours **1A**

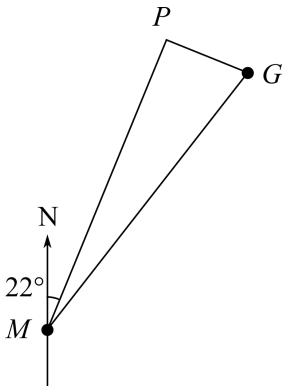
d. Melbourne is about 15 hours ahead of New York. This means that when Corrine's flight leaves Melbourne, the time in New York is 11:15 am – 15 hours = 8:15 pm on the 20th of December.
From 8:15 pm on the 20th of December to 4:30 pm on the 21st of December there are 20 hours 15 minutes.
Flight time = 20 hours 15 minutes **1A**

Question 2 (4 marks)

a. $\theta = 360° - 65° = 295°\text{T}$ **1A**

b. Using Pythagoras theorem, $MG = \sqrt{1.2^2 + 0.24^2} = 1.22$ km **1A**

c.



$$\angle PMG = \tan^{-1}\left(\frac{0.24}{1.20}\right) = 11° \quad \textbf{1A}$$

Bearing = 22° + 11° = N33°E **1A**

Question 3 (3 marks)

a. $\text{Area}_{\text{large sector}} = \frac{104°}{360°} \times \pi \times 30^2 = 816.814 \text{ m}^2$

$\text{Area}_{\text{small sector}} = \frac{104°}{360°} \times \pi \times 16^2 = 232.338 \text{ m}^2$

$\text{Area} = \text{Area}_{\text{large sector}} - \text{Area}_{\text{small sector}}$
$= 816.814 - 232.338$
$= 584.476 \text{ m}^2$ **1A**

$\text{Volume} = 584.476 \times 0.1$
$= 58.4476$
$\approx 58 \text{ m}^3$ **1A**

b. Arc length = $\frac{\theta^\circ}{360^\circ} \times 2\pi \times \text{radius}$

$\theta^\circ = \frac{360^\circ \times \text{Arc length}}{2\pi \times \text{radius}}$

$= \frac{360^\circ \times 9}{2\pi \times 65}$

$= 7.93$

$\approx 8^\circ$ **1A**

Module 4 – Graphs and relations

Question 1 (4 marks)

a. Time = 1 hour and 36 minutes = 1.6 hours

Average speed = $\frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{120}{1.6}$

$= 75 \text{ kmh}^{-1}$ **1A**

b. The volume of petrol per kilometre = $\frac{10-5}{0-50}$

$= -0.1$

= decrease of 0.1 L per km **1A**

c. Before 50 km, from **part b.**, the change in the volume of petrol per kilometre = –0.1

After 50 km, the change in the volume of petrol per kilometre = $\frac{25-20}{50-120}$

$= -0.07$ L per km

The car used more petrol per kilometre in the first 50 km. **1A**

d. The tank was filled with petrol from 5 L to 25 L. **1A**

Question 2 (4 marks)

a. Substitute $t = 1$ and $d = 3.5$ into $d = kt^2$.

$3.5 = k \times 1^2$

$k = 3.5$ **1A**

b. Average speed = $\frac{87.5-0}{5-0}$

$= 17.5 \text{ ms}^{-1}$ **1A**

c. $56 = 3.5 \times t^2$

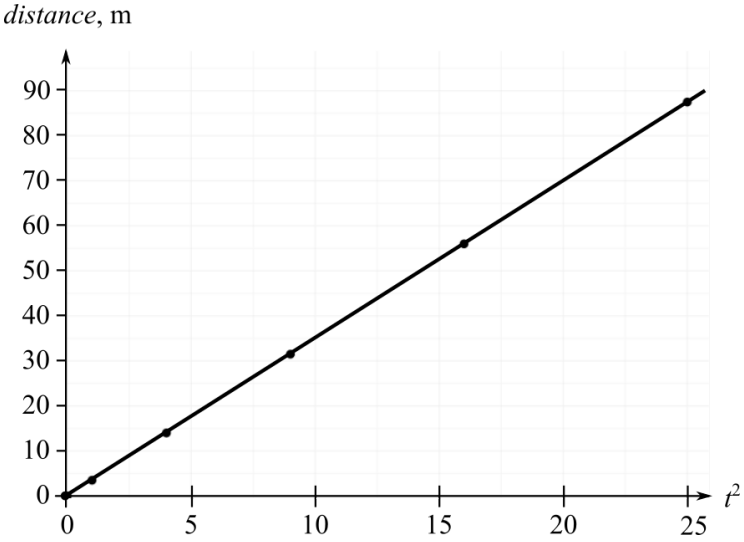
$t^2 = \frac{56}{3.5}$

$= 16$

$t = \sqrt{16}$

$= 4$ s **1A**

d.



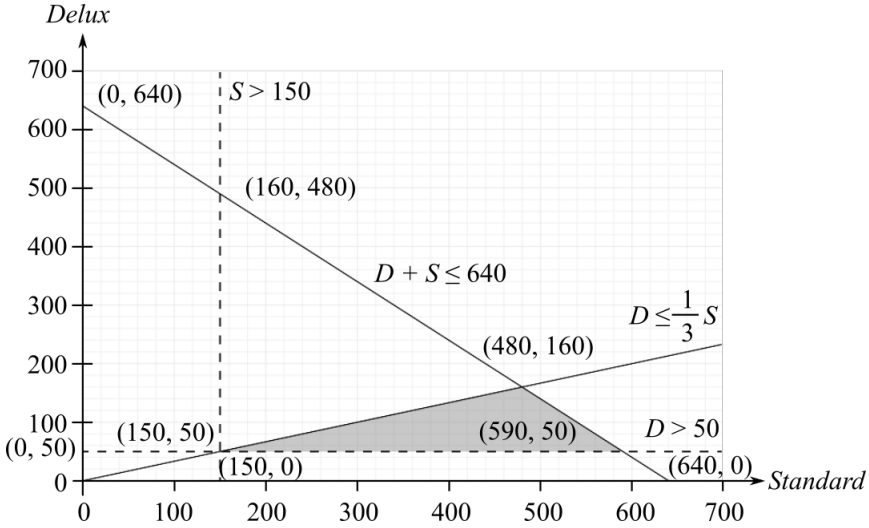
1A

Question 3 (4 marks)

a. Revenue = $300 \times 150 + 500 \times 50 = \$70000$
Cost = $45000 + 120 \times 150 + 140 \times 50 = \$70000$
Cost = Revenue for 150 *standard* snowboards per day and 50 *delux* snowboards per day as required. **1M&A**

b. $S \geq 3D$ **1A**

c.



2A