Neap:

Trial Examination 2018

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 E

The question asks for an average so **D** is incorrect. **A**, **B** and **C** are incorrect calculations. Removing the 22.0 figure (which is an outlier) allows the mean of 4.5, 1.3, 3.2, 1.1, 3.2 and 5.9 to be found.

Question 2 D

Seasonally adjusting the raw figures by dividing the raw figures by the relevant seasonal index shows the adjusted sales figure for Thursday is 200 sales, which is the smallest adjusted figure.

Question 3 C

A scatterplot is used for bivariate numerical data, a histogram for continuous numerical data, a stem and leaf plot shows patterns in numerical data and a time series plot is bivariate (numerical data versus time). This leaves a frequency table as the best method to represent a categorical set of data.

Question 4 C

6 out of 10 males voted yes in the 20–30 group, which is 60%.

Question 5 D

The oldest person is 79. The range in the male ages is 79 – 1 = 78 and for female 78 – 1 = 77. Neither set of data is symmetrical. There is no mode for males, and the female data is bimodal. Finding the median of both sets gives 51.

Question 6 E

Correlation refers to the strength of the relationship between two variables. The solution is not **A** because, although the relationship is negative, it is not strong. **B** refers to a causal relationship; r^2 would need to be calculated to measure the casual relationship = 36%. **C** is incorrect because there is no regression equation. **D** is incorrect because both variables could be positive, but the actual data is not given.

Question 7 D

The IQR is 5 for 2017 and is 3 for 2016. Therefore **D** is correct.

Question 8 D

95% of normally distributed data lies within two standard deviations of the mean. 4 cm = 40 mm so the distribution is 40 – 3 × 2 = 34 mm to 40 + 3 × 2 = 46 mm.

Question 9 C

Enter the bivariate data into a calculator and find the value of r^2.

Question 10 B

For the raw unchanged data, Q_1 = 15 and the IQR is 20. When the data is changed, Q_1 = 18 and the IQR is 17.

Question 11 E

Enter the raw figures into calculator and then change the x values to $\frac{1}{x}$. Determine the least squares regression equation for the transferred data and read the value of r.

Question 12 D

The midpoint of each group in the data must be found first as an estimate for the group, giving 3, 8, 13, 18 and 23. $n = 30$ so the median will be between the fifteenth and sixteenth data points, both of which are 13 so the median is 13. Enter the data into a calculator using the midpoint and frequency and calculate the one-variable statistics to find the standard deviation.

Question 13 B

The mean of the 2, 3, 4 and 5 figures = 130. The mean of the 3, 4, 5 and 6 figures = 125. The solution requires the mean centred on the fourth figure, $\frac{130 + 125}{2} = 127.5$.

Question 14 C

Substitute 10 into the trend line. $d = 104 + 6n$ becomes $d = 104 + 6 \times 10 = 164$.

Question 15 B

Substituting a *reading age* of 15 into the regression equation predicts a value of 14.35. The residual is $11 - 14.35 = -3.35$.

Question 16 B

Both *shoe size* and *reading age* are responding to a third variable; age. This is therefore the common response correlation.

Recursion and financial modelling

Question 17 D

Substituting $t_0 = 8$ into the equation to find $t_1 = 3(8) - 16 = 8$. Alternatively, use your calculator to generate the values using the recursive function.

Question 18 A

Using a calculator or otherwise, $n = 180$, $I = 4.5$, payments per year and compounding periods per year are both 12 and the future value is 0.

Question 19 E

The rate is 100 – 6.4 = 93.6% or 0.936. The rate is applied to the reducing balance, so option **E** is the correct solution.

Question 20 B

The rate is 4% per annum applied quarterly, so 1% per quarter. $r = 1.01$ and is applied only once at the end of the first quarter. The question requires the balance and not the interest earned.

Question 21 B

The question requires the recursive relation, not the equation of the line of best fit. The value of the next term can be found by applying the formula $t_{n+1} = \frac{t_n}{2} + 1$.

Question 22 C

At the end of each year interest of 3% is added and a sum of $5000 is withdrawn.

Question 23 D

Calculate the compounding daily interest using an interest rate of 12% per annum; this becomes $\frac{12}{365}\%$ per day and this rate is applied for 30 days.

For the compounding monthly interest use, the interest rate of 12% per annum becomes $\frac{12}{12}\%$ per month, or 1%, and this rate is applied for one month.

This is represented by $3500\left(1 + \frac{\frac{12}{365}}{100}\right)^{30} - 3500\left(1 + \frac{1}{100}\right)$.

Question 24 E

A flat rate of 12% is calculated using the simple interest formula. However, this is not a true representation because the repayments being made each month are reducing the balance. Interest should be charged on a lesser amount.

SECTION B – MODULES

Module 1 – Matrices

Question 1 **E**

The matrix only contains ‘0’s and ‘1’s, and so is binary. It is diagonal with all ‘0’s except the main diagonal. It is a 5 × 5, so it is square, and since the figures along the diagonal are all ‘1’s, then it is a unit matrix. It is also symmetrical.

Question 2 **D**

The sales of milk can be found using 17 + 12 + 22 + 20 + 18 = 89. The income from the sales of milk is 89 × \$3.40 = \$302.60.

Question 3 **B**

Multiplying a 4 × 4 matrix by a 4 × 1 produces a 4 × 1 matrix. Options **D** and **E** can be eliminated.

Multiplying the first row by the column matrix gives $0 \times S + 0 \times E + 1 \times A + 0 \times L$ gives A.

Multiplying the second row by the column matrix gives $0 \times S + 1 \times E + 0 \times A + 0 \times L$ gives E.

Multiplying the third row by the column matrix gives $1 \times S + 0 \times E + 0 \times A + 0 \times L$ gives S.

Multiplying the fourth row by the column matrix gives $0 \times S + 0 \times E + 0 \times A + 1 \times L$ gives L.

Question 4 **E**

Using p to represent the number of pies and s the number of soft drinks, the simultaneous equations are $3p + 2s = 23$ and $2p + s = 14$.

Question 5 **C**

The question asks for S_2, not S_1. The rule must be applied twice.

$$\begin{bmatrix} 0.8 & 0.6 \\ 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 300 \\ 400 \end{bmatrix} - \begin{bmatrix} 50 \\ 60 \end{bmatrix} = \begin{bmatrix} 430 \\ 450 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.6 \\ 0.5 & 0.9 \end{bmatrix} \begin{bmatrix} 430 \\ 450 \end{bmatrix} - \begin{bmatrix} 50 \\ 60 \end{bmatrix} = \begin{bmatrix} 564 \\ 560 \end{bmatrix}$$

Question 6 **C**

To find the inverse matrix, swap the top-left to bottom-right diagonal elements and change the sign of the other two elements. This must then be multiplied by the reciprocal of the determinant of the matrix.

$$\frac{1}{ad - bc} = \frac{1}{2w \times 4z - -3y \times x} = \frac{1}{8wz + 3xy}$$

Question 7 A

Applying the transition matrix $\begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.2 & 0.1 & 0.3 \\ 0.6 & 0.4 & 0.4 \end{bmatrix}$ to $S_1 = \begin{bmatrix} 40 \\ 50 \\ 50 \end{bmatrix}$ according to $T^n S_1$ or a large value of n shows that the total sales stay constant at 140.

There is a slight increase in *Asif* and a bigger increase in *Cinderella*, meaning *Brilliant* has a decrease in sales. Notice that each column adds to 1, meaning the overall sales remain constant.

Question 8 C

$A \times B$ gives $2 \times 3 \times 3 \times 1$, producing a 2×1 matrix.

The $2 \times 1 \times \ 1 \times 3$ produces a 2×3 matrix.

Module 2 – Networks and decision mathematics

Question 1 B

For a graph with n vertices to be connected, there must be $n - 1$ edges.

The number of edges for this graph to be connected is $11 - 1 = 10$.

Question 2 D

Let x be the number of vertices. The number of faces is twice the number of vertices so $2x$.

For a planar graph $v + f = e + 2$ so

$$x + 2x = 22 + 2$$
$$3x = 24$$
$$x = 8$$

The number of faces is $2x$, so 16.

Question 3 D

The graph is not a tree since it contains cycles.

The graph is connected as it is possible to reach any vertex from another vertex.

The graph does not contain a Hamiltonian circuit but does contain one Hamiltonian path being *GABCDEHF* (or *FHEDCBAG*).

Question 4 B

For a connected graph to have an Eulerian trail it can have only two vertices with odd degree.

The degrees of the vertices in this graph are listed below.

Vertex	Degree
A	2
B	3
C	3
D	2
E	2
F	1
G	1

Adding *GH* gives vertex *G* degree 2 and vertex *H* degree 5, leaving 4 vertices with odd degree. Adding this edge would not create an Eulerian trail.

Adding any of the other edges would leave two vertices with odd degree.

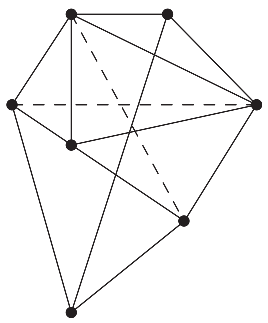
Question 5 E

The adjacency matrix for the graph is shown below:

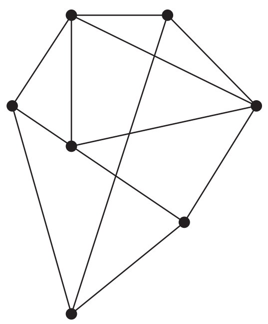
$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Of the 16 elements, five are ‘0’, nine are ‘1’ and two are ‘2’.

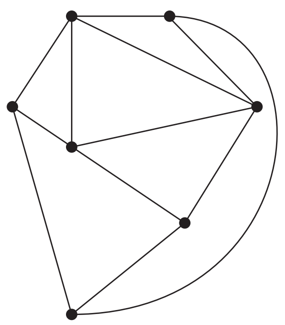
Question 6 B



The removal of the two edges shown leaves the following graph.



This graph is planar as it can be redrawn as shown below.



Question 7 B

Cut C is not a valid cut so this can be disregarded.

The cut capacity for each of the other cuts is:

Cut A: 30

Cut B: 20

Cut D: 15

Cut E: 16

The minimum cut gives the maximum flow through the network, so the maximum flow is 15.

Question 8 C

Activity *C* is the immediate predecessor to activity *E*.

The earliest start time for activity *E* is 24 hours.

Since activity *E* takes 4 hours, the latest start time for activity *E* is 4 hours before the earliest start time of activity *F*, so 31 hours.

The float time for activity *E* is 31 – 24 = 7 hours.

This means that activity *C* could be delayed by up to 7 hours without affecting the earliest start time of activity *E*.

Module 3 – Geometry and measurement

Question 1 B

shaded area = area of square – area of circle

$= 20 \times 20 - \pi \times 10^2$

$= 85.8407$

$= 86$

Question 2 D

$AE = 6.4$ cm

scale factor $= \dfrac{6.4}{4.1}$

$BC = 4.9 \times$ scale factor

$= 7.64878$

$= 7.6$

Question 3 C

radius of larger pipe = 3.75

radius of small pipe = 1.5

$\pi \times 3.75^2 \times 10 - \pi \times 1.5^2 \times 10 = 371.101$

$= 371$

Question 4 A

Use Heron's formula:

$s = \dfrac{7 + 8 + 10}{2} = 12.5$

$\sqrt{12.5(12.5 - 10)(12.5 - 8)(12.5 - 7)} = 27.8107$

$= 28$

Question 5 E

$a = BC$, $b = 7$, $c = 9$, angle at $A = 72°$

Using the cosine rule:

$BC^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \times \cos(72°)$

$BC = \sqrt{7^2 + 9^2 - 2 \times 7 \times 9 \times \cos(72°)}$

Question 6 A

Matilda is 30° east. 15° = 1 hour time difference.

30° to the east = 2 hours ahead.

Therefore, at 8.05 pm in Auckland it is 6.05 pm in Geelong.

Question 7 A

volume of a sphere = $\frac{4}{3} \times \pi \times r^3$

Solve $\frac{4}{3} \times \pi \times r^3 = 8000$ for r.

$r = 12.407$

$2r = 24.814$

Question 8 B

angle of BAC = 70° (bearing of C – bearing of B)

Therefore ACB = 50°.

Using sine rule:

$\frac{50\sin(50)}{\sin(70)} = 40.7604$

Alternatively, use a calculator to solve $\frac{x}{\sin(50)} = \frac{50}{\sin(70)}$ for x.

Module 4 – Graphs and relations

Question 1 **C**

Substitute $x = 2$ and $y = 16$ into $y = kx^2$:

$$k \times 2^2 = 16$$
$$k = 4$$

Question 2 **B**

intercept = 70

slope = 55

Question 3 **D**

$$\text{gradient of line} = -\frac{4}{2}$$

$m = -2$

equation of line: $y = -2x + 9$

For the x-intercept when $y = 0$, solve:

$$0 = -2x + 9$$
$$x = 4.5$$

Question 4 **C**

Solve the simultaneous equations for wins (w) and losses (l):

$8w + 2l = 440$

$6w + 6l = 150$

$w = 65$, $l = -40$

Solve $65w - 40(9 - w) = 0$ for w:

$w = 3.42857$

Therefore 4 wins.

OR

Use trial and improvisation:

$3 \times 65 - 6 \times 40 = -45$

$4 \times 65 - 5 \times 40 = 60$

Therefore 4 wins.

Question 5 **A**

Substitute (–5, 2) into inequalities.

For option **A**, $-5 + 4 = -1$.

This satisfies the inequality.

Therefore, the answer is **A**.

Question 6 **C**

Find the gradient:

$$\frac{-2-10}{2--2}=-3$$

Solve $y-10=-3(x--2)$ for y:

$y=-3x+4$

Question 7 **C**

The cost for Tom is 6 × \$65.

The cost for Holly is 15 × \$50.

The total cost is (6 × \$65) + (15 × \$50) = \$1140.

Question 8 **E**

Students may solve this by inspection.

OR

Students may test a coordinate in the region such as (1, 0).