

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2018 Trial Examination

SOLUTIONS

SECTION A: Core Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation: By completing one variable statistics the answer is easily found. From this list it can be determined that the standard deviation is equal to 5.4.

	A	B	C
			=OneVar...
1	3	Title	One-Va...
2	4	\bar{x}	10.3636...
3	7	Σx	114.
4	7	Σx^2	1476.
5	8	$s_x := s_{n-1}$	5.42720...
6	10	$\sigma_x := \sigma_{n-1}$	5.17453...
7	11	n	11.
8	13	MinX	3.
9	13	Q ₁ X	7.
10	17	MedianX...	10.
11	21	Q ₃ X	13.
12		MaxX	21.
13		$SSX := \Sigma$	294.545...

Question 2

Answer: A

Explanation: The sum of the seasonal indices must be 12 because there are 12 “seasons” in this data.

$$1.5 + 1.2 + 0.9 + 0.9 + 0.6 + 0.8 + 0.6 + 0.6 + 0.7 + 0.9 + a + b = 12$$

$$\therefore a + b + 8.7 = 12$$

Since b is 0.3 larger than a then:

$$a + a + 0.3 + 8.7 = 12$$

$$2a = 3$$

$$a = 1.5 \Rightarrow b = 1.5 + 0.3 = 1.8$$

Question 3

Answer: E

Explanation: To deseasonalise you need to divide the actual figure by the seasonal index:

$$\frac{2168}{0.9} = 2408.888... \approx 2409$$

Question 4

Answer: C

Explanation: Bar charts are used for categorical data, not numerical data. The data is univariate, so a scatter plot is of no use, and a dot plot will not distinguish between boys and girls grades, thus back to back stem and leaf plot is most appropriate.

Question 5

Answer: E

Explanation: The correlation coefficient is negative which means that the regression line will have a negative gradient, thus answer **E**

Question 6

Answer: B

Explanation:

$$\frac{(5.917 + 5.749 + 5.692 + 5.666)}{4} = 5.756 \quad \text{and} \quad \frac{(5.749 + 5.692 + 5.666 + 5.711)}{4} = 5.7045$$

$$\text{Centred value is } \frac{5.756 + 5.7045}{2} = 5.73025 \approx 5.730$$

Question 7

Answer: C

Explanation: Quarter number of Q1 of 2015 will be 1, so then Q3 for 2018 will be 15. Substitute this value into the regression equation:

$$6.182 - 0.061 \times 15 = 5.267$$

Question 8

Answer: **E**

Explanation: The number 32 will change to 23, a smaller value which will reduce the size of the mean. The median will remain unchanged, as will the IQR and range, thus answer E.

Question 9

Answer: **D**

Explanation: 85 is 1 standard deviation below the mean, so 34% of data lies between 85 and 120. 190 minutes is two standard deviations above the mean, so 47.5% of data will lie between 120 and 190. This means that 81.5% of the students will study between 85 minutes and 190 minutes.

$$\frac{81.5}{100} \times 124 = 101.06 \text{ thus we should expect 101 students to study this amount of time.}$$

Question 10

Answer: **C**

Explanation: Need to check the data for outliers. $Q1=24, Q3=32$, thus $IQR = 8$.

Upper fence = $32 + 1.5 \times 8 = 44$ thus 50 is an outlier. Check for skewed data:

Median – min = 9 compared with Max – median = 23 (better to exclude outliers, $41 - 27 = 14$) and median - $Q1 = 27 - 24 = 3$ compared with $Q3 - \text{median} = 32 - 27 = 5$

The data has a slight positive skew, thus answer C.

Question 11

Answer: **A**

Explanation: Transform time data using CAS:

Then determine the correlation coefficient using one of the methods available like:

```
LinRegBx logtime,weight,1: CopyVar stat.Rep
┌───────────┬───────────┐
│ "Title"    │ "Linear Regression (a+bx)" │
│ "RegEqn"   │ "a+b· x"                   │
│ "a"        │ 2.3443017488               │
│ "b"        │ 4.3307995064               │
│ "r²"       │ 0.996185297876            │
│ "r"        │ 0.998090826466            │
│ "Resid"    │ "(...)"                   │
└───────────┴───────────┘
```

	A time	B weight	C logtime
=			=log(time)
1	1.	2.3	0.
2	2.	3.7	0.30102...
3	3.	4.5	0.47712...
4	4.	4.8	0.60205...
5	5.	5.4	0.69897...
6	6.	5.8	0.77815...
7	7.	6.	0.84509...
8	8.	6.2	0.90308...

Question 12

Answer: D

Explanation: Add the frequencies shown in the graph. $2+1+6+3+2+5+3=22$

Question 13

Answer: D

Explanation: The median is found half way between the 11th and 12th data. This lies in the 1-2 on the log(brightness) scale. To calculate brightness:

$$\begin{aligned} \log(\text{brightness}) = 1 &\Rightarrow \text{brightness} = 10^1 = 10 \\ \log(\text{brightness}) = 2 &\Rightarrow \text{brightness} = 10^2 = 100 \end{aligned} \quad \text{Thus answer is between 10 and 100 units}$$

Question 14

Answer: D

Explanation: First we need to calculate the brightness for each star:

$$\begin{aligned} \text{Adhara brightness} &= 10^{-0.7} = 0.1195 \\ \text{Sun brightness} &= 10^{5.7} = 501187.23 \\ \frac{\text{Sun brightness}}{\text{Adhara brightness}} &= \frac{501187.23}{0.1195} = 2511886.34 \approx 2.5 \text{ million} \end{aligned}$$

Question 15

Answer: E

Explanation: From the formula sheet $a = \bar{y} - b\bar{x}$ $b = r \frac{s_y}{s_x}$

$$\begin{aligned} \bar{x} = 63.2 \quad s_x = 20.1 \quad \bar{y} = 53.8 \quad s_y = 22.2 \quad r = 0.765 \\ b = 0.765 \times \frac{22.2}{20.1} = 0.844925 \approx 0.845 \\ a = 53.8 - 0.844925 \times 63.2 = 0.4007 \approx 0.401 \end{aligned}$$

Question 16

Answer: A

Explanation: Substitute into equation and solve for SAC1 grade:

$$\left| \begin{array}{l} \text{solve}(73=0.401+0.845 \cdot x,x) \\ x=85.9159763314 \end{array} \right.$$

$$\text{Residual} = 82.5 - 85.9 = -3.4$$

Core: Recursion and financial modelling**Question 17**

Answer: D

Explanation: Geometric recurrence relations are of the form: $V_{n+1} = R \times V_n$ and will be decaying if $0 < R < 1$, thus answer D.

Question 18

Answer: B

Explanation: The rule for calculating the value of the van is $V_n = 44800 - 0.35n$. Use the solve function on CAS to determine the value of n :

$$\left| \begin{array}{l} \text{solve}(10000=44800-0.35 \cdot n,n) \\ n=99428.5714286 \end{array} \right.$$

Question 19

Answer: A

Explanation: Determine the effective interest rate for each option:

So Bank A $r_{\text{eff}} = 18.73\%$

Bank B $r_{\text{eff}} = 18.67\%$

Bank C $r_{\text{eff}} = 18.67\%$ The best option is thus Bank A, as it has the highest interest rate by 0.06%

$\text{eff}(17.2,52)$	18.7340775543
$\text{eff}(17.24,12)$	18.6696419109
$\text{eff}(17.49,4)$	18.6709296065

Question 20

Answer: **A**

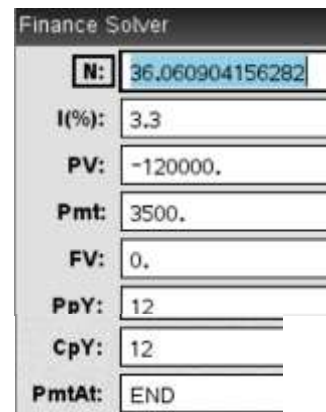
Explanation: Recurrence equation will be of the form $V_{n+1} = R \times V_n - D$ where

$$R = 1 + \frac{3.3}{100} = 1.00275 \text{ and } D = 3500$$

Question 21

Answer: **E**

Explanation: Use CAS and finance solver to determine N (in months)
 $36 \div 12 = 3$ years



Question 22

Answer: **D**

Explanation: For this recurrence relation $P_0 = 24500, P_{n+1} = 1.017 \times P_n - 5000$.

$$P_1 = 1.017 \times 24500 - 5000 = 19916.50 \text{ so the value is reducing thus not a perpetuity.}$$

If an annuity is compounded monthly with interest rate of 1.7% then $R \neq 1.017$.

If a reducing balance loan at 6.8% compounding monthly, then $R = 1 + \frac{6.8}{100} \neq 1.017$.

If it were an investment with deposits then the recurrence relation would have +5000 on the end.

Answer D is a reducing balance loan with quarterly payments so it will have -5000 at

the end. Also with quarterly payments at 6.8% p.a., $R = 1 + \frac{6.8}{100} = 1.017$

Question 23

Answer: B

Explanation: Need to complete further lines in the amortisation table. This can be done by calculating the interest each time or by using the amortisation table function on CAS.

$$\text{Interest for 4}^{\text{th}} \text{ payment: } 314583.10 \times \frac{6.21 \div 26}{100} = 751.37$$

Payment Number	Payment	Interest	Principle Reduction	Balance
0	-	-	-	\$ 315,000.00
1	\$891.00	\$ 752.37	\$ 138.63	\$ 314,861.37
2	\$891.00	\$ 752.03	\$ 138.97	\$ 314,722.40
3	\$891.00	\$ 751.70	\$ 139.30	\$ 314,583.10
4	\$891.00	\$ 751.37	\$ 139.63	\$ 314,443.47
5	\$891.00	\$ 751.04	\$ 139.96	\$ 314,303.51

Or using amortisation table function on CAS: Syntax for TIInspire is

`amortTbl (table entries, N, I, PV, Pmt, (FV), Ppy, Cpy, , rounding)` rounding is set to 2 for 2 decimal places.

```
amortTbl(5,30.26,6.21,315000,-891,,26,26,▶
[
0  0.    0.    315000.
1 -752.37 -138.63 314861.37
2 -752.03 -138.97 314722.4
3 -751.7  -139.3  314583.1
4 -751.37 -139.63 314443.47
5 -751.04 -139.96 314303.51
]
```

Question 24

Answer: C

Explanation: Need to use Finance solver for this to firstly determine the original repayment value:

So repayments are initially \$806.19. Next we need to determine how much is still owed after 6 months, so using finance solver again, we get \$22762.37. Then use finance solver once again to determine how much longer it takes to pay off the loan with \$900 payments. The result is an additional 27 months after the initial 6 months.

The loan was originally for three years (36 months), so it is being reduced by a total of 3 months.

Finance Solver	
N:	36.
I(%):	4,75
PV:	27000.
Pmt:	-806,18710627047
FV:	0.
PpY:	12
CpY:	12
PmtAt:	END

Finance Solver	
N:	6
I(%):	4,75
PV:	27000
Pmt:	-806,19
FV:	-22762,368377282
PpY:	12

Finance Solver	
N:	26,701619224932
I(%):	4,75
PV:	22762,37
Pmt:	-900.
FV:	0.
PpY:	12

SECTION B: Modules Multiple-choice questions (1 mark each)

Module 1 - Matrices

Question 1

Answer: E

Explanation: All matrices can be transposed.

Question 2

Answer: B

Explanation: Since matrix C is a 2×3 matrix, matrix X must be a $? \times 2$ matrix. Thus only matrix B (2×2) or matrix D (3×2) can be multiplied by matrix C this way.

Question 3

Answer: C

Explanation: For an inverse to exist the matrix must be square and not singular. Matrices A and B are square and neither has a zero determinant.

Question 4

Answer: D

Explanation: To multiply C by matrix Y, means that Y must be a $3 \times ?$ matrix. If the result can be added to matrix E, then the result of $C \times Y$ must be 2×4 , meaning that Y must be a 3×4 matrix.

Question 5

Answer: A

Explanation: From the transition matrix T it can be seen that 0.22 represents the number from C that will change to I, thus 22% of customers at Colesworth in one week will shop at IGAmart the following week.

Question 6

Answer: B

Explanation: Care must be taken with this question as the in the first week 80% of customers shop at Colesworth, thus the initial state matrix S_0 represents week 1, which means that week 4 will be represented by S_3 . To obtain figures for week 4, there need to be 3 transitions. On CAS:

$$\begin{bmatrix} 0.78 & 0.14 \\ 0.22 & 0.86 \end{bmatrix}^3 \cdot \begin{bmatrix} 80 \\ 20 \end{bmatrix} \quad \begin{bmatrix} 49.66592 \\ 50.33408 \end{bmatrix}$$

Thus the percentage expected to shop at Colesworth is 49.665% which is rounded to 50%.

Question 7

Answer: D

Explanation: To find the long term proportion use CAS to find S_{50}

$$\begin{bmatrix} 0.78 & 0.14 \\ 0.22 & 0.86 \end{bmatrix}^{50} \cdot \begin{bmatrix} 80 \\ 20 \end{bmatrix} \quad \begin{bmatrix} 38.8888888973 \\ 61.1111111027 \end{bmatrix}$$

This indicates that in the long term the Colesworth share is 38.9%. If IGAmart's share was 68% prior to the new store opening which means Colesworth was 32%. Thus there has been an increase of about 7% of the population now regularly shopping at Colesworth.

Question 8

Answer: B

Explanation: Set up the simultaneous equations in matrix form:

$$\begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} 272.75 \\ 210.80 \end{bmatrix}$$

$$\therefore \begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 272.75 \\ 210.80 \end{bmatrix}$$

Using CAS to find the inverse:

$$\begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}^{-1} \quad \begin{bmatrix} -3. & 4. \\ 4. & -5. \end{bmatrix}$$

Thus $\begin{bmatrix} t \\ s \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 272.75 \\ 210.80 \end{bmatrix}$ so answer B

Module 2 – Networks and decision mathematics

Question 1

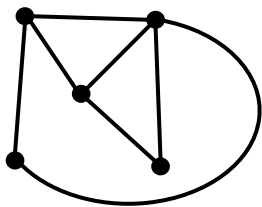
Answer: A

Explanation: The graph is connected as there are no isolated vertices, and planar since no edges cross over. It is not a simple network because there are multiple edges between two vertices and it is not complete since each vertex is not directly connected to every other vertex.

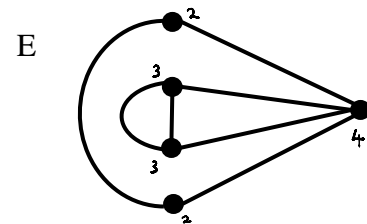
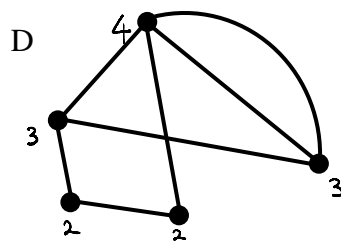
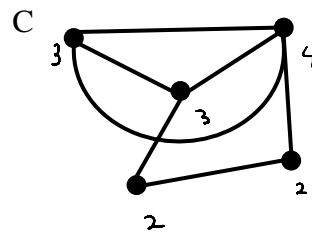
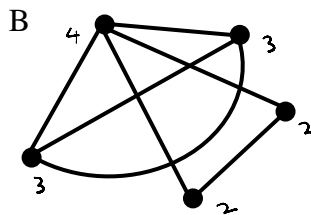
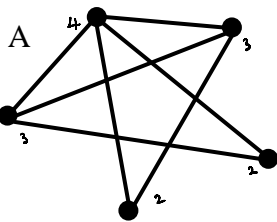
Question 2

Answer: A

Explanation: The easiest way to determine if networks are isomorphic is to mark the degree of each vertex and compare the connections:



For this network the vertices with degree 2 are not connected to one another. The networks in option B, C, D and E all have the two degree 2 vertices joined together.



Question 3

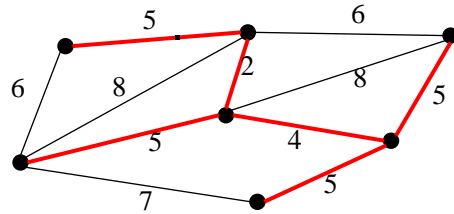
Answer: C

Explanation: Each of the networks needs to be checked for the number of unique paths from one vertex to another. Only network B satisfies the requirements of the adjacency matrix.

Question 4

Answer: B

Explanation: The minimum spanning tree is shown below:



Length of the minimum spanning tree $5+2+5+4+5+5=26$

Question 5

Answer: B

Explanation: This answer can be “reasoned” out by identifying which person must complete each task in the initial optimum.

	A	B	C	D
Warren	10	13	19	14
Xian	15	14	19	20
Yarra	11	18	17	20
Zeb	12	15	17	21

The shaded boxes represent the optimal allocation of tasks. Now by adjusting Zeb’s time for task C a new optimum can be determined.

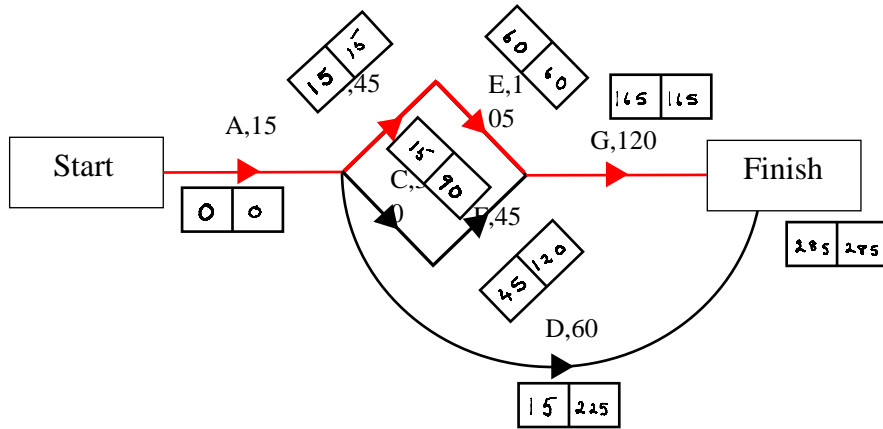
	A	B	C	D
Warren	10	13	19	14
Xian	15	14	19	20
Yarra	11	18	17	20
Zeb	12	15	21	21

The new optimum can be determined using the Hungarian algorithm or by reasoning out. Since Yarra also takes 17 minutes to complete task C and Zeb takes just one minute more on Yarra’s task, A, the increased time is just 1 minute.

Question 6

Answer: E

Explanation: Minimum completion time is obtained by completing a forward scan of the activity network:



The critical path is $A \rightarrow B \rightarrow E \rightarrow G$ with a completion time of 285 hours.

Question 7

Answer: D

Explanation: To determine the latest FINISHING time of activity C, we are actually finding the latest starting time for activity F. A backward scan of the network is now required. Job F needs to be completed by 165 hours, so latest starting time for F is $165 - 45 = 120$.

Question 8

Answer: A

Explanation: Prim's algorithm needs to be applied starting at vertex A. Shortest path from A is 18 (the path A-E) is the first added to the minimum spanning tree. Next we look at remaining paths from both A and E. The minimum now is 25 (the path E-F), the second added. Now we need to look at the remaining paths from A, E and F. The third minimum is 27, (path A-D).

Module 3 – Geometry and measurement

Question 1

Answer: C

Explanation: We are given that length AC is 12.3m and XC is 4.7m, thus length AX is 7.6m.

Using similar triangles we can determine the length AB:

$$\frac{AB}{CD} = \frac{AX}{CX}$$

$$\frac{AB}{6.2} = \frac{7.6}{4.7}$$

$$\therefore AB = 6.2 \times \frac{7.6}{4.7} = 10.0255 \approx 10.0 \text{ m}$$

Question 2

Answer: B

Explanation: First we need to know the circumference of the circular base of the cone, this will then give us the length of the curved edge of the sector, and thus the fraction of the full circle from which the pointed section of the cone is constructed from.

circumference = $\pi \times \text{diameter} = 5\pi$ for the base of the cone.

circumference = $2\pi \times \text{radius} = 2\pi \times 3.5 = 7\pi$ for the full circle for the sector

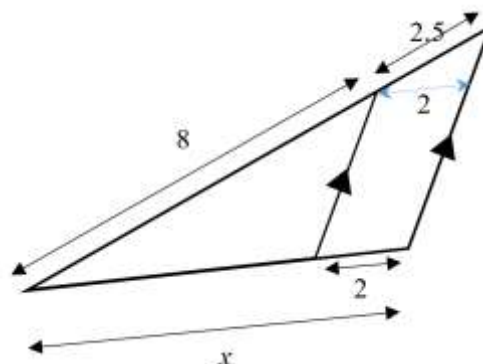
fraction = $\frac{5\pi}{7\pi} = \frac{5}{7}$ of the circle used to create the sector.

area = $\frac{5}{7} \times \pi \times \text{radius}^2 = \frac{5}{7} \times \pi \times (3.5)^2 = 27.489 \text{ cm}^2$

Question 3

Answer: D

Explanation: There are various approaches that could be used here. Use of similar triangle ratios is shown below:



$$\frac{8}{x-2} = \frac{10.5}{x} = \frac{2.5}{2}$$

$$\therefore x = 8.4$$

Question 4

Answer: D

Explanation: The walls at the ends of the doll house have dimensions $(78-0.5)$ by 44.5 cm. the partitioning walls will require both sides to be wall papered. The back walls have a total size of $(100-0.5)$ by $(78-0.5)$. then the areas of the doors (both sides) need to be subtracted. $4 \times 77.5 \times 44.5 + 99.5 \times 77.5 - 4 \times 9 \times 31 = 20390.25 \approx 20400 \text{ cm}^2$

Question 5

Answer: C

Explanation: Angles can be calculated from the bearings given.

Next determine distance of A from P:

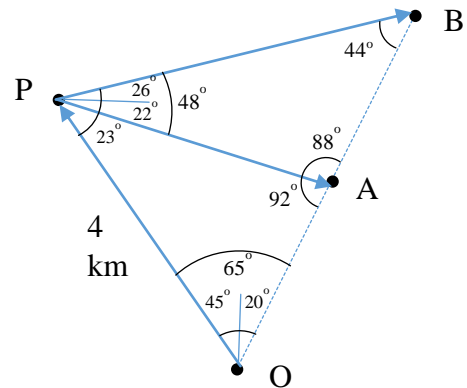
$$\frac{PA}{\sin 65^\circ} = \frac{4\text{km}}{\sin 92^\circ}$$

$$PA = \sin 65^\circ \times \frac{4}{\sin 92^\circ} = 3.627 \text{ km}$$

Now calculate distance from A to B:

$$\frac{AB}{\sin 48^\circ} = \frac{3.627}{\sin 44^\circ}$$

$$AB = \sin 48^\circ \times \frac{3.627}{\sin 44^\circ} = 3.8806 \approx 3.881 \text{ km}$$



Question 6

Answer: B

Explanation: Dallas is $151^\circ + 97^\circ = 248^\circ$ west of Sydney. Each 1° west means that the time will be 4 minutes EARLIER. Thus the time in Dallas when the Prime minister is due to leave Sydney will be $248 \times 4 = 992$ min earlier than 2:40pm on Monday 4th June 2018. The flight will take 15 hours 20 minutes = 920 minutes. The time in Dallas when the PM's plane is due to land will be $992 - 920 = 72$ min EARLIER than the time of departure, so approximately 1 hour 12 minutes earlier. $2:40\text{pm} - 1\text{hr } 12 \text{ min} = 1:28\text{pm}$ Monday 4th June 2018. Thus the correct answer is B.

Question 7

Answer: B

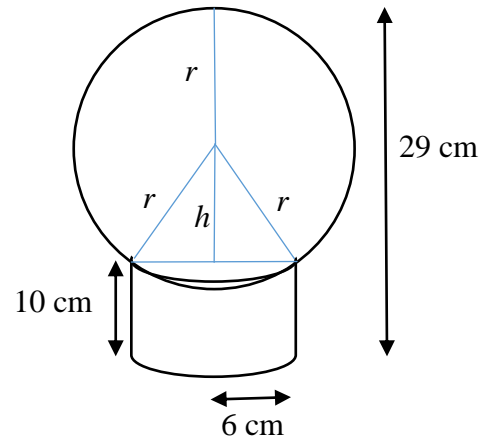
Explanation: Using Pythagoras theorem, the height h can be determined:

$$r^2 = h^2 + 6^2$$

$$h^2 = r^2 - 36$$

$$h = \sqrt{r^2 - 36}$$

Total height is 29, so: $r + h + 10 = 29$
 $\therefore r + \sqrt{r^2 - 36} + 10 = 29$



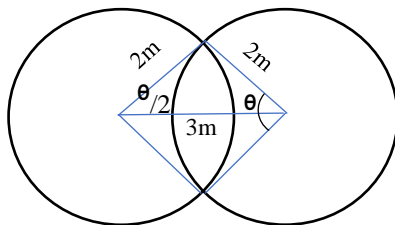
```
solve(r+sqrt(r^2-36)+10=29,r)
r=10.4473684211
```

Now this can be solved using CAS

Question 8

Answer: C

Explanation: The shape of the area grazed by the two goats will be the shape of two intersecting circles as shown here:



There are several approaches that could be taken to determine the total area grazed. The isosceles triangle in the diagram can be used to determine the size of the angle at the centre of the circle, θ .

$$\frac{\theta}{2} = \cos^{-1} \left(\frac{2^2 + 3^2 - 2^2}{2 \times 3 \times 2} \right) = 41.41^\circ$$

$$\therefore \theta = 82.82^\circ$$

Heron's formula can be used to determine the area of the triangle:

$$s = \frac{2+2+3}{2} = 3.5$$

$$Area = \sqrt{3.5 \times (3.5-2) \times (3.5-2) \times (3.5-3)} = 1.984$$

The Total area grazed is made up of two triangles and two sectors:

$$Total\ Area = 2 \times \text{triangle} + 2 \times \text{sector}$$

$$= 2 \times 1.984 + 2 \times \left(\pi \times 2^2 \times \frac{(360^\circ - 82.82^\circ)}{360^\circ} \right)$$

$$= 3.968 + 19.35079 \approx 23.319\text{ m}^2$$

Module 4 – Graphs and relations**Question 1****Answer: D**

Explanation: The equation of the line can be found easily. The y intercept is 8 and the gradient is

$$\frac{-1}{3}. \text{ Solve the equation for } y = 0. \quad y = 8 - \frac{1}{3}x \Rightarrow 0 = 8 - \frac{1}{3}x \quad \therefore x = 24$$

Question 2**Answer: D**

Explanation: Each set of coordinates can be substituted into the inequality to find which is larger than 2480:

$13 \cdot 126 + 27 \cdot 31$	2475.
$13 \cdot 174 + 27 \cdot 8$	2478.
$13 \cdot 189$	2457.
$13 \cdot 144 + 27 \cdot 23$	2493.
$13 \cdot 136 + 27 \cdot 26$	2470.

Question 3**Answer: E**

Explanation: Although it is difficult to accurately determine the price, we can estimate from what we see on the graph and then select the closest answer. For 1000 business cards, the price is 9 cents per card. For 1500 business cards the cost appears to be about 6.5 cents per card: $1000 \times \$0.09 + 1500 \times \$0.065 = \$187.50$ which is closest to \$183.75.

Question 4**Answer: E**

Explanation: From the options given, the equation must be either $y = kx^2$ or $y = kx^3$. The resulting linear equation after the transformation should have a gradient of k . If the equation is of the form $y = kx^2$ then k can be calculated: $108 = k \times 3^2 \Rightarrow k = 12$. When $x^2 = 1$, y should be equal to 12, so not A. When $x^2 = 3$ then $y = 12 \times 3 = 36$ so answer E is also incorrect.

If the equation was of the form $y = kx^3$ then $108 = k \times 3^3 \Rightarrow k = 4$. If $x^3 = 1$ then $y = 4 \times 1$ so answer C is correct.

Question 5

Answer: D

Explanation: The number of hours binding must be less than or equal to 4500, so not A or C. If x is the number of paperback books and each of these takes 2.25 hours to cover and y is the number of hardback books and each of these take 6.75 hours to cover then the equation must contain $2.25x$ and $6.75y$, thus answer D.

Question 6

Answer: B

Explanation: The shaded section is the feasible region. This is smaller than $x = 14$ and less than $y = 6$ so $x \leq 14$ is one equation and $y \leq 6$ is another, thus answers A, D and E are incorrect. The equation of the sloped line can be found, but it is easier to substitute in a point within the feasible region and determine which of the equations in answers B and C it satisfies, eg. Using the point (1,1):

$$3x + 20y \leq 140$$

$$3 + 20 = 23 \text{ which is less than } 140, \text{ so Answer is B}$$

Question 7

Answer: B

Explanation: The CAS can be used to show the feasible region and the “corner points” can be found.

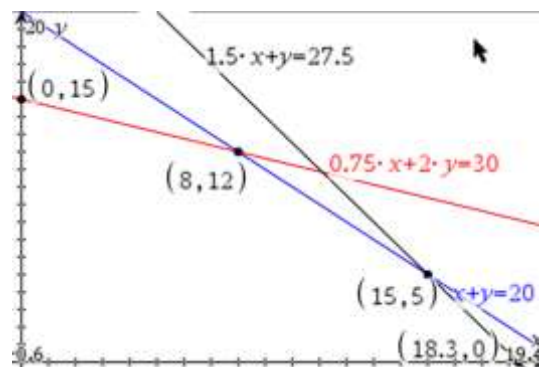
Substituting these coordinates into the profit function will determine the maximum profit:

$$(0,15) \quad 400 \times 0 + 300 \times 15 = 4500$$

$$(8,12) \quad 400 \times 8 + 300 \times 12 = 6800$$

$$(15,5) \quad 400 \times 15 + 300 \times 5 = 7500$$

$$(18,0) \quad 400 \times 18.3 + 300 \times 0 = 7320$$



Thus 15 of component A and 5 of component B gives maximum profit.

Question 8

Answer: D

Explanation: We are given that 800 business cards cost \$136, so by substituting these values into the third equation we can determine the value of k .

$$k + 0.12 \times 800 = 136$$

$$k = 40$$

$n = a$ it shouldn't matter which of the second or third equation are used, the cost should be the same. So:

$$20 + 0.16a = 40 + 0.12a$$

$$0.04a = 20$$

$$a = 500$$