

 **FURTHER MATHEMATICS**

 **TRIAL** **EXAMINATION 2**

 **SOLUTIONS**

 **2019**

## SECTION A - Core

## Data analysis

# Question 1 (8 marks)

1. **i.** one (i.e. *annual turnover*)

**(1 mark)**

1. two (i.e. *client code* and *markets*)

 **(1 mark)**

|  |  |
| --- | --- |
|  | size of business |
| markets | small | medium |
| domestic only | 4 | **2** |
| overseas only | 3 | 3 |
| domestic & overseas | 1 | **9** |
| Total | 8 | **14** |

 **(1 mark)**

1. Using 1-Var stats find .



So mean annual turnover is $3 100 000 (correct to two significant figures).

**(1 mark)**

1. 

  **(1 mark)**

1. Method 1 – using 1-Var stats from part **c.**

Median = 3.15

So the median annual turnover is $3 150 000.

**(1 mark)**

Method 2 – using the dot plot

There are 22 dots. The middle value lies halfway between 3.1 and 3.2 i.e. 3.15.

So the median annual turnover is $3 150 000.

**(1 mark)**

1. Both the mean and the median are reliable measures of the centre of this distribution because the distribution is approximately symmetrical and has no outliers.

**(1 mark)**

1. 

 

The upper fence is $7 400 000.  **(1 mark)**

**Question 2** (2 marks)

1. From the segmented bar chart we see that 40% of small businesses had overseas only markets.



So 136 of these small businesses had overseas only markets. **(1 mark)**

1. Both the small and medium sized businesses had approximately the same percentage of domestic only markets i.e. 30% for the small businesses and 32% for the medium businesses.  **(1 mark)**

**Question 3** (9 marks)

1. We are looking for the median ‘*y*’ value, which is 0.42.

**(1 mark)**

1. The association is strong, positive and linear.

 **(1 mark)**

(Note that the ‘strong’ is confirmed using



This value of *r* confirms a strong assocation.)

**c.** The explanatory variable is *age*.

 **(1 mark)**

**d.**



 **(1 mark)**

1. The vertical axis intercept of the least squares line predicts that a company starting up, that is with an age of zero, will have a debt ratio of 0.282.

**(1 mark)**

1. The slope of the regression line tells us that for every increase of one year in the age of a company, there will be a predicted increase of 0.012 in the debt ratio of the company.

 **(1 mark)**

1. From the scatterplot, we obtain the data point (14, 0.4).

So the actual value of the debt ratio when  is 0.4.

To find the predicted value of the debt ratio when , we calculate

 

So the predicted value when  is 0.45. **(1 mark)**



**(1 mark)**

1. The coefficient of determination tells us that 76.8% of the variation in the *debt ratio* can be explained by the variation in the *age* of the company. **(1 mark)**

**Question 4** (5 marks)

1. Enter the data into your CAS.

Note that the ‘*x*’ variable is *year of operation*.

The ‘*y*’ variable is .



or



where values are expressed correct to three significant figures.

 **(1 mark)** for 5.78

**(1 mark)** for −0.136

1. The response variable is 

 **(1 mark)**

1. Since 



The capital expenditure of the company in its 15th year is predicted to be $5495 (correct to four significant figures).

 **(1 mark)**

1. The 15th year of operation is outside the range of data that was used to calculate the least squares regression line equation. This means we would be extrapolating beyond the data range and therefore the predicted value may not be reliable.

**(1 mark)**

**Recursion and financial modelling**

**Question 5** (4 marks)

1. $15 000

**(1 mark)**

1. 

**(1 mark)**

1. 

**(1 mark)**

1. Generate the sequence on your CAS.

15 000, 15 150, 15 301.50, 15 454.51, … 17 942.21, 18 121.63, …

Counting the terms in the sequence we note that 

(Remember that )

So after 19 quarters the balance first exceeds $18 000.

 **(1 mark)**

**Question 6** (4 marks)

1. **i.** 

 

The value is $6 950.

**(1 mark)**

* 1. annual flat rate of depreciation



**(1 mark)**

1. **i.** Since , the annual percentage rate of depreciation is

****

**(1 mark)**

1. ****

**(1 mark)**

**Question 7** (4 marks)

1. Using TVM:



Joyce’s monthly repayment will be $1731.59. **(1 mark)**

1. Using TVM:



This means that after 4 years (48 months) of paying $2105 each month, the balance of the loan is -22.0833. This means that Joyce owes an extra $22.08.

Her last payment will have to be 

**(1 mark)**

1. Using TVM – find the amount Joyce owes after 2 years:



She owes $47 611.21 after 2 years.

After 3 years:



She owes $24 504.84 after 3 years.

She has reduced the balance of the loan by  during the third year of the loan.

**(1 mark)**

She has paid  during the third year.

So  is the amount of interest she has paid during the third year.

 **(1 mark)**

**SECTION B - Modules**

**Module 1 - Matrices**

**Question 1** (3 marks)

1. The order of matrix *G* is  (i.e. 4 rows and 1 column).

**(1 mark)**

1. If the matrix product  exists and produces a total, i.e. a matrix of order , then matrix *P* must be of the order .

****

So .

**(1 mark)**

1. From part **b.**, the matrix product  gives us the total number of grants provided this year. The matrix product  gives us the total number of grants provided last year, i.e. it adds all the elements in matrix *G*.

The given matrix expression gives the total number of grants for last year and this year.

**(1 mark)**

**Question 2** (3 marks)

1. The soccer club and the football club have never made a joint application together.

 **(1 mark)**

1. The leading diagonal gives the number of joint applications that:
* *C* makes with *C*
* *F* makes with *F*
* *L* makes with *L*

and so on.

A club cannot make a joint application with itself hence all the numbers along the leading diagonal must be zero.

**(1 mark)**

1. The netball club made a total of  joint applications. Of these, 3 were with the football club.

So  of joint applications made by the netball club were made with the football club.

**(1 mark)**

**Question 3** (4 marks)

1. 0.3 or 30% of funds allocated to **clubs** in the first year will be allocated to **schools** in the second year i.e. 

 **(1 mark)**

1. 

 **(1 mark)**



In the third year $13 420 was allocated to *I*.

Using *T* we see that 0.2 or 20% of what is allocated to individuals one year is allocated to individuals the next year.



 **(1 mark)**



We have a steady state, so over the long term, there will be $41 666.66 allocated to schools.

The proportion of the total funds available that this represents is



**(1 mark)**

**Question 4** (2 marks)

1. Method 1

The amount of money to be allocated each year remains constant i.e. $120 000. Therefore, the sum of all the elements in the state matrices should be 120 000.

Since matrix *A* is added to the state matrix , then its elements must total zero.



**(1 mark)**

Method 2



The sum of the elements in  should be 120 000.



**(1 mark)**



So $30 400 is to be allocated to schools in the third year of this new allocation model.

**(1 mark)**

**Module 2 - Networks and decision mathematics**

**Question 1** (3 marks)

1. Four vertices have an even degree (i.e. *C*, *E*, *F* and *G*). **(1 mark)**
2. *A B G H F E D C A*

or *A C B G F H E D A*

There are many others. Each route must start and finish at *A* and pass through each of the other vertices only once.

**(1 mark)**

1. Vesna could not follow an Eulerian trail because the graph does not have exactly two vertices that have an odd degree. (It has 4 vertices with an odd degree).

 **(1 mark)**

**Question 2** (3 marks)

1.  **(1 mark)**

Note that the edge marked 6 passes from right to left across the cut instead of from left to right and therefore we don’t count it.

1. Method 1



Starting with the upper path, there is a maximum of ten trucks that can pass.

The middle path has seven trucks that can pass and the lower path has three.

The ordered pairs on the edges coming from the subdivision vertex and going to the dump facility vertex give (initial capacity, final flow).

The total number of trucks coming from the subdivision is .

The total number of trucks going to the dump facility is .

The maximum number of trucks is therefore 20.  **(1 mark)**

Method 2



The minimum cut is. Note that the edges marked 9 and 6 pass from right to left (or more from top to bottom!!!!!) across the cut instead of from left to right and therefore we don’t count them. **(1 mark)**

1. Looking at the original graph, the greatest number of trucks that can pass from the subdivision to the dump facility along one route is ten. This is found along the upper path. The middle path is limited to eight. The bottom path is limited to seven.

**(1 mark)**

**Question 3** (4 marks)

1. Do a forward and backward scan for the network.



The earliest start time for activity G is 8 weeks.

 **(1 mark)**

1. Using the graph above, the critical path is *C D G K*.

**(1 mark)**

1. Again, using the graph above, the four activities with a float time of three days are *A*, *E*, *F* and *H*.

**(1 mark)**

1. It means for this second subdivision of land that activities *B*, *D* and *E* are immediate predecessors of activity *J*. (Activity *I* is also an immediate predecessor of activity *J.*)

**(1 mark)**

**Question 4** (2 marks)

1. 



The graph has 4 faces.

**(1 mark)**

1. Ideally the camera would follow an Eulerian circuit whereby it would start and finish at the same vertex and pass through every pipe just once.

An Eulerian circuit exists if every vertex has an even degree. Our graph has four vertices *N*, *M*, *J* and *K* with odd degrees.

By effectively adding two edges, the graph will have an Eulerian circuit and the two shortest edges connecting these vertices are *JN* and *KM*. So the camera will have to pass over these edges twice. One possible route could be *I N M L K M K J N J I*.

The minimum distance is therefore.

**(1 mark)**

### Module 3 - Geometry and measurement

**Question 1** (3 marks)

1. 

  square centimetres

 **(1 mark)**

****



The length of the flute is 68 cm, to the nearest centimetre.

 **(1 mark)**

1. Greta’s carry on luggage has volume 



Depth of carry on luggage is 20 cm so depth of checked in luggage is 40 cm.

**(1 mark)**

**Question 2** (4 marks)

1. When Greta left Melbourne at 2.15pm on Thursday, it was 7.15am on Thursday in Athens i.e. Athens is seven hours **behind** Melbourne because Melbourne is further east.

Greta arrived at 8.55am on Friday in Athens so her travel time was 25 hours and 40 minutes.

 **(1 mark)**

1. difference in longitude is .



 **(1 mark)**



The distance required is the length of an

arc from Louisville to the North Pole.



Distance is 5808 km (to the nearest km).

**(1 mark)**

**(1 mark)**



1. The difference in longitude between

Louisville and Athens is .



Distance is 9682 km (to the nearest km).

**(1 mark)**

**(1 mark)**

**Question 3** (2 marks)



The bearing of the recital hall

from the hotel is .

 **(1 mark)**

1. There are two possible places where the café could be as shown in the diagrams below.



This scenario is referred to as “the ambiguous case” of the sine rule.



**(1 mark)**

**Question 4** (3 marks)

1. In ,



So  (correct to 2 decimal places).

 **(1 mark)**

1. area of sector *ABC*



**(1 mark)**

area of 





Required area is 238 m2 (to nearest square metre). **(1 mark)**

**Module 4 - Graphs and relations**

**Question 1** (3 marks)

1. From the graph, the discount is 15%.

**(1 mark)**

1. Again from the graph Joe needs to purchase a minimum of 11 tins of paint to get a discount of 10%.

**(1 mark)**

1. Again from the graph, the maximum number of tins Joe could purchase would be 45.

**(1 mark)**

**Question 2** (3 marks)

1. From the graph, the amount charged was $17 000.

**(1 mark)**

(i.e. the point (17 000, 500) lies on the graph)

1. **i.** The intercept is 2000.

**(1 mark)**

* 1. The amount charged for a job has to be more than $2000 for a bonus to be received.

 **(1 mark)**

**Question 3** (2 marks)



 **(1 mark)**

1. 



**(1 mark)**

**Question 4** (4 marks)

1. **i.**  is the required inequality. Note that the inequality could **not** be 

because the point must satisfy it.

**(1 mark)**

* 1. Joe has a contractual arrangement to paint at least one house per month for the builder.

**(1 mark)**



The feasible region is shown above.

The integer points contained in this region (which has borders included) are



There are eight integer points.

**(1 mark)**

1. The profit *P* made on average by the business is given by .

Using some of the points found in part **b.**, (i.e. clearly (0, 4) will give a greater profit than (0,3) etc.)



Maximum profit per month will be achieved by painting one apartment and three houses.

**(1 mark)**