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YEAR 12 *Trial Exam Paper*

2019

FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:
➢ worked solutions
➢ explanatory notes
➢ tips.

SECTION A – Core

Data analysis

Question 1

Answer: D

Explanatory notes

There are 16 babies born with a *weight* between 2.0 and 4.0 kilograms.

$$\frac{16}{25} \times 100 = 64\%$$

Question 2

Answer: C

Explanatory notes

There are 25 data values, and the median (middle value) will therefore be the thirteenth value, as illustrated below.

birth weight   key: 0 | 1 = 0.1

0	
1	7 9
2	2 4 6 7 8
3	1 2 3 3 4 ⑤ 5 6 6 7 9
4	2 3 3 5 7 8
5	1

Answer = 3.5

Tip

- *Remember that determining the median of a data set will depend on whether there are an odd or even number of data values. An odd number of data points will result in your median being one of your original data values. An even number of data points will result in your median being between two of your data points. To find the median, add them together and divide by 2.*

Question 3

Answer: D

Explanatory notes

Reading from the graph:

$$24 + 20 + 12 + 4 = 60\%$$

SECTION A

Question 4

Answer: E

Explanatory notes

$$z = \frac{x - \bar{x}}{s}$$
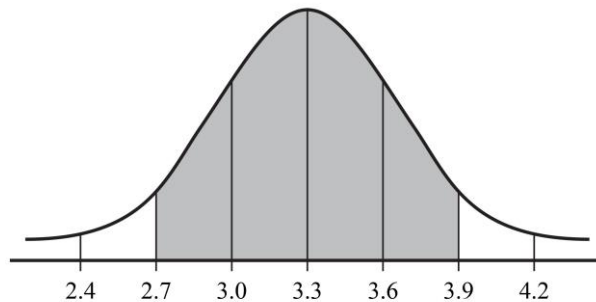
$$0.8 = \frac{x - 3.3}{0.3}$$

$$x - 3.3. = 0.24$$

A CAS calculator can be used to solve this.

$$x = 3.54$$

Question 5

Answer: D

Explanatory notes

Use the 68 – 95 – 99.7% rule together with the given standard deviation.

The normal curve is shown below, with relevant sections shaded.

2.7 and 3.9 are both two standard deviations away from the mean. Therefore, 95% of the babies born lie between these values.

SECTION A

Question 6

Answer: E

Explanatory notes

The normal curve is shown below, with the required sections highlighted.
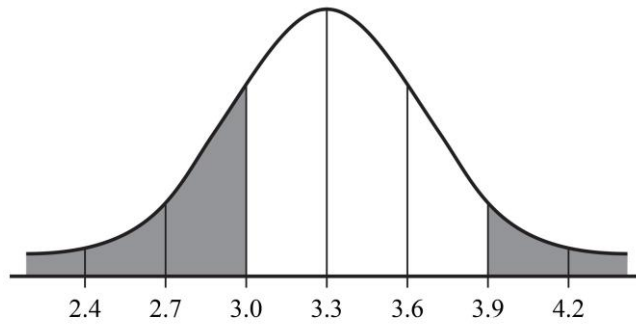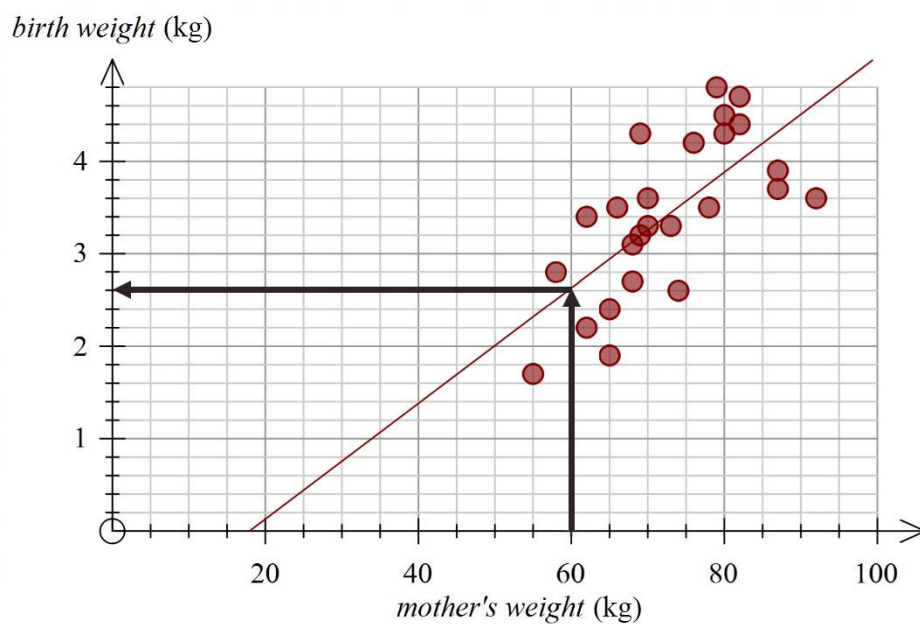

The percentage required is $16 + 2.5 = 18.5\%$

Therefore, the predicted number of babies born within these weight ranges will be 18.5% of 150.

$$\frac{18.5}{100} \times 150 = 27.75 \approx 28$$

Question 7

Answer: A

Explanatory notes

Reading from the graph:

Answer $= 2.6$

SECTION A

Question 8

Answer: E

Explanatory notes

Option A is incorrect. *Mother's weight* is the explanatory variable; in this equation it is written as the response variable.

Option B is incorrect. *Mother's weight* is the explanatory variable; in this equation it is written as the response variable.

Option C is incorrect. This equation has both a negative slope and negative *y*-intercept, but this graph has a positive slope.

Option D is incorrect. This equation has both a positive slope and positive *y*-intercept, but this graph has a negative *y*-intercept.

Question 9

Answer: E

Explanatory notes

To find the value of *r*, find the square root of the coefficient: $r = \sqrt{0.6877} = 0.829276$

Tip

- *It is important to remember that the square root of a number can either be a positive or a negative. When calculating r from r^2 you must look at the graph or equation that you have been given. If the regression line has a positive gradient (slope), then the value of r will be positive. If it is negative, then r will also be negative.*

Question 10

Answer: B

Explanatory notes

Residual = actual value – predicted value

Reading from the graph, the actual value for *birth weight* when the *mother's weight* is 92 kilograms is 3.6 kilograms.

The line predicts that the *birth weight* will be 4.6 kilograms.

This is a residual value of –1.0 kilograms.

Question 11

Answer: E

Explanatory notes

$$b = r \times \frac{s_y}{s_x} = 0.98 \times \frac{18.77}{4.47} = 4.11512$$

$$a = \overline{y} - b\overline{x} = 50.53 - 4.115 \times 8 = 17.61$$

Therefore, $y = a + bx$ is $y = 17.6 + 4.1x$.

Question 12

Answer: B

Explanatory notes

Option B is correct. These transformations are the only ones that could linearise the data.

Question 13

Answer: D

Explanatory notes

When the y^2 transformation is performed, the equation obtained is $y^2 = 73661.5 - 8115.6x$.

Substitute $x = 5$ and find the value of y.

$$y^2 = 73661.5 - 8115.6x$$

$$y^2 = 73661.5 - 8115.6 \times 5$$

$$y = \sqrt{73661.5 - 8115.6 \times 5}$$

$$y = 181.89 \approx 182$$

Tip

- *When performing a data transformation on the response variable, as above, it is important to note that when finding the least squares line, you will not have an equation in the form of $y = a + bx$. The variable y has been transformed, which means there will be an additional step in your working.*

Question 14

Answer: D

Explanatory notes

Three-mean smoothing using values for June, July and August gives:

$$\frac{89 + 120 + 105}{3} = 104.6667 \approx 105$$

Question 15

Answer: C

Explanatory notes

There are twelve seasons, so the seasonal indices should add to 12.

We are missing one of the indices and can therefore subtract our known values from 12 to find the unknown.

$12 - 1.29 - 1.17 - 0.77 - 0.70 - 0.92 - 1.24 - 1.08 - 0.99 - 0.80 - 0.95 - 1.22 = 0.87$

Question 16

Answer: E

Explanatory notes

$\text{actual value} = \text{predicted value} \times \text{seasonal index}$

$\text{actual value} = 99 \times 1.24$

$\qquad\qquad = 122.76 \approx 123$

SECTION A

Recursion and financial modelling

Question 17

Answer: E

Explanatory notes
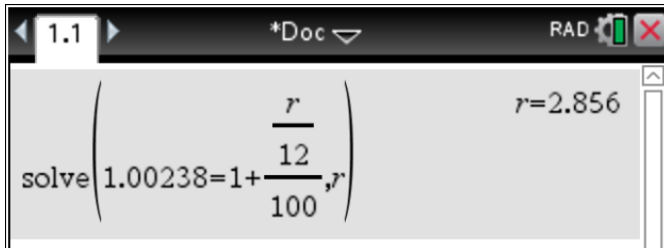
The general relation for a reducing balance loan is:

$V_0 =$ the principal, $V_{n+1} = RV_n - D$

where $R = 1 + \dfrac{r}{100}$

r is the annual interest rate, and in our relation we have a monthly calculation.

Therefore, $R = 1 + \dfrac{r/12}{100}$

In this relation, $R = 1.00238$. Use this to solve for r.

Therefore $r = 2.86\%$

Tip

- *When you have compound interest (or a reducing balance loan) you may need to change the annual interest rate.*
  The following information is useful when doing this:
  - *Compounding bi-yearly*   *divide by 2*
  - *Compounding quarterly*   *divide by 4*
  - *Compounding monthly*   *divide by 12*
  - *Compounding fortnightly*   *divide by 26*
  - *Compounding weekly*   *divide by 52*

SECTION A

Question 18

Answer: C

Explanatory notes

Use recursion to find the value of the loan after six months.

This is best done using a CAS calculator.

Question 19

Answer: B

Explanatory notes

Options A and C are incorrect. The balance does not increase by a regular amount each year, and therefore cannot be simple interest or a perpetuity.

Options D and E are incorrect. The balance continues to increase; therefore, no withdrawals are being taken.

Question 20

Answer: E

Explanatory notes

Options A and B are incorrect. They do not take into account the interest added prior to the payment, that is used to reduce the balance. The value of R must be calculated using,

$$R = 1 + \frac{r/n}{100} .$$

Option C is incorrect. The payment added to balance of the loan, instead of having it subtracted; therefore, it does not reduce the balance of the loan at all.

Option D is incorrect. It is a relation that can be used to model flat rate depreciation. In a reducing balance loan, interest must be added before the payment is deducted.

SECTION A

Question 21

Answer: D

Explanatory notes

The printer has depreciated by $1200 - 765 = 435$ total over the three years.

This is equal to $435 \div 3 = 145$ each year.

The cost per page printed is $145 \div 1740 = \$0.08338$ cents per page.

Question 22

Answer: C

Explanatory notes

The amount that Kate has paid into her loan is $5 \times 12 \times 3100 = 186\,000$.

The amount that Kate has actually reduced the balance of her loan by is $425\,000 - 340\,089.56 = 84\,910.40$.

Therefore, the interest that Kate has paid is $186\,000 - 84\,910.40 = 101\,090$.

Tips

- *Remember that if money is being invested, the value should be entered as a negative into the finance solver.*
- *Carefully read the question – do not assume that interest or payments are added monthly.*

SECTION A

Question 23

Answer: E

Explanatory notes

Use the finance solver.

Finance Solver	
N:	48
I(%):	4.95
PV:	319256
Pmt:	-7345.0113562978
FV:	0
PpY:	12

Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...
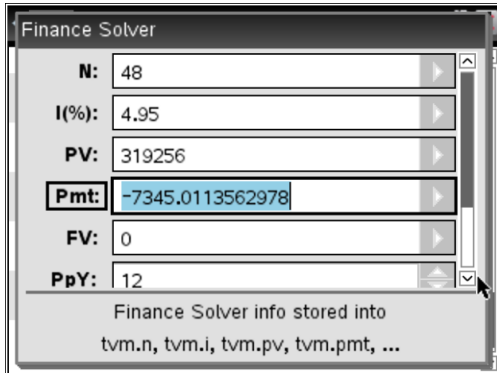
Question 24

Answer: C

Explanatory notes

The principal reduction between the fifteenth and sixteenth payment is

$346117.98 - 344788.03 = 1329.95$.

The payment is \$2700; therefore, the difference is the interest paid: $2700 - 1329.95 = 1370.05$.

SECTION A

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Section B – Modules

Module 1 – Matrices

Question 1

Answer: E

Explanatory notes

Add the elements in the Evening (*E*) column.
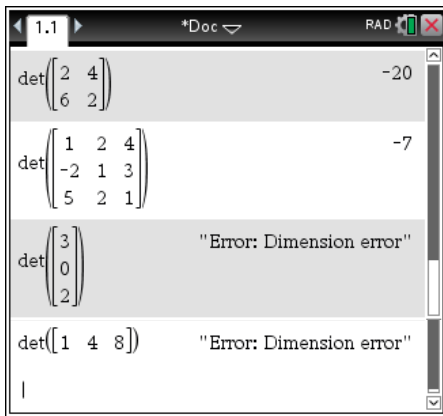
$13 + 21 + 8 = 42$

Question 2

Answer: C

Explanatory notes

Only matrices that have a determinant that does not equal zero will have an inverse.

A CAS calculator can be used to find the determinant.

Only matrices A and B have a determinant that is not zero or is 'undefined'.

Tip

- *If you have a 2×2 matrix, you can use* $\det = ad - bc$*, which can often be quicker than using your calculator. This is a judgement you need to make.*

Question 3

Answer: B

Explanatory notes

The number of staff that Brad reports to can be found by adding the elements in row 2.
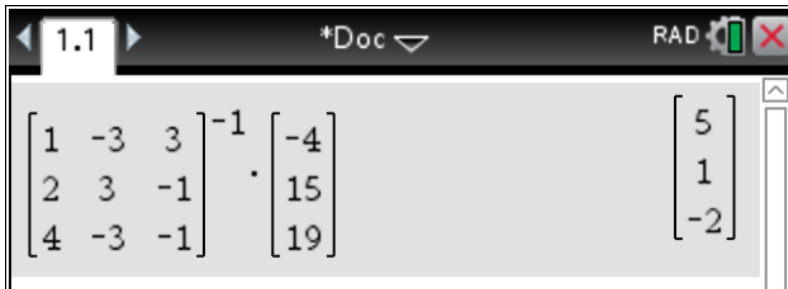
Question 4

Answer: D

Explanatory notes

A 1×1 matrix is required to provide a total. Option D is only the option that does this.

In Option D we have 1×6 and a 6×1 matrix. These will multiply to give a 1×1 matrix, as shown below.

$$\begin{bmatrix} 12 & 10 & 8 & 6 & 10 & 5 \end{bmatrix}\begin{bmatrix} 0.65 \\ 0.78 \\ 1.16 \\ 1.35 \\ 5.35 \\ 12.26 \end{bmatrix} = \begin{bmatrix} 12\times0.65+10\times0.78+8\times1.16+6\times1.35+10\times5.35+5\times12.26 \end{bmatrix} = \begin{bmatrix} 147.78 \end{bmatrix}$$

Question 5

Answer: D

Explanatory notes

A CAS calculator can be used to quickly find the answer.

SECTION B – Module 1

Question 6

Answer: D

Explanatory notes

From the transition diagram, enter the following elements in the transition matrix.

$$
D = \begin{array}{c} \textit{this year} \\ \begin{array}{ccc} A & B & C \end{array} \\ \left[\begin{array}{ccc} & 0.30 & 0.65 \\ 0.45 & & 0.10 \\ 0.20 & 0.50 & \end{array}\right] \begin{array}{l} A \\ B \\ C \end{array} \textit{next year} \end{array}
$$

Provided is the proportion of change from each state back to itself; for example, A to A.

The columns in the transition matrix should add to 1.

Therefore, subtract the known values from 1 to obtain the remaining element.

Column *A*: $1 - 0.45 - 0.20 = 0.35$

Column *B*: $1 - 0.30 - 0.50 = 0.20$

Column *C*: $1 - 0.65 - 0.10 = 0.25$

Now complete the matrix.

$$
D = \begin{array}{c} \textit{this year} \\ \begin{array}{ccc} A & B & C \end{array} \\ \left[\begin{array}{ccc} 0.35 & 0.30 & 0.65 \\ 0.45 & 0.20 & 0.10 \\ 0.20 & 0.50 & 0.25 \end{array}\right] \begin{array}{l} A \\ B \\ C \end{array} \textit{next year} \end{array}
$$

Tip

- *Remember that in a transition matrix, the columns will need to add to 1, unless it is indicated that the number of elements in the system can change.*

Question 7

Answer: A

Explanatory notes

$G_4 = T \times G_3$

so, $G_3 = T^{-1} \times G_4$

$$G_3 = \begin{bmatrix} 0.62 & 0.15 & 0.08 & 0.05 & 0.01 \\ 0.22 & 0.55 & 0.22 & 0.07 & 0.07 \\ 0.07 & 0.18 & 0.53 & 0.42 & 0.14 \\ 0.05 & 0.07 & 0.13 & 0.38 & 0.32 \\ 0.04 & 0.05 & 0.04 & 0.08 & 0.46 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \\ 10 \\ 6 \\ 4 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 2.28 \\ 2.00 \\ 11.55 \\ 5.91 \\ 6.25 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 12 \\ 6 \\ 6 \end{bmatrix}$$

Therefore, two students received a B on the third maths test.

Question 8

Answer: C

Explanatory notes

To find the steady state, two consecutive matrices with the same elements are required.

Therefore, five students can be expected to receive an A on their maths test.

SECTION B – Module 1

Module 2 – Networks and decision mathematics

Question 1

Answer: A

Explanatory notes
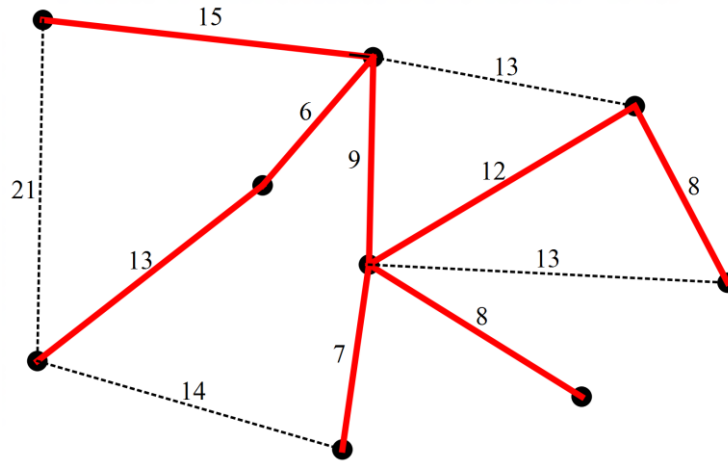
Length of minimum spanning tree is $15 + 6 + 13 + 9 + 7 + 8 + 12 + 8 = 78$

Tip

- *It is important to know the process of finding the minimum spanning tree (Prim's algorithm).*

Question 2

Answer: D

Explanatory notes

Using Euler's formula, $v - e + f = 2$, test each option by substituting values for v, e and f.

Option A is incorrect. $4 - 6 + 5 = 3$, not equal to 2

Option B is incorrect. $4 - 6 + 6 = 4$, not equal to 2

Option C is incorrect. $4 - 6 + 7 = 5$, not equal to 2

Option D is correct. $4 - 6 + 4 = 2$

Option E is incorrect. $4 - 6 + 3 = 1$, not equal to 2

Question 3

Answer: C

Explanatory notes

A planar graph can be re-drawn with no edges crossing.

It is not possible to do this with option C.

Question 4

Answer: E

Explanatory notes

In order for an Euler trail to be possible within this network, exactly two vertices with an odd degree are required.

The degree of each of the vertices is shown below.

deg(A) = 3   deg(B) = 3

deg(C) = 4   deg(D) = 4

deg(E) = 3   deg(F) = 3

deg(G) = 6

There are currently four odd vertices, and therefore an edge can be added between any two of these. Doing so will result in the two vertices becoming even, leaving exactly two odd vertices.
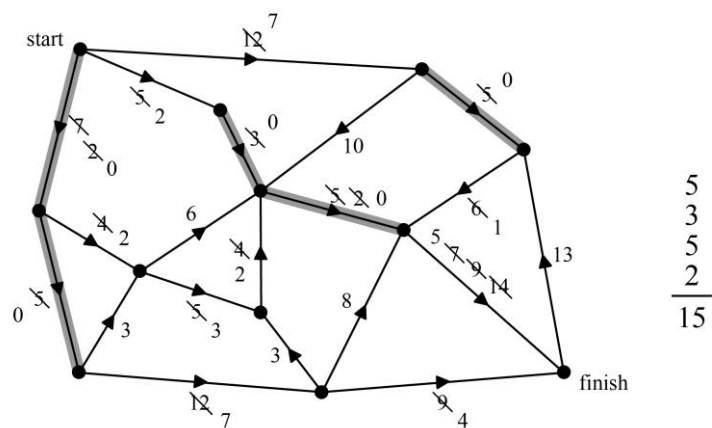
Adding an edge to vertex G, as option E suggests, would make this odd, which would still leave 3 odd vertices.

Question 5

Answer: C

Explanatory notes

One way of finding the maximum flow is to use 'tracking' through the directed graph, as shown below. Cuts can also be used.
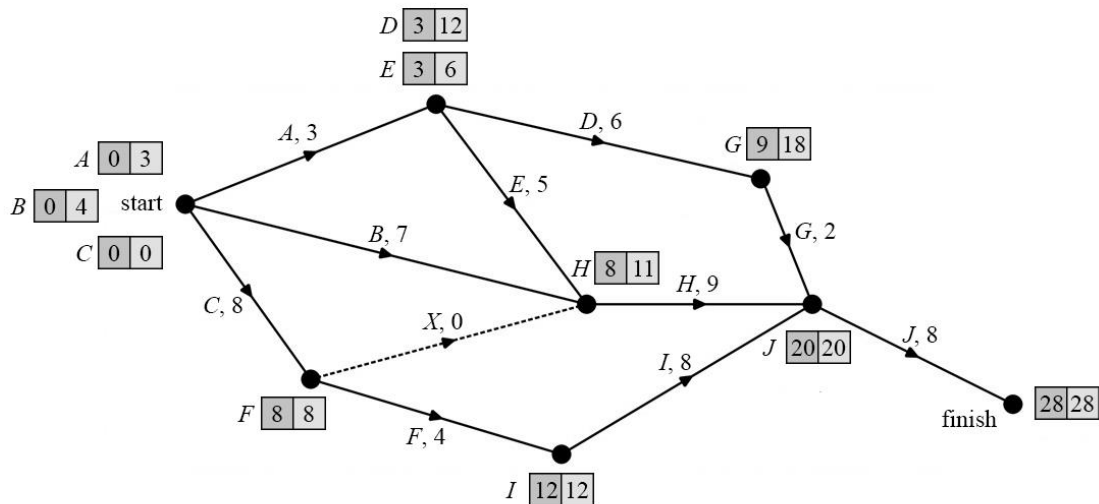
SECTION B – Module 2

Question 6

Answer: D

Explanatory notes

Activities A, B, C and E must all occur before activity H.

Activity D does not have any effect on activity H.

Question 7

Answer: E

Explanatory notes

See working below.

Tip

- *Remember that, although they do not have a weighting, 'dummy' activities indicate that a certain activity must be completed before another.*

Question 8

Answer: D

Explanatory notes

The diagram accompanying **Question 7** can also be used to find the critical path and the minimum completion time.

The critical path is comprised of all activities which have a 0 float time – that is, there is no difference between the EST and LST.

The activities that make up the critical path are: C – F – I – J.

The length of this path is 28; therefore, the minimum completion time is 28 days.

SECTION B – Module 2

Module 3 – Geometry and measurement

Question 1

Answer: D

Explanatory notes

Use Pythagoras' theorem to find the length of the longer side.
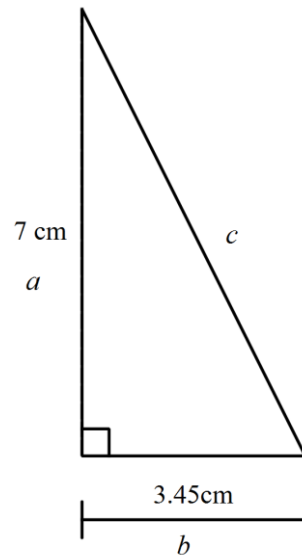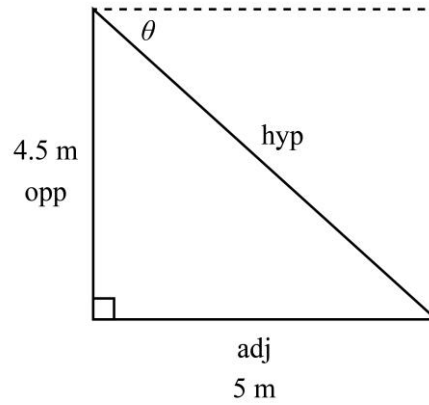

$$c^2 = a^2 + b^2$$
$$c^2 = 7^2 + 3.45^2$$
$$c = \sqrt{7^2 + 3.45^2}$$
$$c = 7.804$$

Question 2

Answer: D

Explanatory notes

Label sides of triangle using opp (opposite), adj (adjacent) and hyp (hypotenuse).

Since measurements are only provided for the opposite and adjacent sides, use the tan ratio.

$$\tan \theta = \frac{opp}{adj}$$

$$\tan x = \frac{4.5}{5}$$

$$x = \tan^{-1}\left(\frac{4.5}{5}\right)$$

$$x = 41.987 = 42°$$

Question 3

Answer: A

Explanatory notes

Area of the larger circle:

$A = \pi r^2$

$A = \pi \times 55^2$

$A = 9505.31777...$

Area of smaller circle (2 halves):

$A = \pi r^2$

$A = \pi \times 36^2$

$A = 4071.504$

Shaded area = larger area – smaller area:

$9505.32 - 4071.504 = 5433.82 \text{ mm}^2$

Tip

- *When calculating the area of a circle, remember to halve the diameter to find the radius.*

Question 4

Answer: E

Explanatory notes

Use the sine rule to find the angle at point *A*. This will help to find the unknown angle at point *B*.

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{9.9}{\sin A} = \dfrac{7.1}{\sin(20)}$

$\sin A = \dfrac{9.9 \times \sin(20)}{7.1}$

$A = \sin^{-1}\left(\dfrac{9.9 \times \sin(20)}{7.1}\right)$

$A = 28.5°$

Now subtract both known angles from 180 degrees, to find the unknown angle, θ.

$x = 180 - 20 - 28.5 = 131.5 \approx 132°$

Question 5

Answer: B

Explanatory notes

$TSA = 2\pi r^2 + 2\pi rh$

$TSA = 2 \times \pi \times 8^2 + 2 \times \pi \times 8 \times (20 - 5)$

$TSA = 1156.11 \approx 1156 \text{ cm}^2$

Question 6

Answer: B

Explanatory notes

$\text{arc length} = 6400 \times \dfrac{\pi}{180} \times (90 - 53)$

$\phantom{\text{arc length}} = 4132.94 \approx 4133 \text{ km}$

Question 7

Answer: E

Explanatory notes

The time difference can be calculated using an approximation of 4 minutes per degree of the longitude.

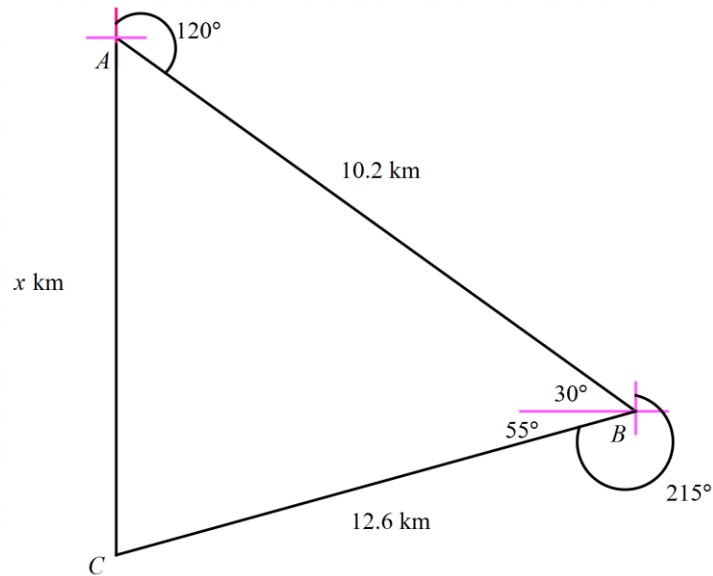$136 - 67 = 69 \text{ degrees}$

$69 \times 4 = 276 \text{ minutes}$

$276 \div 60 = 4.6 \approx 5 \text{ hours}$

Kabul is west of Osaka, so the sun will rise five hours later.

Question 8

Answer: E

Explanatory notes

Start by drawing a diagram. In the diagram below, A is the starting point, B is the lake and C is the drinking fountain.

The diagram below is **not** drawn to scale.

Use the information provided to find the remaining angles in the triangle, as shown above.

Use the sine rule to find the unknown side.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b = \sqrt{12.6^2 + 10.2^2 - 2 \times 12.6 \times 10.2 \times \cos 85}$$
$$b = 15.5048$$

SECTION B – Module 3

Module 4 – Graphs and relations

Question 1

Answer: D

Explanatory notes

First, find the gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{8-6}{2-1}$$
$$m = \frac{2}{1} = 2$$

Now, find the equation, first in gradient-intercept form.

$$y - y_1 = m\ (x - x_1)$$
$$y - 6 = 2(x-1)$$
$$y - 6 = 2x - 2$$
$$y = 2x - 2 + 6$$
$$y = 2x + 4$$

Rearrange this equation to get the correct form.

$$y = 2x + 4$$
$$2x - y = -4$$

Question 2

Answer: D

Explanatory notes

The charge of $150 is a constant.

The cost then increases by $70 per hour.

Therefore, 150 is the *y*-intercept and 70 is the gradient or slope.

$$C = 70t + 150$$

SECTION B – Module 4

Question 3

Answer: B

Explanatory notes

From the graph, the *y*-intercept is –4.

Find the gradient using the points (1, –1) and (2, 2).

$$m = \frac{rise}{run} = \frac{+3}{+1} = 3$$

Therefore, the equation is $y = 3x - 4$.
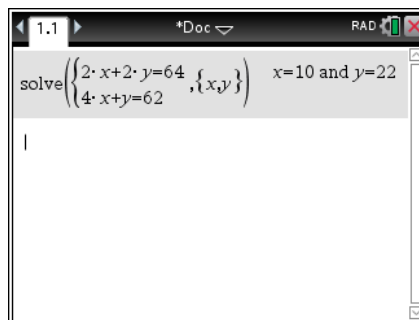
Question 4

Answer: C
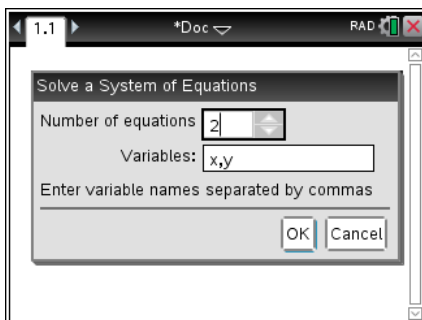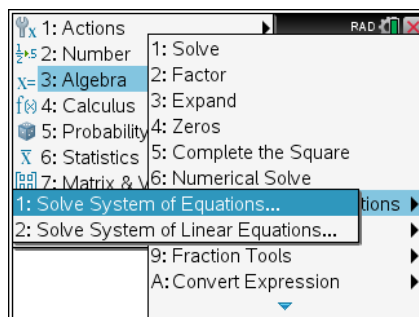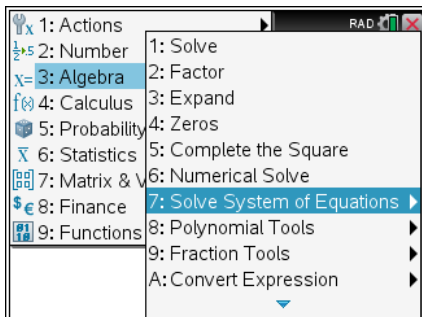
Explanatory notes

First, write two equations to represent each of the family's purchases.

Jackson family: $2x + 2y = 64$

Learner family: $4x + y = 62$

Then solve these simultaneously.

This can be done using a CAS calculator.

Therefore, one serve of small fries costs $10 and one serve of large fries costs $22.

SECTION B – Module 4

Question 5

Answer: D

Explanatory notes

We must test each coordinate to ensure that it satisfies all constraints.

$(4, 6)$, $x > 3$, $y \leq 10$, $3 + 10 < 15$ YES

$(5, 8)$, $x > 3$, $y \leq 10$, $5 + 8 < 15$ YES

$(6, 4)$, $x > 3$, $y \leq 10$, $6 + 4 < 15$ YES

$(7, 9)$, $x > 3$, $y \leq 10$, $7 + 9 < 15$ NO

$(8, 3)$, $x > 3$, $y \leq 10$, $8 + 3 < 15$ YES

Question 6

Answer: A

Explanatory notes

Since the line of the equation is dotted, no point on it is shared with the shaded region. Further, all points in the shaded area are less than all points on the equation. Hence the required relation is $<$.

Question 7

Answer: E

Explanatory notes

The area of the first piece of art is $15 \times 12 = 180$ cm^2.

The cost of this artwork is $50 + 180 \times 15 = \$2750$.

The area of the second piece of art is $25 \times 25 = 625$ cm^2.

The cost of this artwork is $20 \times 625 = \$12\,500$.

Total cost $= 2750 + 12\,500 = \$15\,250$.

Question 8

Answer: C

Explanatory notes

Substitute the point (10, 25), into the equation $y = kx^2$ to find the value of k.

$25 = k \times 10^2$

$25 = 100k$

$k = \dfrac{25}{100} = \dfrac{1}{4}$

END OF WORKED SOLUTIONS