TSSM™
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2019 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER									Letter

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A					36
B			4	2	24
					Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 27 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Core: Data analysis

Question 1. (9 marks)

The length of new born full term female babies in Australia is normally distributed. The "normal" range of lengths is considered to be from 45.7 cm up to 52.7 cm.

a. Given that 95% of female babies are born with lengths between 45.7 cm and 52.7 cm, calculate:

i. the mean length of new born full term female babies in Australia.

ii. the standard deviation of new born babies in Australia.

1 + 1 = 2 marks

b. Two daughters are born to a family. Both babies are full term; the first measured 49.0 cm at birth and the other 50.2 cm.

i. Determine the z scores for each child. Give answers correct to two decimal places

SECTION A – Question 1 - continued

ii. Given that adult female heights are normally distributed with a mean of 165 cm and a standard deviation of 9 cm; predict the adult heights of the two sisters using the z scores calculated from their birth lengths. Give answers to the nearest centimetre.

2 + 2 = 4 marks

c. The two sisters are now aged in their early 20's and considered to have reached their full adult heights. The elder sister has a height of 167 cm and the younger is 175 cm tall.

i. Determine the residual for each using your predicted heights.

ii. Comment on the reliability of predicting adult height using length at birth based on these two results.

2 + 1 = 3 marks

SECTION A- continued
TURN OVER

Question 2. (8 marks)

The back to back stem leaf plot below shows the daily maximum temperatures recorded in February in 2018 and 2019 in Melbourne.

Key
$19|2 = 19.2^oC$

2018	Stem	2019
8	19	2 7
	20	2 4
2	21	2 5
5 2 1	22	3 7 8
6 3	23	1 2 2 2 3 6
9 2 0	24	2
7 7 4 3	25	7 8
5	26	9
5	27	5
1	28	3
9 5 3 0	29	0
9 3	30	
6 3 0	31	
	32	9
5	33	
	34	2 4 7
	35	
	36	8
4	37	
	38	2

a. From the information above, determine

i. the median maximum daily temperature in 2019

____

ii. the percentage of days in February 2018 when the maximum daily temperature was at least 30^oC

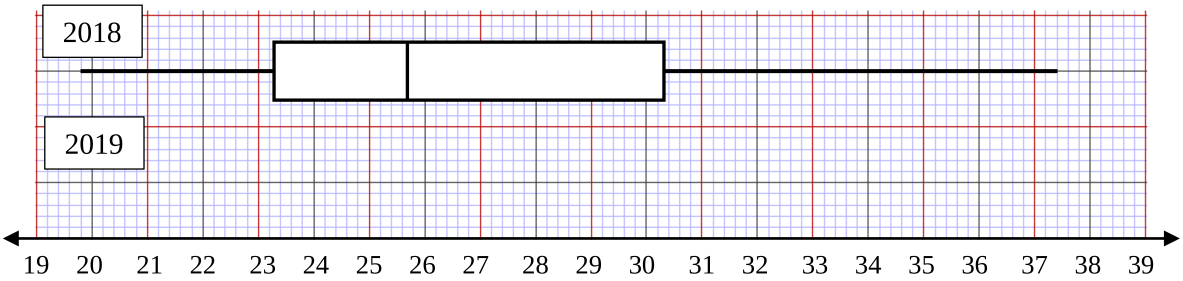
____

____

1 + 1 = 2 marks

SECTION A- Question 2 - continued

b. The maximum daily temperatures in February 2018 have been represented on a box plot below. Complete the box plot for maximum daily temperatures in February 2019 in the space provided.



2 marks

c. Many people consider this last summer to have been hotter than the previous summer. Do the statistics for February support this contention? Provide supporting evidence for your answer.

1 mark

d. What would the maximum daily temperature need to be if it were an outlier in February 2019? Show appropriate working.

2 marks

e. Maximum daily temperature is continuous numerical data. What type of data is the date in February?

1 mark

SECTION A- continued
TURN OVER

Question 3. **(7 marks)**

The following data is the result of a study into monomolecular absorption. The absorption volume was measured as different pressures were applied.

Pressure	77.6	114.9	141.1	190.8	239.9	289	332.8	378.4	434.8	477.3	536.8	593.1	689.1	760
Volume	10.07	14.73	17.94	23.93	29.61	35.18	40.02	44.82	50.76	55.05	61.01	66.4	75.47	81.78

a. Determine the equation of the least squares line that can be used to predict the volume from the pressure applied.
Round your answers to three significant figures.

Volume = $\boxed{}$ + $\boxed{}$ × *pressure*

2 marks

b. Interpret the slope of the regression line in terms of the variables *Volume* and *pressure.*

____

____

1 mark

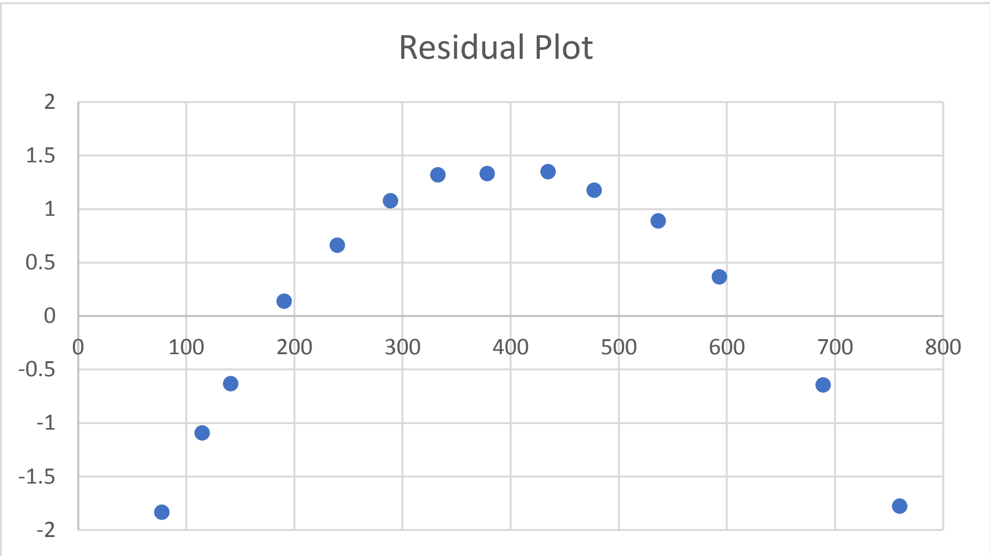
c. State the coefficient of determination for this data. Give your answer correct to four decimal places.

____

____

1 mark

The following diagram is the residual plot for this data.



SECTION A- Question 3 - continued

d. Interpret the residual plot.

1 mark

The best transformation for this data is obtained using a y^2 transformation. The equation is given below:

$$(volume)^2 = -1242.37 + 9.60 \times pressure$$

e. Calculate the correlation coefficient for the transformed data

1 mark

f. Comment on the effectiveness or otherwise of the transformation to improve the accuracy of predictions made.

1 mark

SECTION A - continued
TURN OVER

Core: Recursion and financial modelling

Question 4. (6 marks)

A club purchases new equipment priced at $8420. A deposit of 15% is made and the remainder is paid in 12 monthly instalments of $620.

a. How much is the deposit for this purchase?

1 mark

b. The total amount paid after each monthly instalment can be represented as a recurrence relation where the initial amount paid represents the deposit. Complete the recurrence equation below.

$P_0 = \boxed{}$ $P_{n+1} = \boxed{} + \boxed{}$

2 marks

c. Determine the total amount paid for the equipment by calculating the value of P_{12}

1 mark

d. Calculate the simple interest rate for this repayment option considering the amount "borrowed" is the amount owing after the deposit has been paid. Give your answer correct to two decimal places.

1 mark

e. The payments can be considered as a reducing balance loan where the amount borrowed is the purchase price less the deposit. Using finance solver or otherwise determine the annual interest rate paid under these circumstances. Give your answer correct to four decimal places.

1 mark

SECTION A – continued

Question 5 (6 marks)

Janet retires at age 68 years and receives a superannuation payment of $643,150. She wants to work out a budget so that she does not run out of money to live on during her retirement.

a. She considers setting up an annuity so that she can withdraw funds regularly to cover living costs. Janet assumes that she will most likely be “pushing up daisies” by age 100 years, so the annuity needs to last 32 years. What would she receive monthly if her superannuation payment is invested in an annuity with interest rate 4.75% per annum, compounding monthly?

1 mark

Janet has tallied estimates of costs of living including Rates and insurance on her home, groceries, utilities, phone, car and entertainment and has determined that she currently spends $1000 per **week**. It becomes clear to her that she will need to rein in her spending if she is to live off her superannuation fund!

b. Assuming there are 4.3 weeks in each month, how much would Janet need to withdraw each month to cover her current spending rate?

1 mark

c. If Janet does not change her spending habits (and assuming the costs of living remain the same!), estimate how old she will be when her super fund is exhausted. Give your answer to the nearest year.

2 marks

SECTION A – Question 5 – continued
TURN OVER

d. How much interest would Janet's annuity earn in the third year of the investment? Round your answer to the nearest cent.

2 marks

END OF SECTION A

SECTION B - Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents

Module 1 – Matrices

Question 1. **(4 marks)**

A small regional airline runs flights between the following list of centres located in Victoria and Tasmania. The Matrix F below shows the connections between the centres.

1. Burnie
2. Essendon
3. Flinders Island
4. King Island
5. Launceston

$$\begin{array}{c} \textit{from} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \end{array}$$

$$F = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \; \textit{to}$$

a. Complete the following sentence.

On this airline, you can fly directly from Flinders Island to $\boxed{}$ and $\boxed{}$

1 mark

b. Explain why the leading diagonal contains only 0's.

____

____

____

1 mark

c. Complete the following matrix multiplication $A \times F$ where $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

1 mark

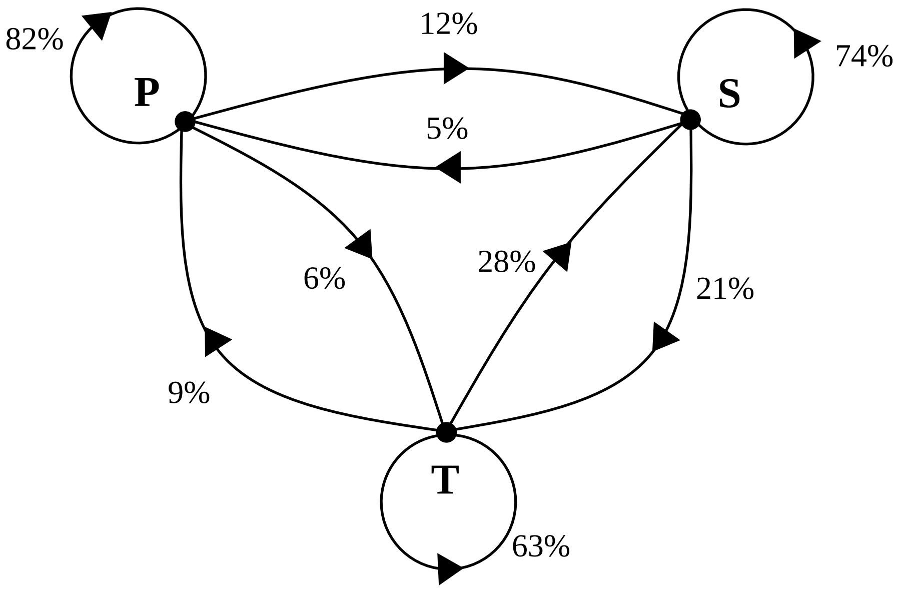
SECTION B - Module 1 – continued

d. In the context of this problem, what information does the result of this matrix multiplication give?

1 mark

Question 2. (4 marks)

A financial planner invests a client's money in property (**P**), shares (**S**) and term deposits (**T**). Income gained in each investment type is paid to the client so that the total amount invested remains the same, year in and year out. Every 12 months the financial planner alters the amount of money in each investment type as shown in the diagram below.



Initially the client's portfolio is made up of investments in dollars represented in the following matrix.

$$S_0 = \begin{array}{c} \begin{bmatrix} 70000 \\ 50000 \\ 400000 \end{bmatrix} \begin{array}{c} T \\ S \\ P \end{array} \end{array}$$

a. How much money will be transferred into a different investment type the following year?

1 mark

SECTION B – Module 1 – continued
TURN OVER

b. The information in the transition diagram has been used to write the transition matrix *T*.
Fill in the missing figures.

$$T = \begin{bmatrix} 0.63 & 0.21 & \text{........} \\ 0.28 & \text{........} & 0.12 \\ \text{........} & 0.05 & \text{........} \end{bmatrix}$$

1 mark

c. If the client chooses to invest an additional $5000 in shares each year from what is earned and the recurrence relation is written in the form $S_{n+1} = TS_n + A$.

i. Write down matrix *A*.

ii. Calculate the amount that will be invested in shares after four years (at the start of the fifth year).

1 + 1 = 2 marks

SECTION B - Module 1 – continued

Question 3. (4 marks)

An AFL merchandise seller stocks scarves (S), beanies (B), guernseys (G) and polos (P). Scarves sell for $23, beanies for $15, guernseys for $50 and polo shirts for $45 each. These are represented in a price matrix, *C*, and sales for the first four home games are recorded in matrix *H* below.

$$C = \begin{array}{c} \begin{bmatrix} 23 \\ 15 \\ 50 \\ 45 \end{bmatrix} \end{array} \begin{array}{c} S \\ B \\ G \\ P \end{array} \qquad H = \begin{array}{c} \begin{array}{cccc} S & B & G & P \end{array} \\ \begin{bmatrix} 53 & 18 & 4 & 40 \\ 10 & 23 & 30 & 28 \\ 22 & 56 & 7 & 7 \\ 2 & 46 & 24 & 28 \end{bmatrix} \end{array} \begin{array}{l} \\ game\ 1 \\ game\ 2 \\ game\ 3 \\ game\ 4 \end{array}$$

a. What is the result of the matrix multiplication $H \times C$ and what does this represent?

2 marks

The total profit in dollars made by the seller for each of these weeks is summarized in the matrix shown below.

$$\begin{bmatrix} 1271.5 \\ 1103 \\ 755 \\ 1075 \end{bmatrix} \begin{array}{l} game\ 1 \\ game\ 2 \\ game\ 3 \\ game\ 4 \end{array}$$

b. Given that the profit made is $*s* for each scarf, $*b* for each beanie, $*g* for each guernsey and $*p* for each polo, calculate the values of *s*, *b*, *g* and *p*.

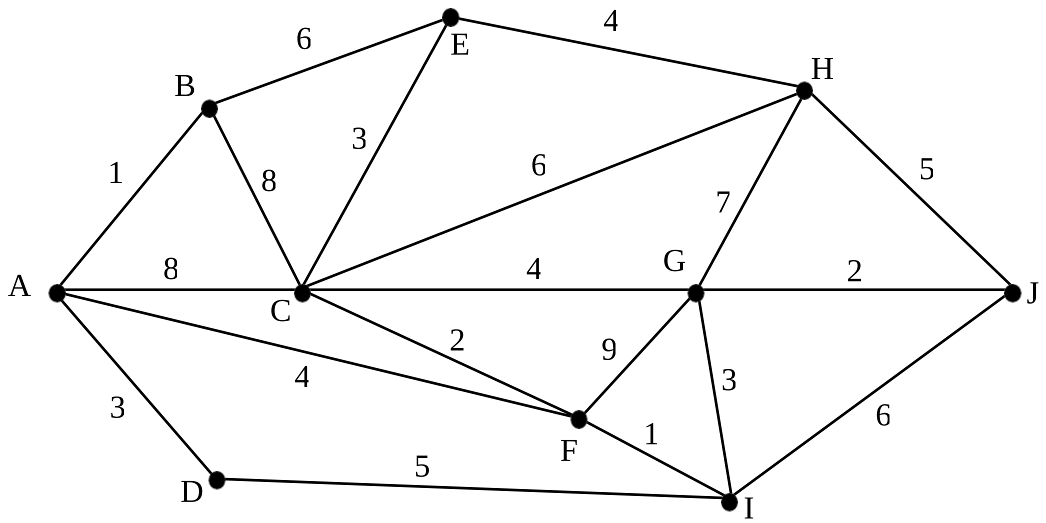
2 marks

End of Module 1 – SECTION B – continued
TURN OVER

Module 2 – Networks and decision mathematics

Question 1. (5 marks)

A group of ten couples are staying at different motels in the same city. The network below shows the distances in kilometres between motels.



a. Friends Jemma and Hayley are staying at motels labelled A and J respectively.

i. Determine the shortest distance between motels A and J

___

ii. If a maxi taxi travels from A to J collecting as many other couples along the way as possible without repeating edges or vertices, how many couples can be collected and dropped off at J?

___

___

___

SECTION B – Module 2: Networks - continued

iii. What is the minimum possible travelling distance to achieve the outcome as stated in part **ii.**? State the path travelled to achieve the minimum distance.

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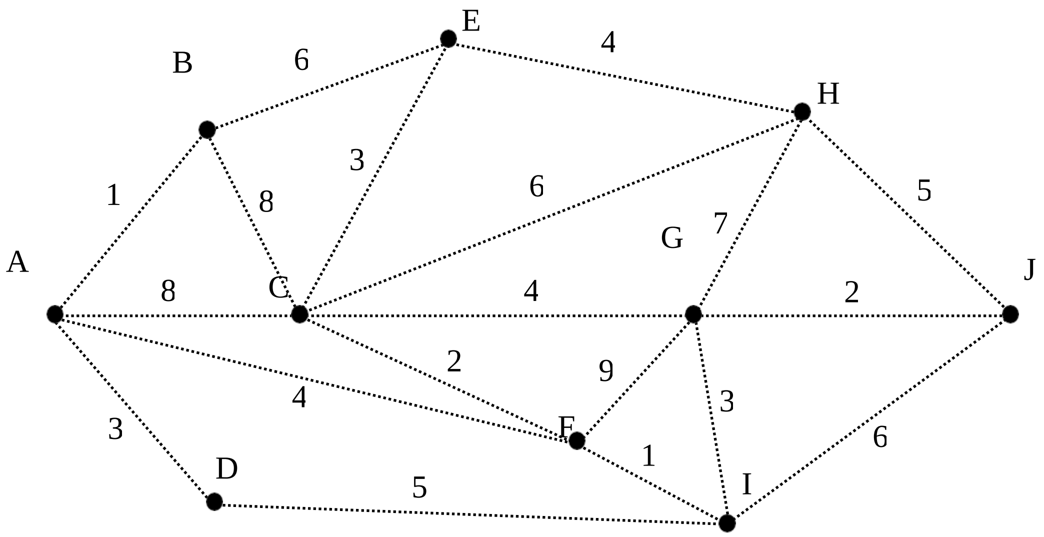
1 + 1 + 2 = 4 marks



b. Determine the minimum spanning tree for this network and show it on the diagram below

1 mark

SECTION B – Module 2: Networks - continued
TURN OVER

Question 2. (7 marks)

A supermarket supervisor needs to have four tasks completed by four floor staff, Ryan, Sophie, Dean and Jess. The tasks are set up a deodorant display (A), transfer items from the "specials" freezer to their usual place (B), pack away Easter display materials (C) and set up of a food tasting station (D). Once each of these tasks is completed, the remaining time during the shift will be spent restocking shelves. From past experience the supervisor predicts the time it will take each worker to complete each task. These are shown in the **Table 1** below.

Table 1

	Worker			
Task	**Ryan**	**Sophie**	**Dean**	**Jess**
A	24	23	26	17
B	35	23	28	18
C	22	18	23	19
D	32	23	35	23

The tasks are to be allocated so that the total time of completing the four tasks is a minimum.
A row reduction is applied to achieve Table 2.

Table 2

	Ryan	**Sophie**	**Dean**	**Jess**
A	7	6	9	0
B	17	5		0
C	4	0	5	1
D	9	0	12	0

a. Complete the row reduction for task B by writing down the number missing from the shaded cell in Table 2.

____

1 mark

b. **i.** Complete a column reduction and show the result in the table below.

	Ryan	**Sophie**	**Dean**	**Jess**
A				
B				
C				
D				

SECTION B – Module 2 - continued

ii. Explain why it is not yet possible to determine an optimal allocation of tasks.

1 + 1 = 2 marks

c. Complete further steps of the Hungarian algorithm in the space below and determine who should complete each task.

Task	Worker
A	
B	
C	
D	

2 marks

d. Given that tasks are allocated with 1.5 hours left in the shift, what total time will be spent restocking shelves?

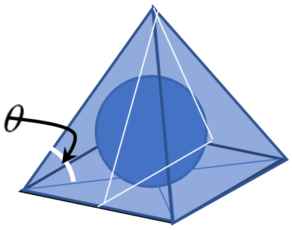
2 marks

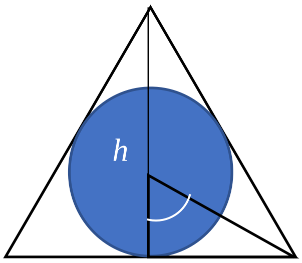
End of Module 1 – SECTION B – continued
TURN OVER

Module 3 – Geometry and measurement

Question 1. (5 marks)

A chocolatier creates a spherical dark chocolate orb which they wish to package in square based pyramid shaped boxes like the one pictured on the left below. The diagram in the middle shows the orb inside the pyramid. The cross-section shown in white in this diagram is an equilateral triangle which is shown on the right with the chocolate orb touching the centres of the sloping faces and the centre of the square base.





Cross section view

a. The chocolate orbs are 5 cm in diameter. The cross section diagram above shows a right angle triangle with its vertical side connecting the centre of the orb to the centre of the square base. Given that the angle subtended at the centre of the orb is 60°, determine the side length of the square base in centimetres correct to one decimal place.

____

____

1 mark

b. Calculate the vertical height of the pyramid, represented by *h* on the diagram above.

____

____

1 mark

c. Determine the angle that the sloping edges make with the base (represented by angle θ on the diagram above). Give your answer to the nearest degree.

____

____

1 mark

SECTION B – Module 3 – continued

d. Calculate the volume of chocolate if the thickness of the walls of the orb are 0.6 cm. Give your answer correct to one decimal place.

1 mark

e. How much unused space, in cubic centimetres, surrounds the chocolate orb inside the box? Round your answer to the nearest whole number.

1 mark

Question 2. (4 marks)

a. Calgary has the same latitude as London but sunrise takes place 7 hours and 36 minutes later than in London on the same day.

i. Is the longitude of Calgary East or West?

ii. Determine the longitude of Calgary given that London has a longitude of 0^{o} (to the nearest degree)

1 + 1 = 2 marks

b. Three friends are enjoying holidays on different continents. Frederick is on holiday in London, Trevor is visiting friends in Calgary, Canada and Allison is travelling in Bogota, Columbia (Lat 5^{o}N, Lon 74^{o}W). The three are wanting to conference call each other, but none of the trio wish to be chatting between the hours of 10 pm and 6 am local time (sleep time).

i. Is it possible for Frederick to have a conversation with Trevor and Allison when the time is 6 am in London?

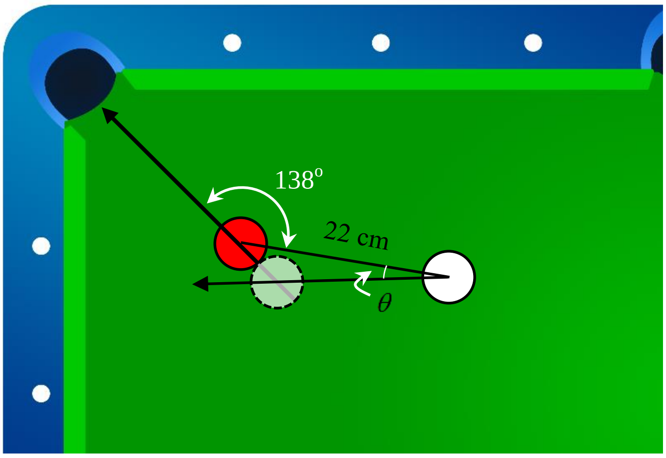
SECTION B – Module 3 – continued
TURN OVER

ii. Between what hours can Frederick make a conference call to both of his friends?

1 + 1 = 2 marks

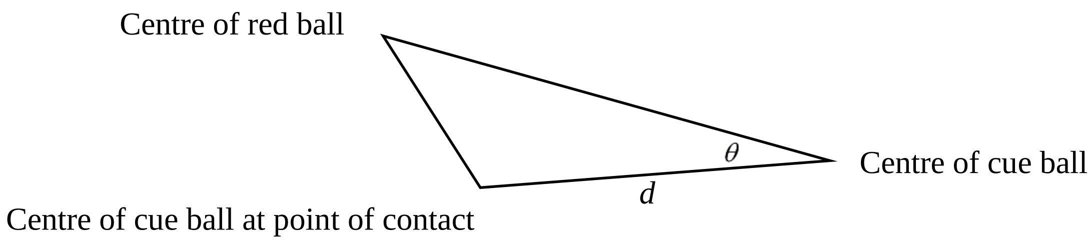
Question 3. (3 marks)

Leonardo loves to play pool. He also loves to use his mathematical talents to assist in the accuracy of each shot he makes.



The diagram above shows the position of the cue ball (white) and the ball (red) he wishes to hit into the corner pocket. It also shows the trajectory that the cue ball needs to be on for the point of contact to be just right. The balls have a diameter of 5 cm.

a. Show all known angles and lengths on the diagram below.



1 mark

b. Calculate the distance (d) the cue ball will travel to the point of contact. Give your answer to the nearest millimetre.

1 mark

SECTION B – Module 3 – continued

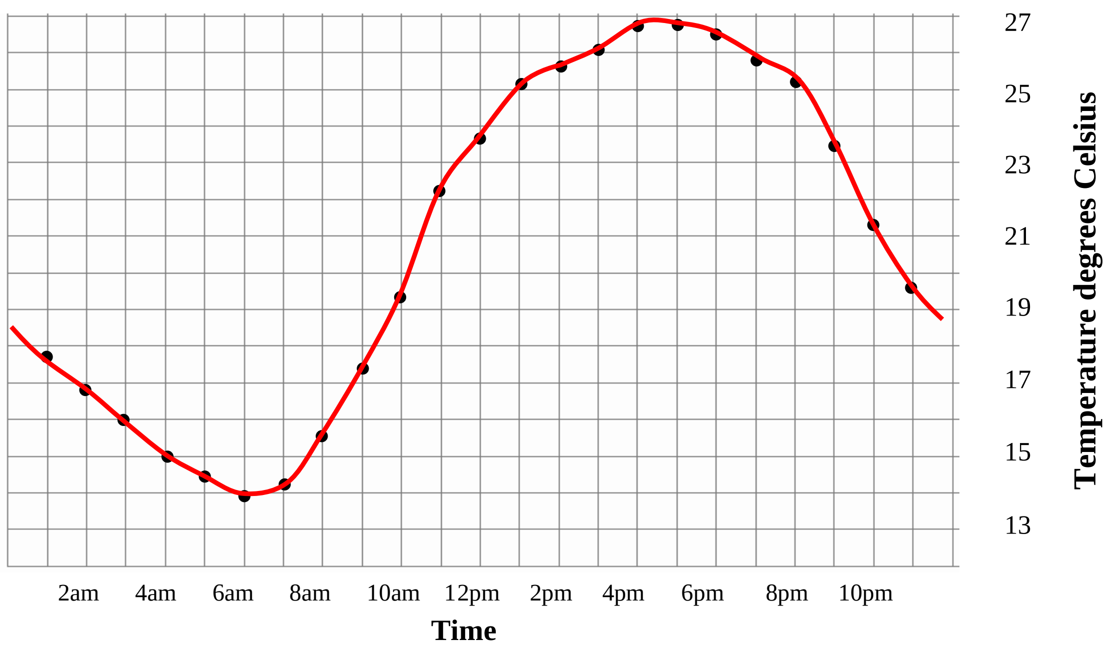
c. Calculate the angle θ, which measures the deviation of the cue ball trajectory from the direct line between the ball centers. Give your answer to the nearest degree.

1 mark

End of Module 1 – SECTION B – continued
TURN OVER

Module 4 – Graphs and relations

Question 1. (2 marks)



The graph above shows the variation of temperature with time over one day.
Use the graph to answer the following questions.

a. What was the minimum temperature and at what time was it recorded?

1 mark

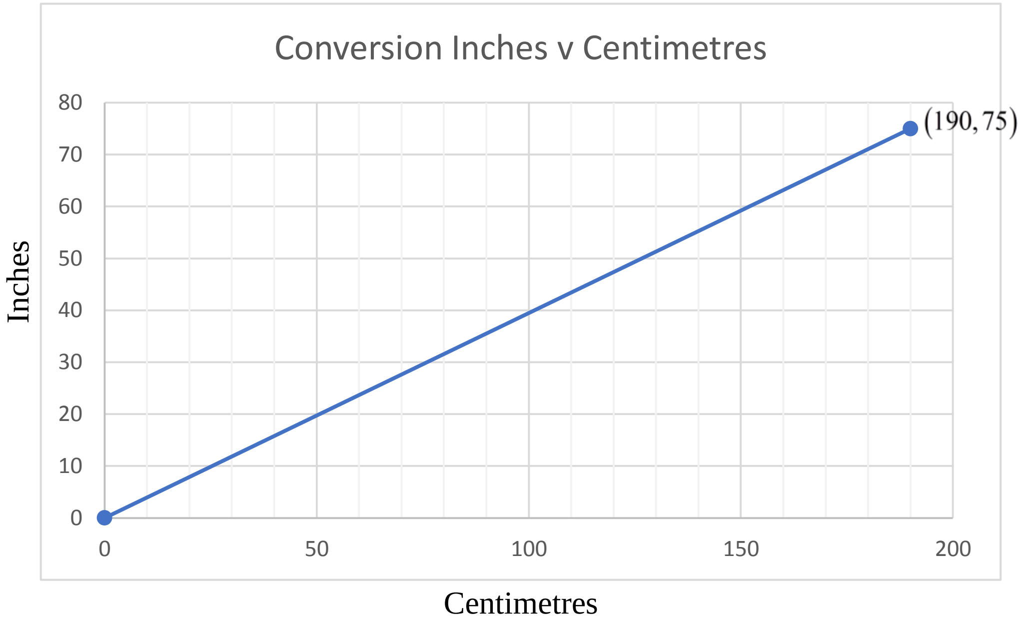
b. Laurence is able to open his bee hives up once the temperature reaches 25°C. Between which hours is he able to work on his hives on the day recorded on the graph above?

1 mark

SECTION B – Module 3 – continued

Question 2. **(3 marks)**

Many online stores give clothing measurements in inches, which aren't very helpful if you know your measurements in centimetres! The graph below shows the conversions of inches and centimetres.



The relationship between inches and centimetres can be represented in the equation

$$inches = m \times centimetres$$

a. Calculate the value of m giving your answer correct to 3 decimal places.

1 mark

b. A website gives measurements in inches. If your waist measurement was 56 cm, what is this to the nearest inch?

1 mark

c. Using the formula, convert a length of 52 inches into centimetres giving your answer correct to the nearest whole number.

1 mark

SECTION B – Module 4 – continued
TURN OVER

Question 3. (7 marks)

The school IT department is constantly checking student devices and updating teacher devices.

Let x represent the number of student devices checked each week and y be the number of teacher devices updated each week.

Each teacher and each student has one device.

It takes 15 minutes of technician time to check a student device and 25 minutes of technician time to update a teacher device.

The constraints on the IT department are as follows:

Constraint 1 $x \geq 20$

Constraint 2 $y \geq 2$

Constraint 3 $15x + 25y \leq 3000$

Constraint 4 $x \leq 14y$

Constraint 5 $x \geq 2y + 20$

Constraint 6 $y \leq 52$

a. Explain the meaning of Constraint 3 in terms of the time available each week to provide the IT maintenance described.

____

____

1 mark

b. Constraint 4 ensures that all staff devices will be updated before all student devices can be checked. If there are 52 teachers, what is the minimum number of students in the school?

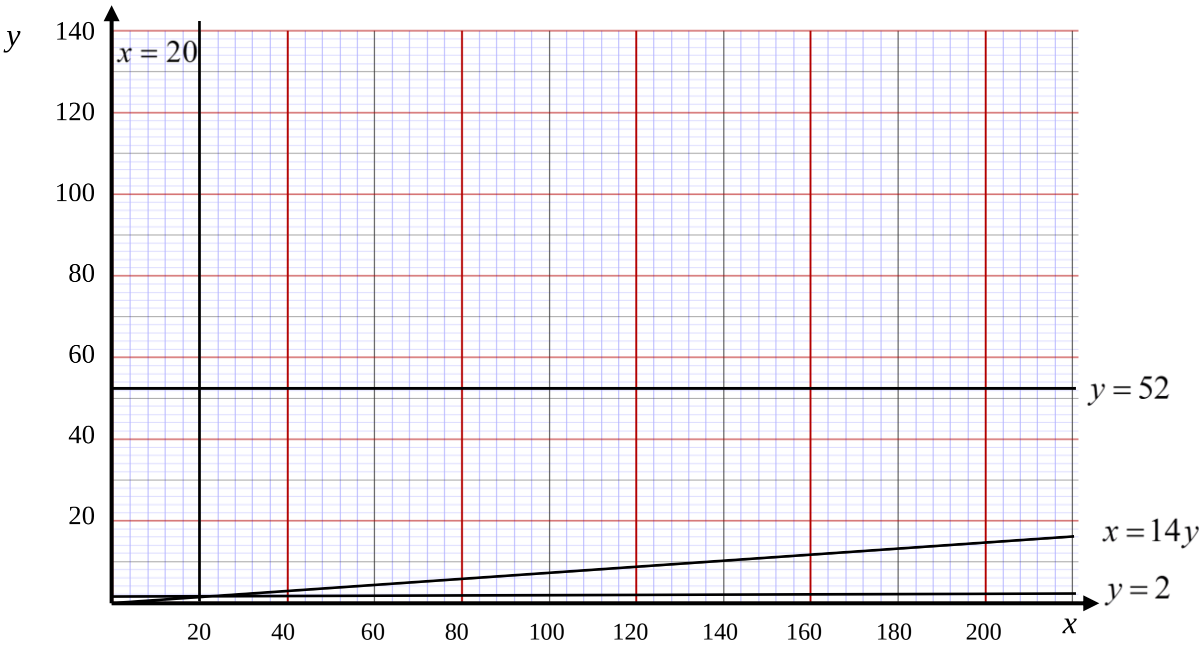
____

____

1 mark

SECTION B – Module 4 – continued

c. Lines representing boundaries of four of the six constraints are shown on the graph below. Add in lines for the other two constraints and shade the feasible region.



2 marks

d. The aim is to complete work on as many devices as possible in the week. Using the objective function $D = x + y$ determine the number of student and teacher devices that should be seen to achieve this.

____

1 mark

e. How many weeks will it take for all teacher devices to be updated?

____

1 mark

The IT department consists of three staff, including two who can spend a total of 3000 hours per week working on devices. If one of these staff goes on leave the amount of time available to service devices will be reduced to 1500.

f. Determine the maximum number of student and teacher devices that could now be serviced in one week with a member of the IT staff on leave.

____

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1 mark

END OF QUESTION BOOKLET