VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2019

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

FURTHER MATHEMATICS

Written examination 2

Thursday 30 May 2019

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	8	8	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 34 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (4 marks)

The table below displays the *average sleep time*, in hours, for a sample of 19 types of mammals.

Type of mammal	***Average sleep time* (hours)**
cat	14.5
squirrel	13.8
mouse	13.2
rat	13.2
grey wolf	13.0
arctic fox	12.5
raccoon	12.5
gorilla	12.0
jaguar	10.8
baboon	9.8
red fox	9.8
rabbit	8.4
guinea pig	8.2
grey seal	6.2
cow	3.9
sheep	3.8
donkey	3.1
horse	2.9
roedeer	2.6

Data: T Allison and DV Cicchetti, 'Sleep in Mammals: Ecological and Constitutional Correlates', in *Science*, American Association for the Advancement of Science, vol. 194, no. 4266, pp. 732–734, 12 November 1976; accessed from OzDASL, StatSci.org, <www.statsci.org/data/general/sleep.html>

SECTION A – Question 1 – continued

a. Which of the two variables, *type of mammal* or *average sleep time*, is a nominal variable? 1 mark

b. Determine the mean and standard deviation of the variable *average sleep time* for this sample of mammals.

Write your answers in the boxes provided below.

Round your answers to one decimal place. 1 mark

mean = $\boxed{}$ hours standard deviation = $\boxed{}$ hours

c. The average sleep time for a human is eight hours.

What percentage of this sample of mammals has an *average sleep time* that is less than the average sleep time for a human?

Round your answer to one decimal place. 1 mark

d. The sample is increased in size by adding in the average sleep time of the little brown bat. Its average sleep time is 19.9 hours.

By how many hours will the range for *average sleep time* increase when the average sleep time for the little brown bat is added to the sample? 1 mark

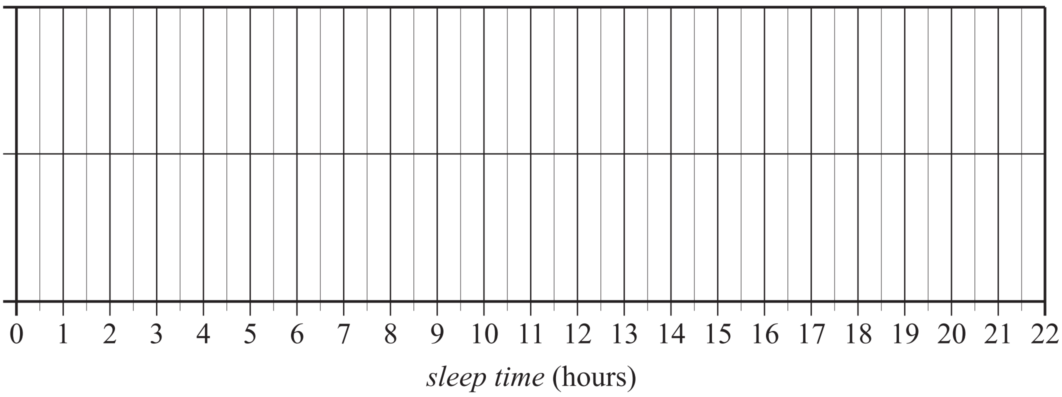
Question 2 (3 marks)

The five-number summary below was determined from the *sleep time*, in hours, of a sample of 59 types of mammals.

Statistic	***Sleep time* (hours)**
minimum	2.5
first quartile	8.0
median	10.5
third quartile	13.5
maximum	20.0

a. Show, with calculations, that a boxplot constructed from this five-number summary will not include outliers. 2 marks

b. Construct the boxplot below. 1 mark



Question 3 (4 marks)

The *life span*, in years, and *gestation period*, in days, for 19 types of mammals are displayed in the table below.

Life span **(years)**	***Gestation period*** **(days)**
3.20	19
4.70	21
7.60	68
9.00	28
9.80	52
13.7	63
14.0	60
16.2	63
17.0	150
18.0	31
20.0	151
22.4	100
27.0	180
28.0	63
30.0	281
39.3	252
40.0	365
41.0	310
46.0	336

a. A least squares line that enables *life span* to be predicted from *gestation period* is fitted to this data.

Name the explanatory variable in the equation of this least squares line. 1 mark

b. Determine the equation of the least squares line in terms of the variables *life span* and *gestation period*.

Write your answers in the appropriate boxes provided below.

Round the numbers representing the intercept and slope to three significant figures. 2 marks

$\boxed{} = \boxed{} + \boxed{} \times \boxed{}$

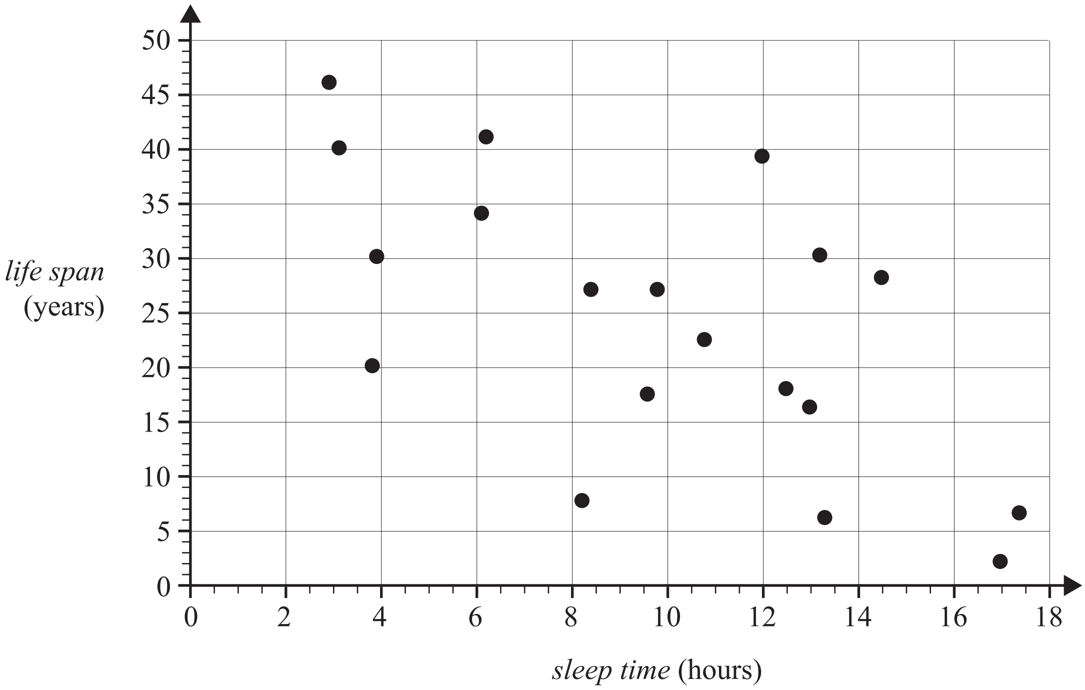
c. Write the value of the correlation coefficient rounded to three decimal places. 1 mark

$r = \boxed{}$

DO NOT WRITE IN THIS AREA

Question 4 (8 marks)

The scatterplot below plots the variable *life span*, in years, against the variable *sleep time*, in hours, for a sample of 19 types of mammals.



On the assumption that the association between *sleep time* and *life span* is linear, a least squares line is fitted to this data with *sleep time* as the explanatory variable.

The equation of this least squares line is

$$\textit{life span} = 42.1 - 1.90 \times \textit{sleep time}$$

The coefficient of determination is 0.416

a. Draw the graph of the least squares line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

b. Describe the linear association between *life span* and *sleep time* in terms of strength and direction. 2 marks

c. Interpret the slope of the least squares line in terms of *life span* and *sleep time*. 2 marks

d. Interpret the coefficient of determination in terms of *life span* and *sleep time*. 1 mark

e. The life span of the mammal with a sleep time of 12 hours is 39.2 years.

Show that, when the least squares line is used to predict the life span of this mammal, the residual is 19.9 years. 2 marks

DO NOT WRITE IN THIS AREA

Question 5 (5 marks)

A random sample of 12 mammals drawn from a population of 62 types of mammals was categorised according to two variables.

likelihood of attack (1 = low, 2 = medium, 3 = high)

exposure to attack during sleep (1 = low, 2 = medium, 3 = high)

The data is shown in the following table.

Likelihood of attack	2	2	1	3	2	3	1	3	1	1	3	3
Exposure to attack	3	1	1	1	3	3	1	3	1	1	3	3

a. Use this data to complete the two-way frequency table below. 1 mark

Likelihood of attack	***Exposure to attack* during sleep**		
	low (=1)	medium (=2)	high (=3)
low (=1)		0	0
medium (=2)		0	
high (=3)		0	

SECTION A – Question 5 – continued

The following two-way frequency table was formed from the data generated when the entire population of 62 types of mammals was similarly categorised.

Likelihood of attack	***Exposure to attack* during sleep**		
	low	medium	high
low	31	8	2
medium	2	0	2
high	1	1	15

b. **i.** How many of these 62 mammals had both a high *likelihood of attack* and a high *exposure to attack* during sleep? 1 mark

ii. Of those mammals that had a medium *likelihood of attack*, what percentage also had a low *exposure to attack* during sleep? 1 mark

iii. Does the information in the table above support the contention that *likelihood of attack* is associated with *exposure to attack* during sleep? Justify your answer by quoting appropriate percentages. It is sufficient to consider only one category of *likelihood of attack* when justifying your answer. 2 marks

DO NOT WRITE IN THIS AREA

Recursion and financial modelling

Question 6 (5 marks)

Marlon plays guitar in a band.

He paid $3264 for a new guitar.

The value of Marlon's guitar will be depreciated by a fixed amount for each concert that he plays.

After 25 concerts, the value of the guitar will have decreased by $200.

a. What will be the value of Marlon's guitar after 25 concerts? 1 mark

b. Write a calculation that shows that the value of Marlon's guitar will depreciate by $8 per concert. 1 mark

c. The value of Marlon's guitar after n concerts, G_n, can be determined using a rule.

Complete the rule below by writing the appropriate numbers in the boxes provided. 1 mark

$$G_n = \boxed{} - \boxed{} \times n$$

d. The value of the guitar continues to be depreciated by $8 per concert.

After how many concerts will the value of Marlon's guitar first fall below $2500? 2 marks

DO NOT WRITE IN THIS AREA

Question 7 (4 marks)

Tisha plays drums in the same band as Marlon.

She would like to buy a new drum kit and has saved \$2500.

a. Tisha could invest this money in an account that pays interest compounding monthly.

The balance of this investment after n months, T_n, could be determined using the recurrence relation below.

$$T_0 = 2500, \qquad T_{n+1} = 1.0036 \times T_n$$

Calculate the total interest that would be earned by Tisha's investment in the first five months.

Round your answer to the nearest cent. 2 marks

Tisha could invest the \$2500 in a different account that pays interest at the rate of 4.08% per annum, compounding monthly. She would make a payment of \$150 into this account every month.

b. Let V_n be the value of Tisha's investment after n months.

Write down a recurrence relation, in terms of V_0, V_n and V_{n+1}, that would model the change in the value of this investment. 1 mark

c. Tisha would like to have a balance of \$4500, to the nearest dollar, after 12 months.

What annual interest rate would Tisha require?

Round your answer to two decimal places. 1 mark

Question 8 (3 marks)

A record producer gave the band $50 000 to write and record an album of songs.

This $50 000 was invested in an annuity that provides a monthly payment to the band.

The annuity pays interest at the rate of 3.12% per annum, compounding monthly.

After six months of writing and recording, the band has $32 667.68 remaining in the annuity.

a. What is the value, in dollars, of the monthly payment to the band? 1 mark

b. After six months of writing and recording, the band decided that it needs more time to finish the album.

To extend the time that the annuity will last, the band will work for three more months without withdrawing a payment.

After this, the band will receive monthly payments of $3800 for as long as possible.

The annuity will end with one final monthly payment that will be smaller than all of the others.

Calculate the **total** number of months that this annuity will last. 2 marks

END OF SECTION A

DO NOT WRITE IN THIS AREA

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1 – Matrices

Question 1 (5 marks)

A total of six residents from two towns will be competing at the International Games.

Matrix A, shown below, contains the number of male (M) and the number of female (F) athletes competing from the towns of Gillen (G) and Haldaw (H).

$$A = \begin{array}{c c} \begin{array}{cc} M & F \end{array} & \\ \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} & \begin{array}{c} G \\ H \end{array} \end{array}$$

a. How many of these athletes are residents of Haldaw? 1 mark

Each of the six athletes will compete in one event: table tennis, running or basketball.

Matrices T and R, shown below, contain the number of male and female athletes from each town who will compete in table tennis and running respectively.

Table tennis	Running
$T = \begin{array}{c c} \begin{array}{cc} M & F \end{array} & \\ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & \begin{array}{c} G \\ H \end{array} \end{array}$	$R = \begin{array}{c c} \begin{array}{cc} M & F \end{array} & \\ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} & \begin{array}{c} G \\ H \end{array} \end{array}$

b. Matrix B contains the number of male and female athletes from each town who will compete in basketball.

Complete matrix B below. 1 mark

$$B = \begin{array}{c c} \begin{array}{cc} M & F \end{array} & \\ \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} & \begin{array}{c} G \\ H \end{array} \end{array}$$

SECTION B – Module 1 – Question 1 – continued

DO NOT WRITE IN THIS AREA

Matrix C contains the cost of one uniform, in dollars, for each of the three events: table tennis (T), running (R) and basketball (B).

$$C = \begin{array}{c} \begin{bmatrix} 515 \\ 550 \\ 580 \end{bmatrix} \begin{array}{l} T \\ R \\ B \end{array} \end{array}$$

c. **i.** For which event will the total cost of uniforms for the athletes be \$1030? 1 mark

ii. Write a matrix calculation, that includes matrix C, to show that the total cost of uniforms for the event named in **part c.i.** is contained in the matrix answer of [1030]. 1 mark

d. Matrix V and matrix Q are two new matrices where $V = Q \times C$ and:

- matrix Q is a 4×3 matrix
- element v_{11} = total cost of uniforms for all female athletes from Gillen
- element v_{21} = total cost of uniforms for all female athletes from Haldaw
- element v_{31} = total cost of uniforms for all male athletes from Gillen
- element v_{41} = total cost of uniforms for all male athletes from Haldaw
- $C = \begin{array}{c} \begin{bmatrix} 515 \\ 550 \\ 580 \end{bmatrix} \begin{array}{l} T \\ R \\ B \end{array} \end{array}$

Complete matrix Q with the missing values. 1 mark

$$Q = \begin{bmatrix} 1 & \_\_ & \_\_ \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ \_\_ & \_\_ & 0 \end{bmatrix}$$

Question 2 (2 marks)

Three television channels, C_1, C_2 and C_3, will broadcast the International Games in Gillen.

Gillen's 2000 residents are expected to change television channels from hour to hour as shown in the transition matrix T below.

The option for residents not to watch television (*NoTV*) at that time is also indicated in the transition matrix.

$$\begin{array}{c} \text{this hour} \\ \begin{array}{cccc} C_1 & C_2 & C_3 & NoTV \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.50 & 0.05 & 0.10 & 0.20 \\ 0.10 & 0.60 & 0.20 & 0.20 \\ 0.25 & 0.10 & 0.50 & 0.10 \\ 0.15 & 0.25 & 0.20 & 0.50 \end{bmatrix} \begin{array}{l} C_1 \\ C_2 \\ C_3 \\ NoTV \end{array} \quad \textit{next hour}$$

The state matrix G_0 below lists the number of Gillen residents who are expected to watch the games on each of the channels at the start of a particular day (9.00 am).

Also shown is the number of Gillen residents who are not expected to watch television at that time.

$$G_0 = \begin{bmatrix} 100 \\ 400 \\ 100 \\ 1400 \end{bmatrix} \begin{array}{l} C_1 \\ C_2 \\ C_3 \\ NoTV \end{array}$$

a. Complete the calculation below to show that 835 Gillen residents are not expected to watch television (*NoTV*) at 10.00 am that day. 1 mark

$$\boxed{} \times 100 + \boxed{} \times 400 + \boxed{} \times 100 + \boxed{} \times 1400 = 835$$

b. Determine the number of residents expected to watch the games on C_3 at 11.00 am that day. 1 mark

DO NOT WRITE IN THIS AREA

Question 3 (2 marks)

The basketball finals will be televised on C_3 from 12.00 noon until 4.00 pm.

It is expected that 600 Gillen residents will be watching C_3 at any time from 12.00 noon until 4.00 pm.

The remaining 1400 Gillen residents will not be watching C_3 from 12.00 noon until 4.00 pm (represented by *Not*C_3).

The transition matrix P below shows how the 2000 Gillen residents are expected to change their viewing habits each hour between watching C_3 and not watching C_3 from 12.00 noon until 4.00 pm.

$$\begin{array}{cccc} & \multicolumn{2}{c}{\textit{this hour}} & \\ & C_3 & NotC_3 & \\ P = & \left[\begin{array}{cc} v & w \\ 0.35 & x \end{array}\right] & & \begin{array}{l} C_3 \\ NotC_3 \end{array} \quad \textit{next hour} \end{array}$$

Write down the values of v, w and x in the boxes provided below.

$v =$ ☐ $w =$ ☐ $x =$ ☐

Question 4 (3 marks)

After 5.00 pm, tourists will start to arrive in Gillen and they will stay overnight.

As a result, the number of people in Gillen will increase and the television viewing habits of the tourists will also be monitored.

Assume that 50 tourists arrive every hour.

It is expected that 80% of arriving tourists will watch only C_2 during the hour that they arrive.

The remaining 20% of arriving tourists will not watch television during the hour that they arrive.

Let W_m be the state matrix that shows the number of people in each category m hours after 5.00 pm on this day.

The recurrence relation that models the change in the television viewing habits of this increasing number of people in Gillen m hours after 5.00 pm on this day is shown below.

$$W_{m+1} = TW_m + V$$

where

$$T = \begin{array}{c} \quad\quad\textit{this hour} \\ \begin{array}{cccc} C_1 & C_2 & C_3 & NoTV \end{array} \\ \begin{bmatrix} 0.50 & 0.05 & 0.10 & 0.20 \\ 0.10 & 0.60 & 0.20 & 0.20 \\ 0.25 & 0.10 & 0.50 & 0.10 \\ 0.15 & 0.25 & 0.20 & 0.50 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ C_1 \\ C_2 \\ C_3 \\ NoTV \end{array} \quad \textit{next hour}, \qquad \text{and} \qquad W_0 = \begin{bmatrix} 400 \\ 600 \\ 300 \\ 700 \end{bmatrix}$$

a. Write down matrix V. 1 mark

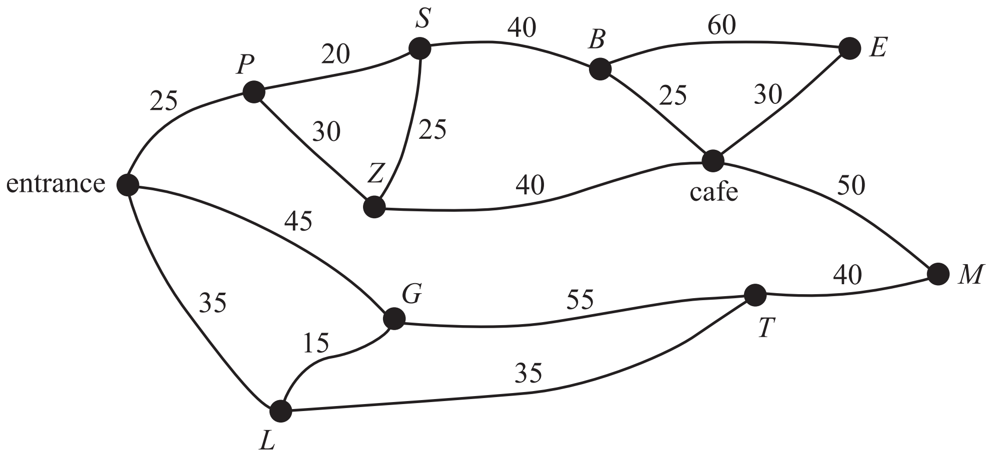
b. How many people in Gillen are expected to watch C_2 at 7.00 pm on this day? 2 marks

Module 2 – Networks and decision mathematics

Question 1 (5 marks)

A zoo has an entrance, a cafe and nine animal exhibits: bears (*B*), elephants (*E*), giraffes (*G*), lions (*L*), monkeys (*M*), penguins (*P*), seals (*S*), tigers (*T*) and zebras (*Z*).

The edges on the graph below represent the paths between the entrance, the cafe and the animal exhibits. The numbers on each edge represent the length, in metres, along that path. Visitors to the zoo can use only these paths to travel around the zoo.

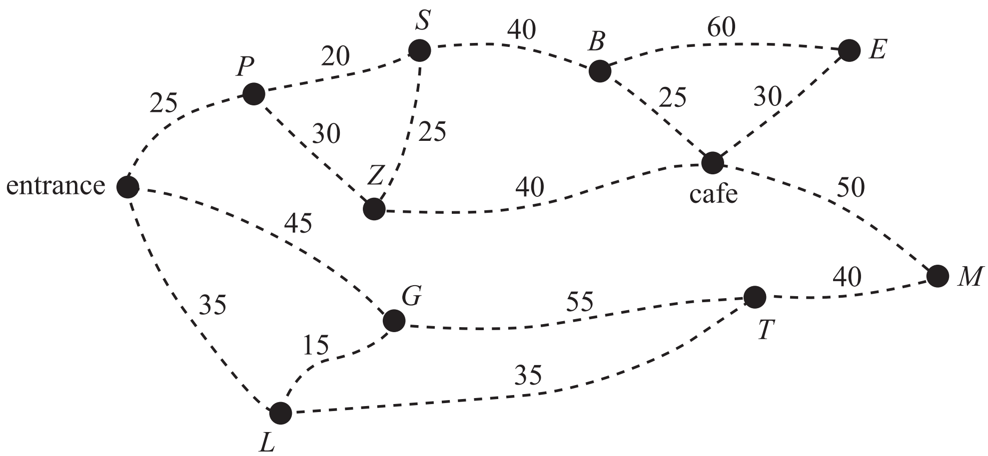


a. What is the shortest distance, in metres, between the entrance and the seal exhibit (*S*)? 1 mark

b. Freddy is a visitor to the zoo. He wishes to visit the cafe and each animal exhibit just once, starting and ending at the entrance.

i. What is the mathematical term used to describe this route? 1 mark

ii. Draw **one** possible route that Freddy may take on the graph below. 1 mark

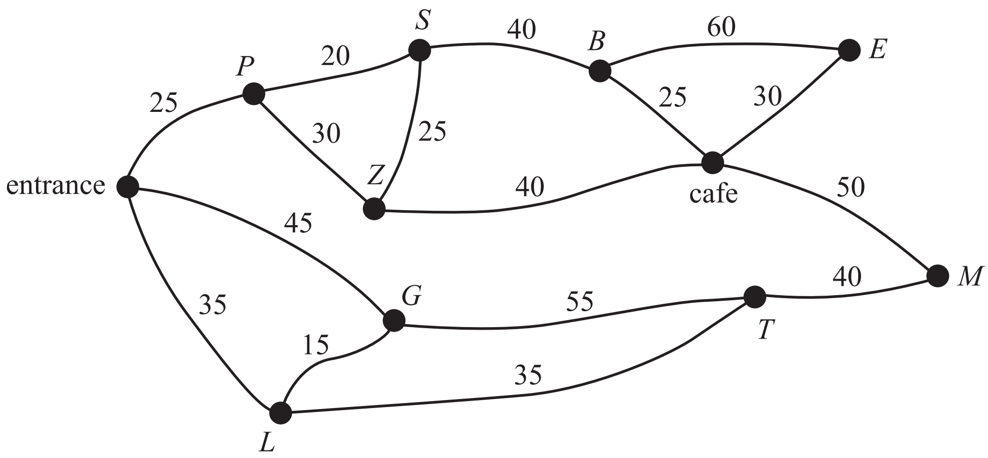


A reptile exhibit (*R*) will be added to the zoo.

A new path of length 20 m will be built between the reptile exhibit (*R*) and the giraffe exhibit (*G*).

A second new path, of length 35 m, will be built between the reptile exhibit (*R*) and the cafe.

c. Complete the graph below with the new reptile exhibit and the two new paths added. Label the new vertex *R* and write the distances on the new edges. 1 mark



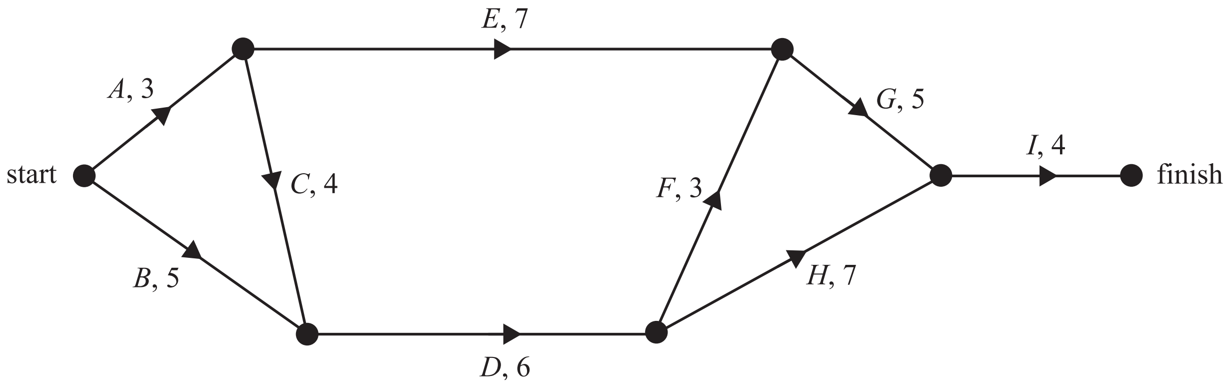
d. The new paths reduce the minimum distance that visitors have to walk between the giraffe exhibit (*G*) and the cafe.

By how many metres will these new paths reduce the minimum distance between the giraffe exhibit (*G*) and the cafe? 1 mark

Question 2 (3 marks)

The construction of the new reptile exhibit is a project involving nine activities, *A* to *I*.

The directed network below shows these activities and their completion times in weeks.



a. Which activities have more than one immediate predecessor? 1 mark

b. Write down the critical path for this project. 1 mark

c. What is the latest start time, in weeks, for activity *B*? 1 mark

Question 3 (4 marks)

The zoo's management requests quotes for parts of the new building works.

Four businesses each submit quotes for four different tasks.

Each business will be offered only one task.

The quoted cost, in $100 000, of providing the work is shown in Table 1 below.

Table 1

	Task 1 Constructing the pathways	**Task 2** Constructing the new reptile exhibit	**Task 3** Heating and lighting the new exhibit	**Task 4** Landscaping the surrounding grounds
Business 1	12	12	5	7
Business 2	10	11	4	7
Business 3	12	8	8	6
Business 4	13	11	9	8

The zoo's management wants to complete the new building works at minimum cost.

The Hungarian algorithm is used to determine the allocation of tasks to businesses.

The first step of the Hungarian algorithm involves **row** reduction; that is, subtracting the smallest element in each row of Table 1 from each of the elements in that row.

The result of the first step is shown in Table 2 below.

Table 2

	Task 1 Constructing the pathways	**Task 2** Constructing the new reptile exhibit	**Task 3** Heating and lighting the new exhibit	**Task 4** Landscaping the surrounding grounds
Business 1	7	7	0	2
Business 2	6	7	0	3
Business 3	6	2	2	0
Business 4	5	3	1	0

The second step of the Hungarian algorithm involves **column** reduction; that is, subtracting the smallest element in each column of Table 2 from each of the elements in that column.

The results of the second step of the Hungarian algorithm are shown in Table 3 below. The values of Task 1 are given as A, B, C and D.

Table 3

	Task 1 **Constructing the pathways**	**Task 2** **Constructing the new reptile exhibit**	**Task 3** **Heating and lighting the new exhibit**	**Task 4** **Landscaping the surrounding grounds**
Business 1	A	5	0	2
Business 2	B	5	0	3
Business 3	C	0	2	0
Business 4	D	1	1	0

a. Write down the values of A, B, C and D. 1 mark

A = ____________ B = ____________ C = ____________ D = ____________

b. The next step of the Hungarian algorithm involves covering all the zero elements with horizontal or vertical lines. The minimum number of lines required to cover the zeros is three.

Draw these three lines on **Table 3 above**. 1 mark

(*Answer on Table 3 above.*)

c. An allocation for minimum cost is not yet possible.

When all steps of the Hungarian algorithm are complete, a bipartite graph can show the allocation for minimum cost.

Complete the bipartite graph below to show this allocation for minimum cost. 1 mark

Business 1 • • **Task 1**

Business 2 • • **Task 2**

Business 3 • • **Task 3**

Business 4 • • **Task 4**

d. Business 4 has changed its quote for the construction of the pathways. The new cost is \$1 000 000. The overall minimum cost of the building works is now reduced by reallocating the tasks.

How much is this reduction? 1 mark

____

____

Module 3 – Geometry and measurement

Question 1 (3 marks)

An engineering conference is being held in Rome. Rome is located at latitude 42° N and longitude 12° E.

Four presenters from different cities will travel to Rome to attend the conference.

The presenters and the coordinates (latitude and longitude) of the city each presenter lives in are shown below.

Presenter	Coordinates
Yvette	(3° S, 12° E)
Anthony	(17° S, 12° E)
Rany	(54° N, 12° E)
Shana	(65° N, 12° E)

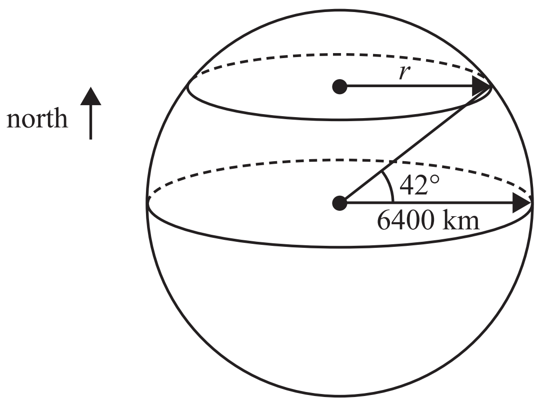
a. Which presenter lives closest to Rome? 1 mark

Rome has a latitude of 42° N.

The diagram below shows the small circle of Earth with latitude 42° N.

The radius of this circle is labelled r.

Assume that the radius of Earth is 6400 km.



b. Calculate the radius of the small circle of Earth with latitude 42° N.

Round your answer to the nearest kilometre. 1 mark

c. The key speaker, Michael, will travel from Auckland to Rome to attend the conference.
Michael left Auckland at 9.40 am on a Monday.
The journey to Rome took 30 hours.
The time difference between Auckland (37° S, 175° E) and Rome (42° N, 12° E) is 10 hours.

On what day and at what time did Michael arrive in Rome? 1 mark

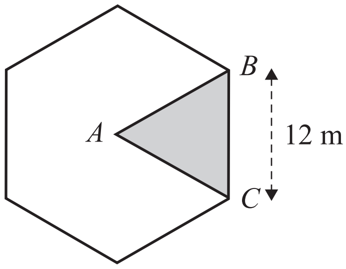
Question 2 (5 marks)

Michael will present a building design at the conference.

The horizontal cross-section of the building is in the shape of a regular hexagon.

Each side of the hexagon is 12 m long.

Triangle *ABC* is shown shaded below.



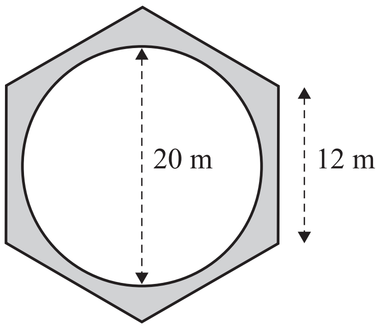
a. **i.** Show, with calculations, that the area of triangle *ABC*, rounded to one decimal place, is 62.4 m^2. 1 mark

ii. Calculate the area of the hexagon.

Round your answer to the nearest square metre. 1 mark

A hemispherical dome will sit on the roof of the building. The dome has a diameter of 20 m.

The view of the building from above is shown in the diagram below.

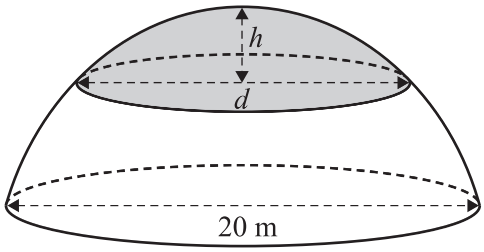


b. The area of the roof that surrounds the dome is shaded in the diagram above.

Calculate this shaded area.

Round your answer to the nearest square metre. 1 mark

The top of the hemispherical dome will be made of glass, as shown shaded in the diagram below.



The glass section has a base diameter of d metres and a height of h metres.
The area of the base of the glass section is one quarter of the area of the base of the dome.

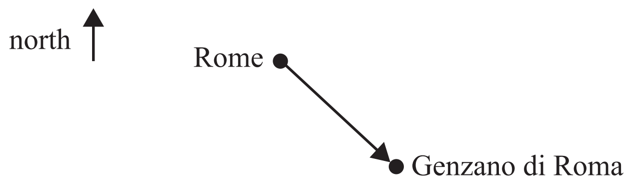
c. **i.** Show, with calculations, that the base diameter of the glass section, d, is 10 m. 1 mark

ii. Calculate the height, h, in metres, of the glass section of the dome.
Round your answer to two decimal places. 1 mark

Question 3 (4 marks)

Some of the presenters organised a helicopter tour of nearby landmarks.

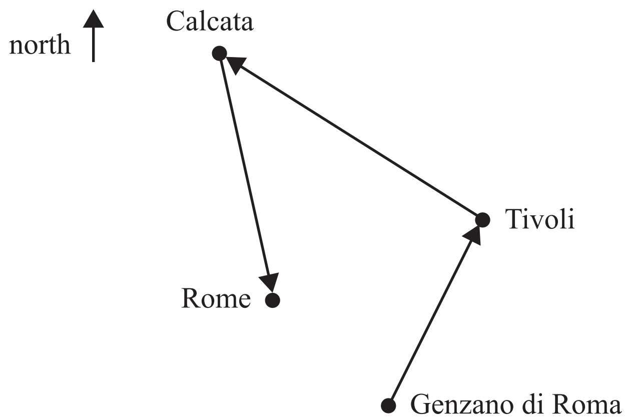
The tour started in Rome and travelled directly to Genzano di Roma, as shown in the diagram below.



Genzano di Roma is 37 km from Rome on a bearing of 153°.

a. How far south of Rome is Genzano di Roma?
Round your answer to the nearest kilometre. 1 mark

From Genzano di Roma, the helicopter then followed a route to Tivoli, Calcata and then back to Rome, as shown in the diagram below.



Tivoli is 25 km from Genzano di Roma on a bearing of 016°.

Calcata is 42 km from Tivoli on a bearing of 309°.

b. Calculate the distance between Genzano di Roma and Calcata.
Round your answer to the nearest kilometre. 2 marks

c. Calcata is 40.3 km north and 11.8 km west of Rome.

Calculate the bearing of Rome from Calcata.
Round your answer to the nearest degree. 1 mark

Module 4 – Graphs and relations

Question 1 (2 marks)

Rumi has a small clothes shop.

He sells men's ties and cravats.

All ties are the same price and all cravats are the same price.

Let t represent the selling price of each tie and let c represent the selling price of each cravat.

One customer, Andy, purchased six ties and four cravats and paid \$230.

A linear relation representing Andy's purchase is $6t + 4c = 230$.

a. A second customer, Barry, purchased four ties and eight cravats and paid \$260.

Write down a linear relation representing Barry's purchase. 1 mark

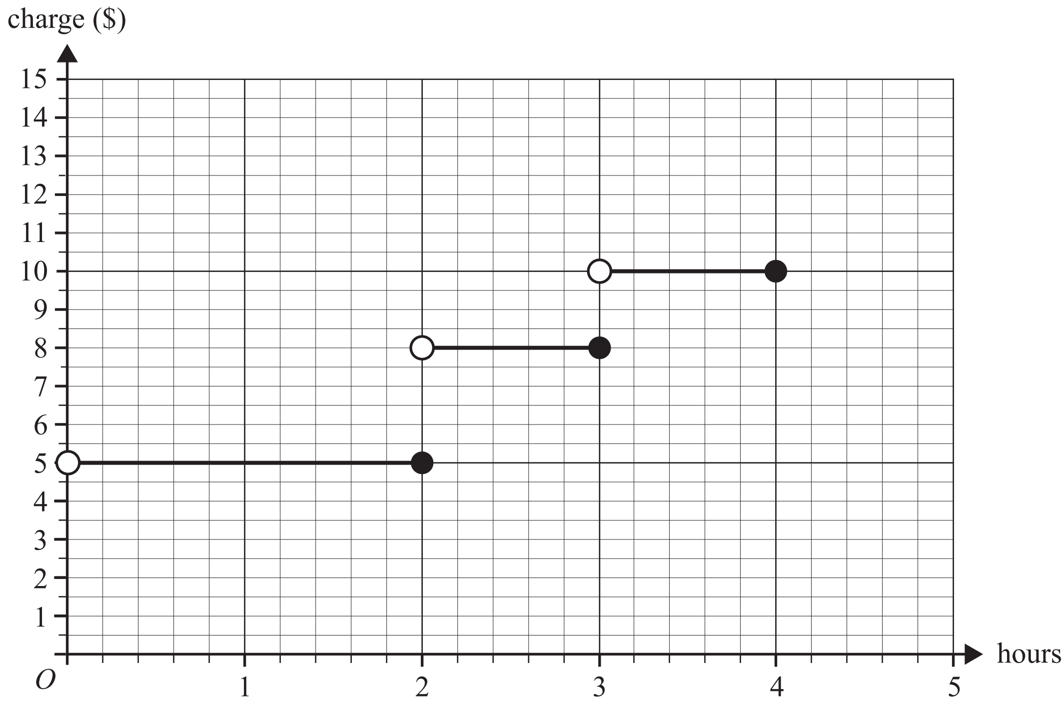
b. The selling price of each cravat in Rumi's shop is \$20.

What is the selling price of each tie? 1 mark

Question 2 (3 marks)

Customers who visit Rumi's shop park their cars in an underground car park.

The graph below shows the total charge for parking, in dollars, according to the number of hours parked in one visit.



a. On Monday, Leon parked in the underground car park for two hours and Lucy parked for one-and-a-half hours.

What was the total combined charge for Leon and Lucy? 1 mark

b. Customers who park for more than four hours are charged $11.50 if they do not stay longer than five hours.

Add this information to the **graph above**. 1 mark

(*Answer on the graph above*.)

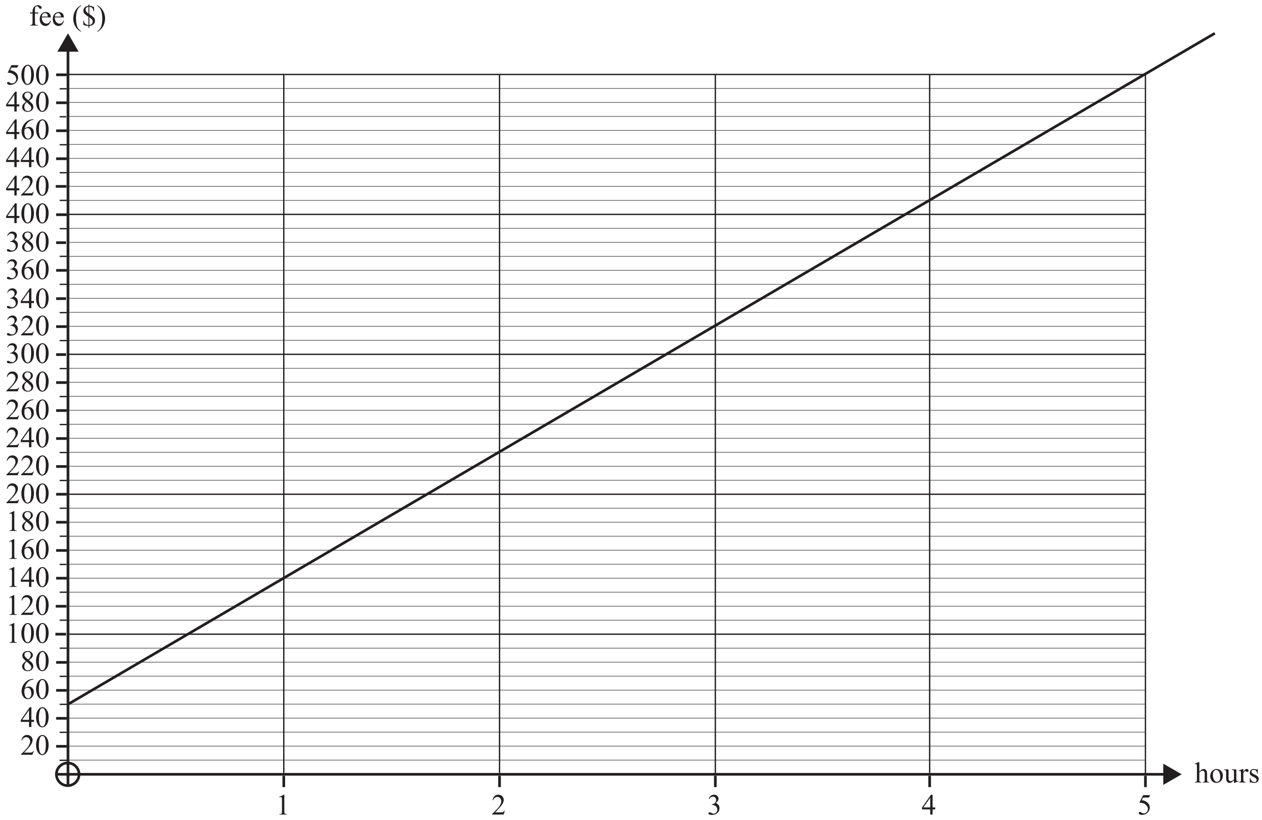
c. Another customer, Pam, parked in the underground car park on both Tuesday and Wednesday. Each day she parked for more than two hours but less than four hours.

Pam was charged less than $20 in total for these two days.

Write down the two possible amounts that Pam could have been charged. 1 mark

Question 3 (2 marks)

Rumi's web designers, Judy and Geoff, charge fees that consist of a fixed amount and a charge per hour. The graph of the fee charged for the number of hours worked by Judy is shown below.



a. Complete the following sentence. 1 mark

Judy charges a fixed amount of $ ☐ and a charge per hour of $ ☐ .

b. Judy and Geoff both charge a fee of $320 for three hours worked.
Geoff's charge per hour is $100.

Draw the linear relation representing the fee charged by Geoff for the number of hours worked **on the graph above**. 1 mark

(*Answer on the graph above*.)

Question 4 (5 marks)

Rumi also sells jackets for men and women.

Let x be the number of jackets for men that Rumi sells in one week.

Let y be the number of jackets for women that Rumi sells in one week.

Sales records suggest constraints on the sale of jackets each week.

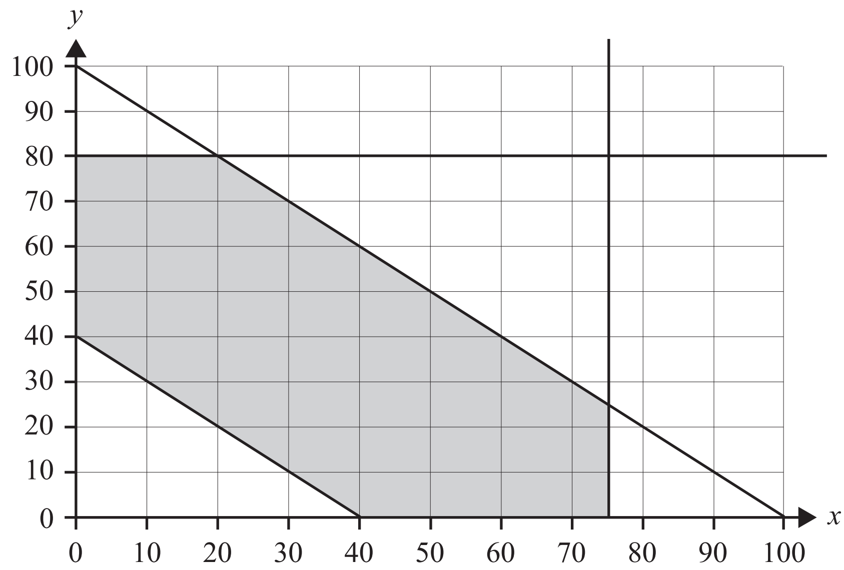
These constraints are represented by Inequalities 1 to 4.

Inequality 1	$0 \le x \le 75$
Inequality 2	$0 \le y \le 80$
Inequality 3	$x + y \ge 40$
Inequality 4	$x + y \le 100$

a. Explain what Inequality 3 and Inequality 4 tell us about the number of jackets that Rumi sells each week. 1 mark

The graph below shows the lines that represent the boundaries of Inequalities 1 to 4.

The feasible region has been shaded.



Rumi makes a profit of $30 on the sale of each jacket for men.

Rumi makes a profit of $50 on the sale of each jacket for women.

b. Determine the maximum profit that Rumi can make on all sales of jackets in one week. 1 mark

c. In a particular week Rumi sold exactly 30 jackets for men.

Determine the maximum profit that Rumi could make on all sales of jackets in that week. 1 mark

d. Later, Rumi finds that the profit he makes on the sale of each jacket for women has changed.

The profit on the sale of each jacket for men remains as $30.

The maximum profit he can make in one week will now be a different amount.

Last week Rumi was able to make this maximum profit by selling 65 jackets for men and 35 jackets for women.

What profit did Rumi make last week? 2 marks

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2019

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2} bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3} \pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3} \pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET