

2020 VCE

Further Mathematics Trial Examination 1 Suggested Solutions



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Section A Core – Data Analysis

Question 1 A

Warburn's data has no outliers, so not B or E.

The bulk of the data is towards the higher values, with a tail heading towards the lower values. This describes a negatively skewed distribution.

Question 2 C

For Yarragrove, $Q_1 = 41$ and $Q_3 = 61$

Upper fence = $Q_3 + 1.5 \times \text{IQR}$

$$= 61 + 1.5 \times (61 - 41)$$

$$= 61 + 1.5 \times 20$$

Question 3 B

$$10 \text{ cm} = 100 \text{ mm} = 10^2 \text{ mm}$$

$$\log_{10}(10^2) = 2$$

By adding the heights of the columns, we get the total number of Redwoods to be 18.

Number of trees with $\log_{10}(\text{diameter}) > 2 = 6$

$$\frac{6}{18} \times 100 \approx 33\%$$

Question 4 D

Number of people at Yarragrove who rated the film as poor = $250 - 125 - 65 = 60$

$$\frac{60}{250} \times 100 = 24\%$$

Question 5 B

To compare rankings at the two cinemas we need the appropriate **percentages** as the audience sizes are not the same size.

Response A does not give percentages. Response C does not compare the two cinemas.

Response D by itself doesn't show any difference between the cinemas.

Response E only mentions audience size, not the ranking given.

Response B shows that a higher percentage of Warburn viewers ranked the film as 'good' compared to that of Yarragrove viewers.

Question 6 C

Use the 68 – 95 – 99.7% rule.

A value of 244.4 mL is 2 standard deviations below the mean (i.e. z score of -2)

2.5% of the values are expected to be less than this.

Expected number of bottles to be rejected

$$= \frac{2.5}{100} \times 3600 = 90$$

Question 7 D

$$z = \frac{x - \bar{x}}{s_x}$$

$$\text{so } x = z s_x + \bar{x}$$

$$\begin{aligned} x &= 1.2 \times 2.3 + 249 \\ &= 251.76 \end{aligned}$$

≈ 251.8

Question 8 C

head circumference is the response variable (vertical axis) so not A or D.

Pick two clear points that the line passes through, for example (4,18) and (13,40).

$$\text{gradient} = \frac{40-18}{13-4} \approx 2.444$$

substitute (4,18) into $y = a + bx$

gives

$$18 = a + 2.444 \times 4$$

$$a \approx 8.224$$

Option C is the closest.

Question 9 A

Using technology, enter the time data. This shows a standard deviation of approximately 4.6 and a mean of 12.3

Row	Label	Value
2	\bar{x}	12.3
3	Σx	98.4
4	Σx^2	1357.48
5	$s_x := s_{n-1}$	4.58507
6	$\sigma_x := \sigma_{n-1}$	4.28894

Question 10 B

When 5 books are read, the actual value for time is 11 minutes.

Predicted value of time =

$$18.65 - 1.08 \times 5 = 13.25$$

Residual = actual value – predicted value

$$= 11 - 13.25$$

$$= -2.25$$

Question 11 E

A reciprocal transformation is applied to *number of books*, so not A, B or C.

Using technology, perform a reciprocal ($1/x$) transformation to *number of books* and calculate the least squares line for

time vs 1/number of books.

	B	C	D	E
	minutes	recipn...		
		=1/number		=LinRegB)
1	21.6	1	Title	Linear Re..
2	16.4	1/2	RegEqn	a+b*x
3	13	1/3	a	7.76342
4	11	1/5	b	14.5631
5	9.7	1/7	r ²	0.978687
E2	="a+b*x"			

Question 12 D

$$r = \pm\sqrt{0.5364} = \pm 0.7324$$

Now from the equation given, we see that the gradient is **negative**, so r will be negative.

So, $r = -0.7324$

Question 13 A

Average price for the year = $\frac{240}{12} = 20$

Seasonal Index for September = $\frac{17.5}{20} = 0.875 \approx 0.88$

Question 14 C

Suppose the cheese price for December is \$100.

The deseasonalized figure

= actual figure / seasonal index

$$= \frac{100}{0.53} = 188.6792\dots$$

This represents an increase of about 88.7%

Question 15 E

Four point average of February to May

$$= \frac{28.4+19.8+26.3+19.9}{4} = 23.6$$

Four point average of March to June

$$= \frac{19.8+26.3+19.9+22}{4} = 22$$

Value centred on April = $\frac{23.6+22}{2} = 22.8$

Question 16 E

Dividing each quarter's value by its seasonal index, we get

Quarter number	1	2	3	4
Deseasonalised sales	1758.57...	1772.727...	2100	1927.27...

Fitting a least squares regression line in the form $y = a + bx$ gives

$$b = 83.337\dots = 83 \text{ (correct to 2 significant figures)}$$

$$a = 1681.2987\dots = 1700 \text{ (correct to 2 significant figures)}$$

So, *deseasonalized sales* = $1700 + 83 \times \text{quarter number}$

Section A Core – Recursion and financial modelling

Question 17 B

Each term is 7 less than 3 times the previous term. Begin with the first term of 2 and repeat the calculation on each term.

2	2
2·3-7	-1
-1·3-7	-10
-10·3-7	-37

Question 18 D

The graph shows the sequence
5, 15, 35, 75, 155

Doubling each term then adding 5 to get the next term produces this sequence.

This is described by the recurrence relation

$$T_0 = 5, \quad T_{n+1} = 2T_n + 5$$

Question 19 A

Interest is added at $\frac{1.8}{12} = 0.15\%$ *per month*

This gives a compounding factor of

$$1 + \frac{0.15}{100} = 1.0015$$

So, Laura's investment amount each month will be 1.0015 times last month's amount less the \$400 she withdraws.

This is given by option A.

Question 20 E

Using finance solver on calculator:

$$N = 12$$

$$I = 1.8$$

$$PV = -25000$$

$$PMT = 400$$

$$FV =$$

$$P/Y = 12$$

$$C/Y = 12$$

This gives $FV = 20613.93\dots$

Value after 12 months is closest to \$20614

Question 21 C

$$\begin{aligned} \text{Principal repaid} &= 266758.78 - 265344.52 \\ &= 1414.26 \end{aligned}$$

$$\text{Interest paid} = \frac{3.8}{1200} \times 266758.78 = 844.74$$

$$\text{Repayment} = 1414.26 + 844.74 = \$2259$$

Question 22 A

In a perpetuity, the investment amount does not change. The amount withdrawn must equal the interest earned.

$$\text{Interest rate per month} = 2.8/12 \%$$

$$\text{Interest} = \frac{2.8}{1200} \times \text{investment amount} = 50$$

$$\text{investment amount} = 50 \times \frac{1200}{2.8} \approx \$21429$$

Question 23 C

Using finance solver

$$N = 2.5 \times 12 = 30$$

$$I = 3.2$$

$$PV = -25000$$

$$PMT =$$

$$FV = 16475$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\text{This gives } PMT = 339.997$$

Miriam withdraws \$340 each month

Question 24 E

Firstly, find the amount of Eddie's repayments before the rate change.

$$N = 25 \times 12 = 300$$

$$I = 5.6$$

$$PV = 150000$$

$$PMT =$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\text{This gives } PMT = -930.11$$

Now work out amount still owing after 3 years

$$N = 36$$

$$I = 5.6$$

$$PV = 150000$$

$$PMT = -930.11$$

$$FV =$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\text{This gives } FV = 141002.29$$

Make this the new present value, alter the interest rate and change the time to pay off the loan to 22 years.

$$N = 264$$

$$I = 6.1$$

$$PV = 141002.29$$

$$PMT =$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

This gives $PMT = -971.50$

So Eddie must make repayments of \$971.50

$$971.50 - 930.11 = \$41.39$$

His repayments increase by \$41.39

End of solutions Section A Core

Module 1 – Matrices**Question 1 C**

A matrix is symmetric if it is a square matrix where the elements are symmetric with respect to the **leading** diagonal.

A, D and E are not square. The elements in B do not mirror each other across the leading diagonal.

Option C is a symmetric matrix.

$$[1 \ 2 \ 4 \ 6 \ 2 \ 1 \ 5 \ 8 \ 4 \ 5 \ 3 \ 8 \ 6 \ 8 \ 8 \ 2]$$

Question 2 D

We have a 5×5 permutation matrix by a 5×1 column matrix. The result will be a 5×1 column matrix.

Row 1 x column 1 = A

Row 2 x column 1 = E

Row 3 x column 1 = T

Row 4 x column 1 = R

Row 5 x column 1 = H

Question 3 E

For a dependent system, there is an infinite number of solutions. The lines are the same.

$$[a \ -1 \ 15 \ -5][x \ y] = [b \ 35]$$

For no unique solutions, determinant = 0

$$-5 \times a = 15 \times -1$$

$$a = 3$$

So,

$$3x - y = b$$

$$15x - 5y = 35$$

These equations represent the same line if $b = 7$

Question 4 C

Let the rule be $m_{ij} = ai + bj + c$

Pick 3 elements, not all from the same column, for example:

$$m_{11} = 4 \quad m_{21} = 5 \quad \text{and} \quad m_{32} = 8$$

This gives us 3 simultaneous equations

$$a + b + c = 4$$

$$2a + b + c = 5$$

$$3a + 2b + c = 8$$

Solving this gives $a = 1$, $b = 2$, $c = 1$

Question 5 A

To find 2-step dominance, square the 1-step dominance matrix.

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Adding the elements of the rows of the 1-step and 2-step dominance matrix gives:

$$A = 2, B = 5, C = 3, D = 4$$

In order from highest to lowest: B,D,C,A

Question 6 D

To find the point value of each card, need to solve:

$$3r + 3y + 4b = 35$$

$$r + 5y + 4b = 33$$

$$7r + y + 2b = 43$$

The matrix equation would be

$$\begin{bmatrix} 3 & 3 & 4 \\ 1 & 5 & 4 \\ 7 & 1 & 2 \end{bmatrix} \begin{bmatrix} r \\ y \\ b \end{bmatrix} = \begin{bmatrix} 35 \\ 33 \\ 43 \end{bmatrix}$$

To solve, pre-multiply both sides by the inverse of the 3 x 3 matrix. This is shown in option D.

Question 7 B

To find long term proportions, find the steady state matrix by raising T to a large power e.g. 99.

In the long term:

	0.3	0.1	0.2	0.3
0.3	0.210256	0.210256	0.210256	0.210256
0.5	0.292308	0.292308	0.292308	0.292308
0.1	0.312821	0.312821	0.312821	0.312821
0.1	0.184615	0.184615	0.184615	0.184615

Proportion of birds on B = 0.292308

so, total number of birds = $\frac{1782}{0.292308} \approx 6096$

Proportion of birds on C = 0.312821

Number of birds on C = $0.312821 \times 6096 \approx 1907$

Question 8 A

8732 penguins nest on Duncan in 2019.

Of these, $0.3 \times 8732 \approx 2620$ nest on Albatross in 2020.

A total of **9641** penguins nest on Albatross in 2020.

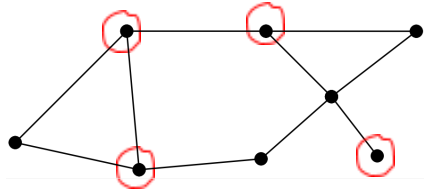
Percentage of these that were nesting on Duncan in 2019

$$= \frac{2620}{9641} \times 100 = 27.17\dots \approx 27\%$$

End of Module 1 Solutions

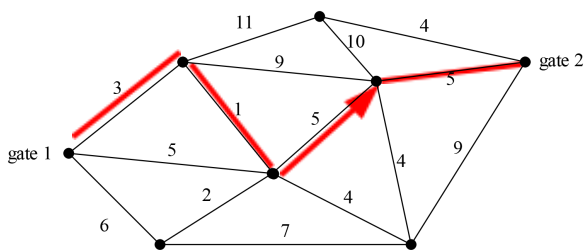
Module 2 – Networks and decision mathematics

Question 1 D



There are 4 vertices of odd degree.

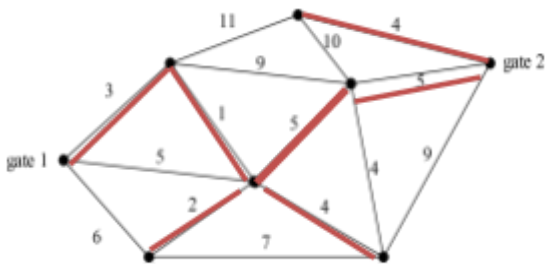
Question 2 C



By inspection, or using Dijkstra’s algorithm the shortest path = $3+1+5+5 = 14$

Question 3 B

We are looking for a minimum spanning tree.



A minimum spanning tree is shown. Its value is:
 $3+2+1+5+4+5+4 = 24$

Question 4 E

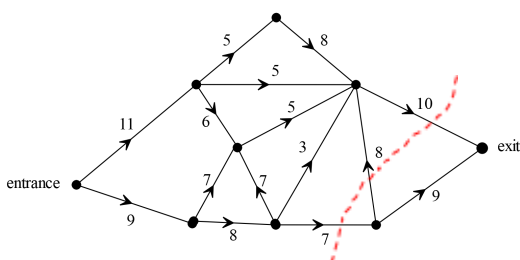
The graph must have 10 vertices, 15 edges and 7 faces.

Only options C and E have 10 vertices.

Option C has 8 faces and 16 edges.

Option E has 7 faces and 15 edges.

Question 5 B



For maximum flow, we need to find the value of the minimum cut.
 The cut shown has a value of $10 + 7 = 17$.
 (Note that the 8 is not counted, as it is flowing back in to the cut)
 Maximum flow of people is 17 per minute.

Question 6 D

performing row reduction gives

48	52	32	14
54	49	38	10
63	55	29	11
50	50	35	12

→

34	38	18	0
44	39	28	0
52	44	18	0
38	38	23	0

Need only 1 line to cover zeros. Cannot make an allocation.

Performing column reduction gives

0	0	0	0
10	1	10	0
18	6	0	0
4	0	0	0

Need 4 lines to cover zeros, so can make an allocation.

Robert must run. This means Jasper must paddle and Ally must cycle. Laura must therefore have to ski.

Question 7 A

If a graph has no loops or multiple edges, the maximum number of edges it can have will be what it has as a **complete** graph.

A complete graph with 7 vertices has $\frac{7 \times 6}{2} = 21$ edges (option C is a correct statement)

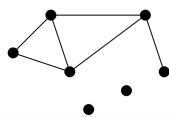
So this graph cannot have a maximum of 28 edges.

Option A is an **incorrect** statement.

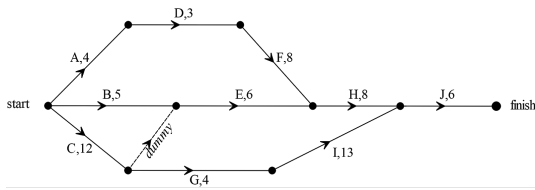
If there are no multiple edges, a cycle must need at least 3 edges, so D is correct.

A tree with n vertices has $n-1$ edges, so E is a correct statement.

B is a correct statement, as the example here shows.



Question 8 D



The critical path is the longest and gives the earliest completion time.

- A-D-F-H-J = 29
- B-E-H-J = 25
- C-E-H-J = 32
- C-G-I-J = 35

In this case, it is C-G-I-J with the earliest completion time being 35 months.

When activities G and I are reduced by the maximum (3 months), C-G-I-J reduces to 32. This is now the same length as C-E-H-J. Reducing activity E would still leave C-G-I-J as the critical path, so will have no effect on completion time. Earliest completion time is 32 months.

End of Module 2 Solutions

Module 3 – Geometry and measurement

Question 1 E

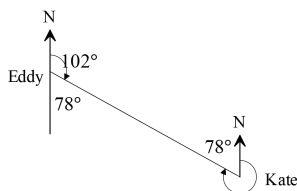
Let d be the side length of the square.
By Pythagoras,

$$d^2 + d^2 = 1.50^2$$

$$2d^2 = 1.50^2$$

$$d = \sqrt{\frac{1.50^2}{2}} \approx 1.06$$

Question 2 A



The diagram above shows the situation. By alternate angles, the bearing of Eddy from Kate is $360 - 78 = 282^\circ$
(note that adding 180° to the first bearing will give the bearing back the other way)

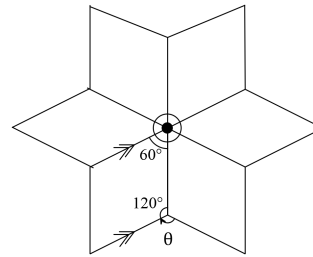
Question 3 C

In the centre of the patchwork pattern we see that 6 identical acute internal angles of the rhombus add to 360° .

Acute internal angle = $360/6 = 60^\circ$

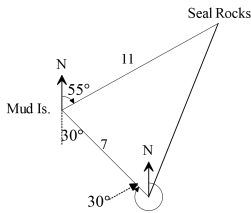
Obtuse internal angle = 120° (supplementary angles)

$\theta = 360 - 120 - 120 = 120^\circ$



Question 4 E

Draw a diagram. The angle between North and Graham's starting direction is 30°



By alternate angles, the internal angle in the triangle at Mud Island is $180^\circ - 55^\circ - 30^\circ = 95^\circ$

Using the cosine rule, the direct distance between Seal Rocks and home is

$$\sqrt{7^2 + 11^2 - 2 \times 7 \times 11 \times \cos(95^\circ)}$$

Question 5 E

Outside radius = 1 cm

Inside radius = $1 - 0.2 = 0.8$ cm

Volume inside =

$\pi \times 0.8^2 \times 9 \approx 18.09$

Closest response is 18 cm^3

Question 6 B

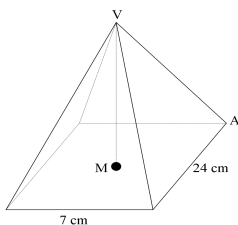
The difference in longitude = $145 - 55 = 90^\circ$

There is 1 hour time difference for every 15 degrees so The Seychelles are $90/15 = 6$ hours behind Melbourne (Melbourne is further east).

9:00 am + 12 hrs 25 min = 9:25 pm Melbourne time.

This is $9:25 - 6 = 3:25$ pm in The Seychelles.

Question 7 A



$V = \frac{1}{3} \text{area of base} \times \text{height}$

$448 = \frac{1}{3} \times 7 \times 24 \times h$

$h = 8$

base diagonal = $\sqrt{24^2 + 7^2} = 25$

□

$AM = 25/2 = 12.5$

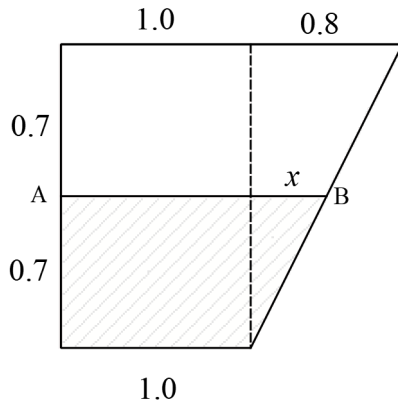
angle VAM = $\left(\frac{8}{12.5}\right) = 32.619^\circ \approx 33^\circ$

Question 8 D

The volume of a prism = the area of the cross-section \times length.

The water and the air occupy prisms of the **same length**, so

volume of water : volume of air = area of water cross-section : area of air cross-section



To find the length of AB, draw a vertical line as shown.

Now, by similar triangles

$$\frac{x}{0.7} = \frac{0.8}{1.4}$$

$$x = 0.4$$

$$AB = 1.0 + 0.4 = 1.4$$

$$\text{Area of water cross-section} = \frac{(1.0+1.4)}{2} \times 0.7 = 0.84$$

$$\text{Area of cross-section of air} = \frac{(1.4+1.8)}{2} \times 0.7 = 1.12$$

$$\text{Volume of water : volume of air} = 0.84 : 1.12 = 3 : 4$$

End of Module 3 Solutions**Module 4 – Graphs and relations****Question 1 C**

Look for the steepest section. This is between 4 and 6 minutes, where the temperature changes from about 46 to 90 degrees.

Question 2 B

The gradient of the graph = $\frac{4-0}{0-3} = \frac{4}{3}$

Graphs will be parallel if they have the same gradient.

By transposing all equations into gradient-intercept form, the only graph without a gradient of $\frac{4}{3}$ is

Option B

Option B has gradient $-\frac{4}{3}$

Question 3 C

$$\text{Gradient} = \frac{-9-3}{-8-8} = \frac{3}{4}$$

Substitute (8,3) into $y = \frac{3}{4}x + c$

$$3 = \frac{3}{4} \times 8 + c \quad 3 = 6 + c \quad c = -3 \quad 3y = \frac{3}{4}x - 3 \quad 4y = 3x - 12 \quad 3x - 4y = 12$$

Alternatively, enter the two points into a graphical calculator and use statistics calculation to perform linear regression to get the equation.

Question 4 D

Let a be the amount, in dollars, charged for a jar of jam.

Then revenue = $a \times 72$

Profit = Revenue – Cost

$$464 = 72a - (5.50 \times 72 + 40)$$

$$72a = 900 \quad a = 12.50$$

Question 5 A

The graph is not linear or reciprocal, so this eliminates options C and D.
Relationships represented by the other three options are:

A. $y = \frac{3}{4}x^2$ B. $2x^3$ C. $y = 6x^2$

The only one of these where the graph passes through the point (2,3) is option A.

Question 6 E

Reading from the table:

$$\text{mass of fibre content} = 0.42x + 0.02y$$

$$\text{mass of fat content} = 0.02x + 0.70y$$

The mixture must contain **at least** 9 grams of fibre, so $0.42x + 0.02y \geq 9$

Also, the mixture must have **no more than** 2 grams of fat, so $0.02x + 0.70y \leq 2$

Question 7 B

Substitute a into the two equations to find expressions for the height of each section at that point.

$$\text{Height of upper section} = 2.5 - \frac{1}{3}a$$

$$\text{Height of lower section} = 0.8 - \frac{1}{10}a$$

The upper section is 1.2 metres higher than the lower section so:

$$0.8 - \frac{1}{10}a + 1.2 = 2.5 - \frac{1}{3}a$$

Solving this gives $a \approx 2.14$

Question 8 D

Start by listing the constraints shown by the feasible region.

$$w + b \leq 18 \quad w \leq 2b \quad w \leq 11 \quad w \geq \frac{1}{3}b$$

Option A: $w + b \geq 18$ incorrect

Option B: $b \leq 11$ is incorrect

Option C: $b \geq 2w$

is incorrect (it should be $b \geq \frac{1}{2}w$)

Option E: $w \geq 3b$ is incorrect.

Option D : $b \leq 3w$ is **correct**. This transposes to

$$\frac{1}{3}b \leq w$$

$$w \geq \frac{1}{3}b$$

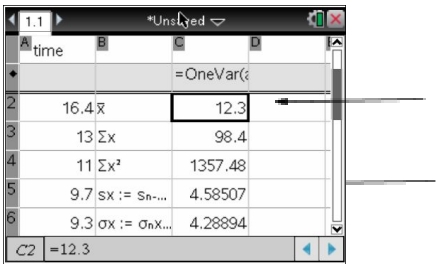
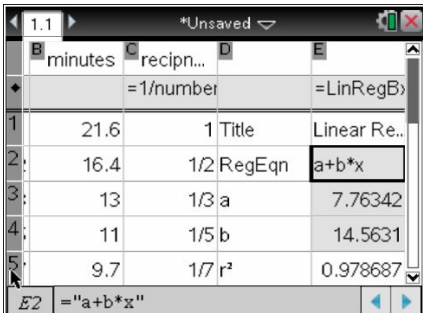
End of Module 4 Solutions

Section A Core – Data Analysis

<p>Question 1 A Warburn’s data has no outliers, so not B or E. The bulk of the data is towards the higher values, with a tail heading towards the lower values. This describes a negatively skewed distribution.</p>	<p>Question 2 C For Yarragrove, $Q_1 = 41$ and $Q_3 = 61$ Upper fence = $Q_3 + 1.5 \times \text{IQR}$ $= 61 + 1.5 \times (61 - 41)$ $= 61 + 1.5 \times 20$</p>
<p>Question 3 B $10 \text{ cm} = 100 \text{ mm} = 10^2 \text{ mm}$ $\log_{10}(10^2) = 2$ By adding the heights of the columns, we get the total number of Redwoods to be 18. Number of trees with $\log_{10}(\text{diameter}) > 2 = 6$ $\frac{6}{18} \times 100 \approx 33\%$</p>	<p>Question 4 D Number of people at Yarragrove who rated the film as poor = $250 - 125 - 65 = 60$ $\frac{60}{250} \times 100 = 24\%$</p>
<p>Question 5 B To compare rankings at the two cinemas we need the appropriate percentages as the audience sizes are not the same size. Response A does not give percentages. Response C does not compare the two cinemas. Response D by itself doesn’t show any difference between the cinemas. Response E only mentions audience size, not the ranking given. Response B shows that a higher percentage of Warburn viewers ranked the film as ‘good’ compared to that of Yarragrove viewers.</p>	<p>Question 6 C Use the 68 – 95 – 99.7% rule. A value of 244.4 mL is 2 standard deviations below the mean (i.e. z score of -2) 2.5% of the values are expected to be less than this. Expected number of bottles to be rejected $= \frac{2.5}{100} \times 3600 = 90$</p>

<p>Question 7 D</p> $z = \frac{x - \bar{x}}{s_x}$ <p>so</p> $x = z s_x + \bar{x}$ $x = 1.2 \times 2.3 + 249$ $= 251.76$ ≈ 251.8	<p>Question 8 C</p> <p><i>head circumference</i> is the response variable (vertical axis) so not A or D. Pick two clear points that the line passes through, for example (4,18) and (13,40).</p> <p>gradient = $\frac{40-18}{13-4} \approx 2.444$</p> <p>substitute (4,18) into $y = a + bx$ gives $18 = a + 2.444 \times 4$ $a \approx 8.224$</p> <p>Option C is the closest.</p>
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Section A Core – Data Analysis

<p>Question 9 A</p> <p>Using technology, enter the time data. This shows a standard deviation of approximately 4.6 and a mean of 12.3</p> 	<p>Question 10 B</p> <p>When 5 books are read, actual value for time is 11 minutes.</p> <p>Predicted value of time = $18.65 - 1.08 \times 5 = 13.25$</p> <p>Residual = actual value – predicted value $= 11 - 13.25$ $= -2.25$</p>
<p>Question 11 E</p> <p>A reciprocal transformation is applied to <i>number of books</i>, so not A, B or C. Using technology, perform a reciprocal ($1/x$) transformation to <i>number of books</i> and calculate the least squares line for <i>time vs 1/number of books</i>.</p> 	<p>Question 12 D</p> $r = \pm\sqrt{0.5364} = \pm 0.7324$ <p>Now from the equation given, we see that the gradient is negative, so r will be negative.</p> <p>So, $r = -0.7324$</p>

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Question 13 A

$$\text{Average price for the year} = \frac{240}{12} = 20$$

$$\text{Seasonal Index for September} = \frac{17.5}{20} = 0.875 \approx 0.88$$

Section A Core – Data Analysis**Question 14 C**

Suppose the cheese price for December is \$100.

The deseasonalised figure

= actual figure / seasonal index

$$= \frac{100}{0.53} = 188.6792\dots$$

This represents an increase of about 88.7%

Question 15 E

Four point average of February to May

$$= \frac{28.4+19.8+26.3+19.9}{4} = 23.6$$

Four point average of March to June

$$= \frac{19.8+26.3+19.9+22}{4} = 22$$

Value centred on April = $\frac{23.6+22}{2} = 22.8$

Question 16 E

Dividing each quarter's value by its seasonal index, we get

Quarter number	1	2	3	4
Deseasonalised sales	1758.57...	1772.727...	2100	1927.27...


Fitting a least squares regression line in the form $y = a + bx$ gives

$$b = 83.337\dots = 83 \text{ (correct to 2 significant figures)}$$

$$a = 1681.2987\dots = 1700 \text{ (correct to 2 significant figures)}$$

So, *deseasonalised sales* = $1700 + 83 \times \text{quarter number}$

Section A Core – Recursion and financial modelling

<p>Question 17 B Each term is 7 less than 3 times the previous term. Begin with the first term of 2 and repeat the calculation on each term.</p> 	<p>Question 18 D The graph shows the sequence 5, 15, 35, 75, 155</p> <p>Doubling each term then adding 5 to get the next term produces this sequence.</p> <p>This is described by the recurrence relation $T_0 = 5, T_{n+1} = 2T_n + 5$</p>
<p>Question 19 A Interest is added at $\frac{1.8}{12} = 0.15\%$ per month This gives a compounding factor of $1 + \frac{0.15}{100} = 1.0015$ So, Laura's investment amount each month will be 1.0015 times last month's amount less the \$400 she withdraws. This is given by option A.</p>	<p>Question 20 E Using finance solver on calculator: $N = 12$ $I = 1.8$ $PV = -25000$ $PMT = 400$ $FV =$ $P/Y = 12$ $C/Y = 12$ This gives $FV = 20613.93\dots$</p> <p>Value after 12 months is closest to \$20614</p>
<p>Question 21 C Principal repaid = $266758.78 - 265344.52 = 1414.26$ Interest paid = $\frac{3.8}{1200} \times 266758.78 = 844.74$ Repayment = $1414.26 + 844.74 = \\$2259$</p>	<p>Question 22 A In a perpetuity, the investment amount does not change. The amount withdrawn must equal the interest earned. Interest rate per month = $2.8/12\%$ Interest = $\frac{2.8}{1200} \times \text{investment amount} = 50$ $\text{investment amount} = 50 \times \frac{1200}{2.8} \approx \\21429</p>

<p>Question 23 C Using finance solver</p> <p>$N = 2.5 \times 12 = 30$ $I = 3.2$ $PV = -25000$ $PMT =$ $FV = 16475$ $P/Y = 12$ $C/Y = 12$ This gives $PMT = 339.997$</p> <p>Miriam withdraws \$340 each month</p>	<p>Question 24 E Firstly, find the amount of Eddie's repayments before the rate change.</p> <p>$N = 25 \times 12 = 300$ $I = 5.6$ $PV = 150000$ $PMT =$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ This gives $PMT = -930.11$</p> <p>Now work out amount still owing after 3 years</p> <p>$N = 36$ $I = 5.6$ $PV = 150000$ $PMT = -930.11$ $FV =$ $P/Y = 12$ $C/Y = 12$ This gives $FV = 141002.29$</p> <p>Make this the new present value, alter the interest rate and change the time to pay off the loan to 22 years.</p> <p>$N = 264$ $I = 6.1$ $PV = 141002.29$ $PMT =$ $FV = 0$ $P/Y = 12$ $C/Y = 12$</p> <p>This gives $PMT = -971.50$ So Eddie must make repayments of \$971.50</p> <p>$971.50 - 930.11 = \\41.39 His repayments increase by \$41.39</p>
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End of solutions Section A Core

Module 1 – Matrices

<p>Question 1 C A matrix is symmetric if it is a square matrix where the elements are symmetric with respect to the leading diagonal. A, D and E are not square. The elements in B do not mirror each other across the leading diagonal. Option C is a symmetric matrix. $[1\ 2\ 4\ 6\ 2\ 1\ 5\ 8\ 4\ 5\ 3\ 8\ 6\ 8\ 8\ 2]$</p>	<p>Question 2 D We have a 5 x 5 permutation matrix by a 5 x 1 column matrix. The result will be a 5 x 1 column matrix. Row 1 x column 1 = A Row 2 x column 1 = E Row 3 x column 1 = T Row 4 x column 1 = R Row 5 x column 1 = H</p>
<p>Question 3 E For a dependent system, there is an infinite number of solutions. The lines are the same. $[a\ -1\ 15\ -5][x\ y] = [b\ 35]$ For no unique solutions, determinant = 0 $-5 \times a = 15 \times -1$ $a = 3$ So, $3x - y = b$ $15x - 5y = 35$ These equations represent the same line if $b = 7$</p>	<p>Question 4 C Let the rule be $m_{ij} = ai + bj + c$ Pick 3 elements, not all from the same column, for example: $m_{11} = 4$ $m_{21} = 5$ and $m_{32} = 8$ This gives us 3 simultaneous equations $a + b + c = 4$ $2a + b + c = 5$ $3a + 2b + c = 8$ Solving this gives $a = 1, b = 2, c = 1$</p>
<p>Question 5 A To find 2-step dominance, square the 1-step dominance matrix. $[0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0]^2 = [0\ 1\ 0\ 0\ 1\ 0\ 2\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0]$ Adding the elements of the rows of the 1-step and 2-step dominance matrix gives: A = 2, B = 5, C = 3, D = 4 In order from highest to lowest: B,D,C,A</p>	

Module 1 – Matrices (continued)**Question 6 D**

To find the point value of each card, need to solve:

$$3r + 3y + 4b = 35$$

$$r + 5y + 4b = 33$$

$$7r + y + 2b = 43$$

The matrix equation would be

$$\begin{bmatrix} 3 & 3 & 4 \\ 1 & 5 & 4 \\ 7 & 1 & 2 \end{bmatrix} \begin{bmatrix} r \\ y \\ b \end{bmatrix} = \begin{bmatrix} 35 \\ 33 \\ 43 \end{bmatrix}$$

To solve, pre multiply both sides by the inverse of the 3 x 3 matrix. This is shown in option D.

Question 7 B

To find long term proportions, find the steady state matrix by raising T to a large power e.g. 99.

In the long term:

Proportion of birds on B = 0.292308

so, total number of birds = $\frac{1782}{0.292308} \approx 6096$

Proportion of birds on C = 0.312821

Number of birds on C = $0.312821 \times 6096 \approx 1907$

Question 8 A

8732 penguins nest on Duncan in 2019.

Of these, $0.3 \times 8732 \approx 2620$ nest on Albatross in 2020.

A total of **9641** penguins nest on Albatross in 2020.

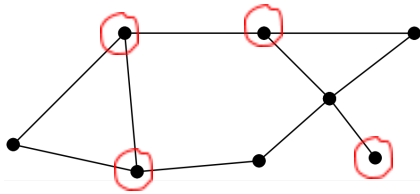
Percentage of these that were nesting on Duncan in 2019

$$= \frac{2620}{9641} \times 100 = 27.17... \approx 27\%$$

End of Module 1 Solutions

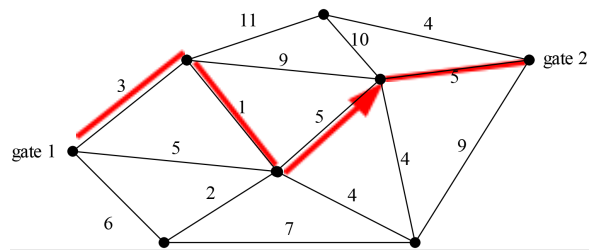
Module 2 – Networks and decision mathematics

Question 1 D



There are 4 vertices of odd degree.

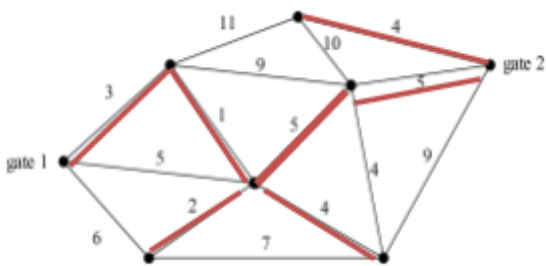
Question 2 C



By inspection, or using Dijkstra's algorithm the shortest path = $3+1+5+5=14$

Question 3 B

We are looking for a minimum spanning tree.



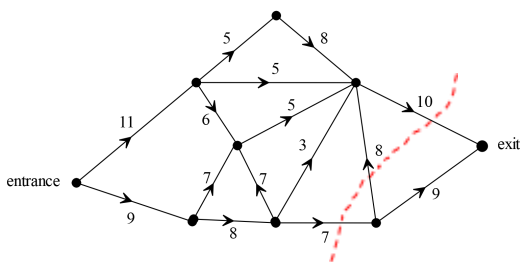
A minimum spanning tree is shown. Its value is:
 $3+2+1+5+4+5+4+5+4 = 24$

Question 4 E

The graph must have 10 vertices, 15 edges and 7 faces.

Only options C and E have 10 vertices.
 Option C has 8 faces and 16 edges.
 Option E has 7 faces and 15 edges.

Question 5 B



For maximum flow, we need to find the value of the minimum cut.

The cut shown has a value of $10 + 7 = 17$.
 (Note that the 8 is not counted, as it is flowing back in to the cut)

Maximum flow of people is 17 per minute.

Module 2 – Networks and decision mathematics (continued)

Question 6 D

48	52	32	14
54	49	38	10
63	55	29	11
50	50	35	12

34	38	18	0
44	39	28	0
52	44	18	0
38	38	23	0

performing row reduction gives



Need only 1 line to cover zeros. Cannot make an allocation.

0	0	0	0
10	1	10	0
18	6	0	0
4	0	5	0

Performing column reduction gives



Need 4 lines to cover zeros, so can make an allocation.

Robert must run. This means Jasper must paddle and Ally must cycle. Laura must therefore have to ski.

Question 7 A

If a graph has no loops or multiple edges, the maximum number of edges it can have will be what it has as a **complete** graph.

A complete graph with 7 vertices has $\frac{7 \times 6}{2} = 21$ edges (option C is a correct statement)

So this graph cannot have a maximum of 28 edges.

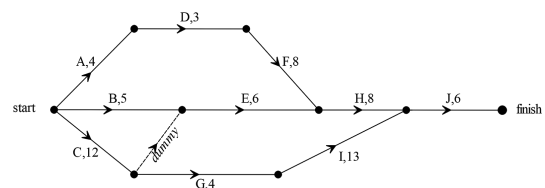
Option A is an **incorrect** statement.

If there are no multiple edges, a cycle must need at least 3 edges, so D is correct.

A tree with n vertices has $n-1$ edges, so E is a correct statement.

B is a correct statement, as the example here shows.

Question 8 D



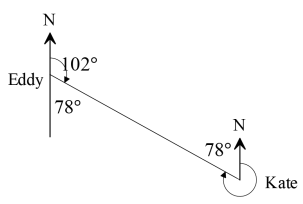
The critical path is the longest and gives the earliest completion time.

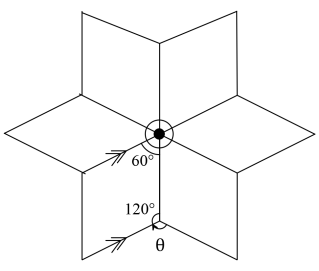
- A-D-F-H-J = 29
- B-E-H-J = 25
- C-E-H-J = 32
- C-G-I-J = 35

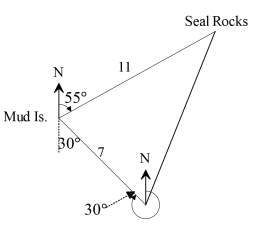
	<p>In this case, it is C-G-I-J with the earliest completion time being 35 months.</p> <p>When activities G and I are reduced by the maximum (3 months), C-G-I-J reduces to 32. This is now the same length as C-E-H-J. Reducing activity E would still leave C-G-I-J as the critical path, so will have no effect on completion time.</p> <p>Earliest completion time is 32 months.</p>
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End of Module 2 Solutions

Module 3 – Geometry and measurement

<p>Question 1 E Let d be the side length of the square. By Pythagoras,</p> $d^2 + d^2 = 1.50^2$ $2d^2 = 1.50^2$ $d = \sqrt{\frac{1.50^2}{2}} \approx 1.06$	<p>Question 2 A</p>  <p>The diagram above shows the situation. By alternate angles, the bearing of Eddy from Kate is $360 - 78 = 282^\circ$ (note that adding 180° to the first bearing will give the bearing back the other way)</p>
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<p>Question 3 C</p> <p>In the centre of the patchwork pattern we see that 6 identical acute internal angles of the rhombus add to 360°. Acute internal angle = $360/6 = 60^\circ$ Obtuse internal angle = 120° (supplementary angles) $\theta = 360 - 120 - 120 = 120^\circ$</p>	
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<p>Question 4 E Draw a diagram. The angle between North and Graham's starting direction is 30°</p>  <p>By alternate angles, the internal angle in the triangle at Mud Island is $180^\circ - 55^\circ - 30^\circ = 95^\circ$</p> <p>Using the cosine rule, the direct distance between Seal Rocks and home is</p>	<p>Question 5 E Outside radius = 1 cm Inside radius = $1 - 0.2 = 0.8$ cm Volume inside = $\pi \times 0.8^2 \times 9 \approx 18.09$</p> <p>Closest response is 18 cm^3</p>
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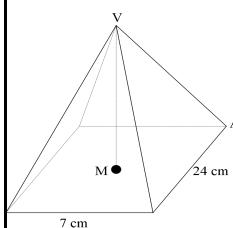
$\sqrt{7^2 + 11^2 - 2 \times 7 \times 11 \times \cos(95^\circ)}$	
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Module 3 – Geometry and measurement (continued)**Question 6 B**

The difference in longitude = $145 - 55 = 90^\circ$
 There is 1 hour time difference for every 15 degrees so The Seychelles are $90/15 = 6$ hours behind Melbourne (Melbourne is further east).

$9:00 \text{ am} + 12 \text{ hrs } 25 \text{ min} = 9:25 \text{ pm}$ Melbourne time.

This is $9:25 - 6 = 3:25 \text{ pm}$ in The Seychelles.

Question 7 A

$$V = \frac{1}{3} \text{area of base} \times \text{height}$$

$$448 = \frac{1}{3} \times 7 \times 24 \times h$$

$$h = 8$$

base diagonal

$$= \sqrt{24^2 + 7^2} = 25$$

$$\square \quad AM = 25/2 = 12.5$$

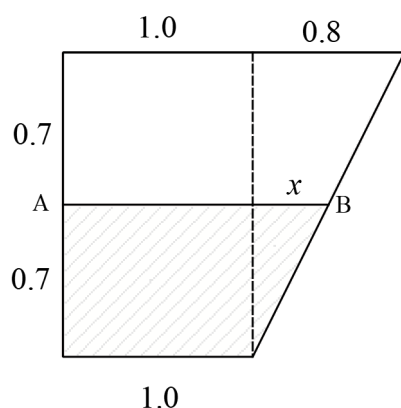
$$\text{angle VAM} = \left(\frac{8}{12.5} \right) = 32.619^\circ \approx 33^\circ$$

Question 8 D

The volume of a prism = the area of the cross-section \times length.

The water and the air occupy prisms of the **same length**, so

volume of water : volume of air = area of water cross-section : area of air cross-section



To find the length of AB, draw a vertical line as shown.
 Now, by similar triangles

$$\frac{x}{0.7} = \frac{0.8}{1.4}$$

$$x = 0.4$$

$$AB = 1.0 + 0.4 = 1.4$$

$$\text{Area of water cross-section} = \frac{(1.0+1.4)}{2} \times 0.7 = 0.84$$

$$\text{Area of cross-section of air} = \frac{(1.4+1.8)}{2} \times 0.7 = 1.12$$

$$\text{Volume of water : volume of air} = 0.84 : 1.12 = 3 : 4$$

End of Module 3 Solutions

Module 4 – Graphs and relations

<p>Question 1 C Look for the steepest section. This is between 4 and 6 minutes, where the temperature changes from about 46 to 90 degrees.</p>	<p>Question 2 B The gradient of the graph = $\frac{4-0}{0-3} = \frac{4}{3}$ Graphs will be parallel if they have the same gradient. By transposing all equations into gradient-intercept form, the only graph without a gradient of $\frac{4}{3}$ is Option B Option B has gradient $-\frac{4}{3}$</p>
<p>Question 3 C Gradient = $\frac{-9-3}{-8-8} = \frac{3}{4}$ Substitute (8,3) into $y = \frac{3}{4}x + c$ $3 = \frac{3}{4} \times 8 + c$ $3 = 6 + c$ $c = -3$ $y = \frac{3}{4}x - 3$ $4y = 3x - 12$</p> <p>Alternatively, enter the two points into a graphical calculator and use statistics calculation to perform linear regression to get the equation.</p>	<p>Question 4 D Let a be the amount, in dollars, charged for a jar of jam. Then revenue = $a \times 72$ Profit = Revenue – Cost $464 = 72a - (5.50 \times 72 + 40)$ $72a = 900$ $a = 12.50$</p>
<p>Question 5 A The graph is not linear or reciprocal, so this eliminates options C and D. Relationships represented by the other three options are: A. $y = \frac{3}{4}x^2$ B. $2x^3$ C. $y = 6x^2$</p> <p>The only one of these where the graph passes through the point (2,3) is option A.</p>	<p>Question 6 E Reading from the table: mass of fibre content = $0.42x + 0.02y$ mass of fat content = $0.02x + 0.70y$</p> <p>The mixture must contain at least 9 grams of fibre, so $0.42x + 0.02y \geq 9$ Also, the mixture must have no more than 2 grams of fat, so $0.02x + 0.70y \leq 2$</p>

Module 4 – Graphs and relations (continued)**Question 7 B**

Substitute a into the two equations to find expressions for the height of each section at that point.

$$\text{Height of upper section} = 2.5 - \frac{1}{3}a$$

$$\text{Height of lower section} = 0.8 - \frac{1}{10}a$$

The upper section is 1.2 metres higher than the lower section so:

$$0.8 - \frac{1}{10}a + 1.2 = 2.5 - \frac{1}{3}a$$

Solving this gives $a \approx 2.14$

Question 8 D

Start by listing the constraints shown by the feasible region.

$$w + b \leq 18 \quad w \leq 2b \quad w \leq 11 \quad w \geq \frac{1}{3}b$$

Option A: $w + b \geq 18$ incorrect

Option B: $b \leq 11$ is incorrect

Option C: $b \geq 2w$
is incorrect (it should be $b \geq \frac{1}{2}w$)

Option E: $w \geq 3b$ is incorrect.

Option D : $b \leq 3w$ is **correct**. This transposes to

$$\frac{1}{3}b \leq w$$

$$w \geq \frac{1}{3}b$$

End of Module 4 Solutions**End of Suggested Solutions 2020 VCE Further Mathematics Trial Examination 1**

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