2020 VCE

Further Mathematics Trial Examination 2

Suggested Solutions

Kilbaha Education

Quality educational content

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Quality educational content

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Data analysis

Question 1

a. (i)

Maximum = 23
Minimum = 15
Range = 23 – 15 = **8**

(1 mark)

a. (ii)

Median will be the 16^{th} value. The dot for the 16^{th} value appears in the column of dots above 19.
Median = **19**

(1 mark)

b. (i)

0 or zero.
There are no days on which exactly 21 eggs were hatched

(1 mark)

b. (ii)

12 dots are representing values above 19.

$$\frac{12}{31}\times 100 \approx 38.7\ \%$$

(1 mark)

Question 2

a.

Bearded Dragon: Negatively skewed with one outlier.
Green Iguana: Positively skewed.

(1 mark)

b.

IQR = 42.5 – 37.5 = 5

Lower fence = 37.5 – 1.5 × 5
= **30**

(1 mark)

c.

The median incubation time of 41 days for the Bearded Dragon is longer than the median incubation time of 34 days for the Green Iguana.

(1 mark)

d.

34 and 39 days are the median and upper quartile respectively.
This represents 25% of the values.

$$\frac{25}{100}\times 252 = 63$$

(1 mark)

Question 3

a. (i)

$15 = \bar{x} + 1s$

16% of values are expected to be above this.

16%.

(1 mark)

a. (ii)

$8.1 = \bar{x} - 2s$

$15 = \bar{x} + 1s$

81.5% of values are expected to be between these.

81.5%

(1 mark)

a. (iii)

$10.4 = \bar{x} - 1s$

16% of values are expected to be less than this.

$$\frac{16}{100} \times 850 = 136$$

(1 mark)

b.

$z = \frac{x - \bar{x}}{s}$

$z = \frac{11.2 - 12.7}{2.3} \approx -0.65$

(1 mark)

Question 4

a.

$gradient = r \times \frac{s_y}{s_x}$

$= 0.87 \times \frac{57.26}{7.28}$

$= 6.842884\ldots$

$intercept = \bar{y} - gradient \times \bar{x} = 339.6983\ldots$

attendance = **340** + **6.84** ×*max temp*.

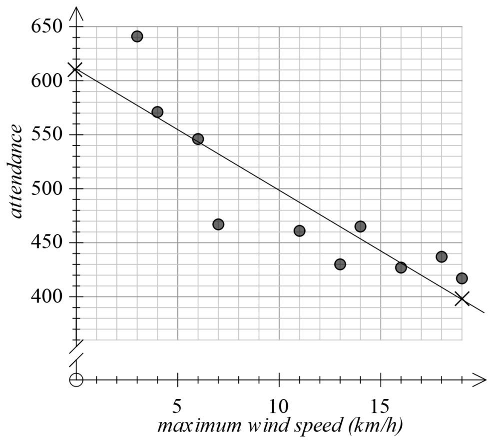
(2 marks)

(only 1 mark if significant figures not correct)

b. (i)

Plot two points at the extremes of the wind speed values. Join the points.
Let *wind speed* = 0.
Attendance = 610.86

Let *wind speed* = 19
Attendance = 610.86 -11.23 × 19 = 397.49



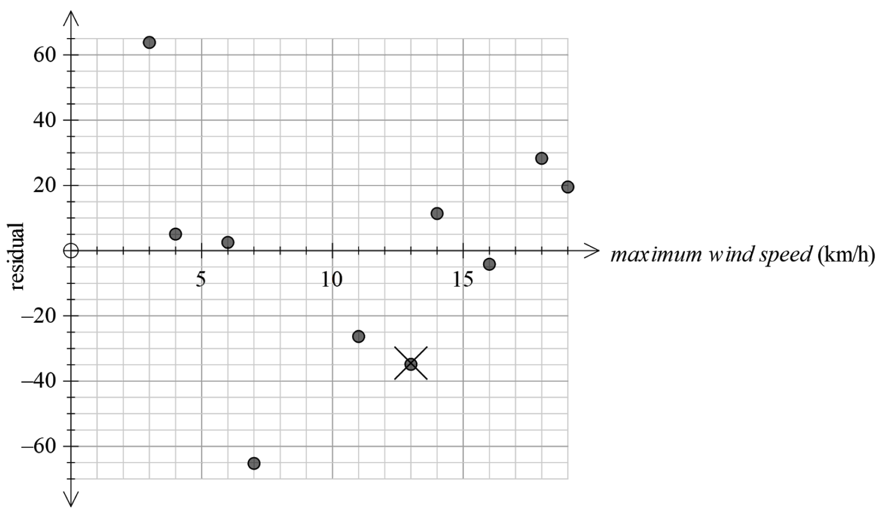
(1 mark)

b. (ii)

On average, for every 1 km/h increase in *maximum wind speed*, the *attendance* is expected to fall by 11.23 people.

(1 mark)

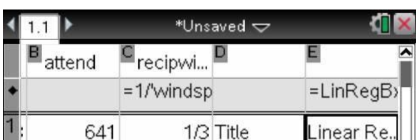
c.



When maximum wind speed = 13 km/h
predicted attendance = 610.86 – 11.23 ×13
= 464.87
Residual = Actual – predicted
= 430 – 464.87 = **-34.87**
(or use technology)

(1 mark)

d.



Apply a reciprocal transformation to the wind speed values and find the least squares regression line equation using technology.

attendance = **387** + **762** × (1/*max wind speed*)

(2 marks)

e.

$$387 + 762 \times \frac{1}{26} \approx 416.3$$

(1 mark)

f.
The prediction is an extrapolation and so may not be reliable.

(1 mark)

Question 5
a.
The four seasonal indices must add to 4.
SI for the third quarter
= 4 – (1.5 + 1.1 + 0.8)
= **0.6**

(1 mark)

b.
deseasonalised value = 43516 ÷ 1.1 = **39560**

(1 mark)

c.

$$\textit{deseasonalised attendance} = 61860 - 9748 \times 4$$

$$= 22868$$

$$\textit{actual attendance}$$

$$= \textit{deseasonalised attendance} \times \textit{seasonal index}$$

$$= 22868 \times 0.8$$

$$= 18294.4$$

18294

(1 mark)

Recursion and financial modelling

Question 6
a.
$B_1 = 48000 - \mathbf{3500} = 44500$
$B_2 = \mathbf{44500 - 3500 = 41000}$

(2 marks)

b.
3500

(1 mark)

c.
The number of balls of wool remaining at the factory will be the initial amount minus 3500 per week.
So,
B_n = **48000 – 3500** × *n*

(1 mark)

d.
After 10 weeks.

Solving the equation

$$48000 - 3500n = 15500$$

gives $n = 9.28571\ldots$

So, after the 9th week there will still be over 15500 balls of wool (16500), but after the **10th** week there will be less than 15500 (13000)

(2 marks)

Question 7
a.

$$r_{effective} = \left[\left(1+\frac{r}{100n}\right)^n - 1\right]\times 100\%$$
$$= \left[\left(1+\tfrac{6.8}{400}\right)^4 - 1\right]\times 100\% = 6.97537\ldots \approx 7.0\%$$

(1 mark)

b.

Depreciation in 10 years = 76000 – 68928 = 7072
Depreciation in 1 year = 7072 ÷ 10 = 707.20

Depreciation per hour =
707.20 ÷ 2080 = 0.34

(1 mark)

c.
Value after 20 years
$= 76000\times\left(1-\tfrac{8}{100}\right)^{20}$

$= 14340.69302$

$14340.69

(1 mark)
(**note**: rounding to 70 cents is an incorrect answer)

Recursion and financial modelling

Question 8
a.
To pay the \$1380 each year without changing the investment value (perpetuity), the annual interest earned must be \$1380.

$$\frac{1380}{32860} \times 100\% = 4.1996\ldots \approx 4.2\%$$

(1 mark)

b.
First find out how long the annuity lasts.

$N = ?$
$I = 4.56$
$PV = -32860$
$PMT = 240$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
This gives N = 193.67221

The value of the annuity after the 193rd payment can be found

$N = 193$
$I = 4.56$
$PV = -32860$
$PMT = 240$
$FV = ?$
$P/Y = 12$
$C/Y = 12$
This gives FV = 160.82019

The final (194th) payment includes one month's interest.
Final payment =

$$\left(1 + \frac{4.56}{1200}\right) \times 160.82019 \approx \$161.43$$

(1 mark)

c.

$4.56\%\ pa = \frac{4.56}{12} = 0.38\%\ per\ month$
So, compounding factor is 1.0038, but 240 is being removed each month.

$\mathbf{A_{n+1} = 1.0038\, A_n - 240}$

(1 mark)

Module 1 – Matrices

Question 1
a.
4 rows and 1 column, so the order is **4×1.**

(1 mark)

b.
Adding the elements in the column gives **160**.

(1 mark)

c. (i)
2nd row by 3rd column = 60 × 0.35 = 21
4th row by 2nd column = 40 × 0.25 = 10

$$M = \begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & \boxed{} & 12 \\ 4 & 5 & 7 & 4 \\ 8 & \boxed{} & 14 & 8 \end{bmatrix} \begin{matrix} 21 \\ \\ 10 \end{matrix}$$

(1 mark)

c. (ii)
The number of jewelers in class C is shown by the element in row 3, column 3.
This is **7**.

(1 mark)

d. (i)
Matrix F (1×4) multiplied by matrix M (4×4) will give a 1×4 matrix showing the total fees collected for each class, so

F × M

(1 mark)

d (ii)
Total amount collected in class D =
80×8 + 70×12 + 65×4 + 95×8 = **$2500** as matrix multiplication shows.

$$\begin{bmatrix} 80 & 70 & 65 & 95 \end{bmatrix} \begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & 21 & 12 \\ 4 & 5 & 7 & 4 \\ 8 & 10 & 14 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2500 & 3125 & 4375 & 2500 \end{bmatrix}$$

(1 mark)

Question 2
a.

$$\begin{bmatrix} 2 & 1 & 3 \\ \mathbf{1} & \mathbf{2} & 1 \\ \mathbf{1} & 1 & 1 \end{bmatrix}$$

(1 mark)

b.

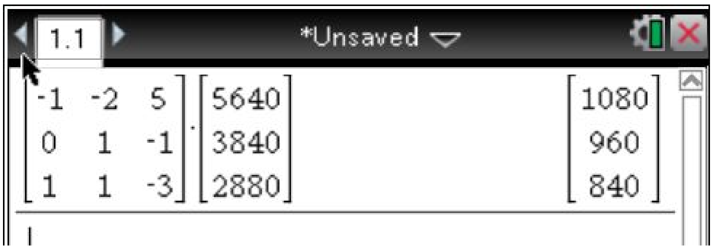
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ \mathbf{1} & \mathbf{2} & 1 \\ \mathbf{1} & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5640 \\ 3840 \\ 2880 \end{bmatrix}$$

$$\text{Now, } \begin{bmatrix} 2 & 1 & 3 \\ \mathbf{1} & 2 & 1 \\ \mathbf{1} & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & \mathbf{-2} & 5 \\ 0 & 1 & -1 \\ \mathbf{1} & 1 & -3 \end{bmatrix}$$

$\boldsymbol{a = -2}$

(1 mark)

c.



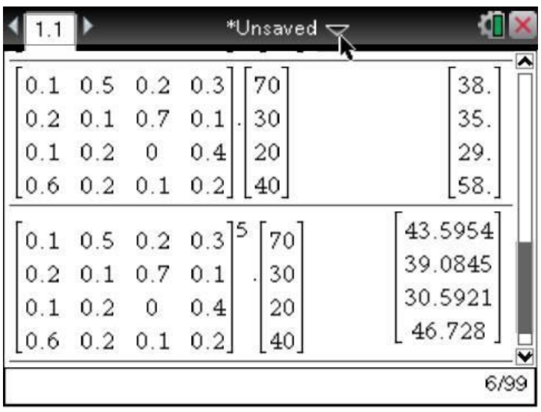
y = 960 and z = 840

Total cost of accommodation for the three women = 960 × 2 + 840 = **$2760**

(1 mark)

Question 3

a.



Pre-multiply the state matrix for the first night by the transition matrix raised to the power of 5 to show the choices for the sixth night.

39 participants are expected to choose Kumara on the sixth night.

(1 mark)

b.

Number choosing Lentil soup on the second night = 29.
Number who had chosen Vegetable soup on the first night who then chose Lentil soup on the second night = $0.4 \times 40 = 16$

$$\frac{16}{29} \times 100 \approx 55\%$$

(1 mark)

c.

Columns of the transition matrix must add to 1.
So, $b = 0.2$
Now the state matrix remains as $\begin{bmatrix} 120 \\ 40 \end{bmatrix}$
So $\begin{bmatrix} 0.8 & a \\ 0.2 & c \end{bmatrix} \times \begin{bmatrix} 120 \\ 40 \end{bmatrix} = \begin{bmatrix} 120 \\ 40 \end{bmatrix}$
$0.8 \times 120 + 40a = 120$
Solving this gives $a = 0.6$
So $c = 0.4$
$\boldsymbol{a = 0.6 \quad b = 0.2 \quad c = 0.4}$

(1 mark)

Module 2: Networks and decision mathematics

Question 1

a.

Vertex D has 4 edges entering or leaving. Degree = **4**

(1 mark)

b. (i)

His journey must start and finish at an odd vertex. Athens is an odd vertex (3). Island F is the only other one with an odd vertex.

F

(1 mark)

b. (ii)

The type of path where every edge is used once and only once is a **Eulerian** path.

(1 mark)

b. (iii)

Islands C, E and G are the only ports visited once on this path. All others are visited twice regardless of which Eulerian path is taken.

So, **5** vertices, including Athens.

(1 mark)

c.

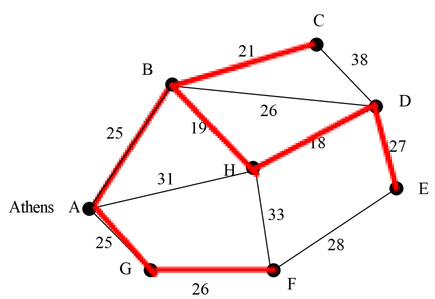
The path from A to E that will incur the minimum cost is A-B-D-E.

The cost will be 75 + 56 + 98 = **$229**

(1 mark)

d.

To get the minimum cost, we need to find a minimal spanning tree in the network.



The minimal spanning tree has a value of

25 + 25 + 26 + 19 + 21 + 18 + 27 = 161.

The least length of channel needed to be dug is **161** km.

(1 mark)

Question 2
a.
Activity C cannot start until activity A finishes, so the earliest starting time is **6 hours.**
(1mark)

b.
If the earliest starting time of activity H is 23 hours, then the 'longest path' from the start to the beginning of H must be 23 hours.
A + C + F =22
B + D = 21
So, B + E + G = 23
Duration of activity G = 23 – 3 – 4 = **16 hours**
(1 mark)

b.
The latest start time for H is the same as its earliest start time, i.e. 23 hours
The earliest start time for D is 4 hours
Float time for D = latest start time for H – earliest start time for D – duration of D
= 23 – 4 – 17 = **2 hours.**
(1 mark)

Question 3
a.

	Athena	**George**	**Makis**	**Yanis**
U	0	0	1	0
V	5	5	0	13
W	4	6	0	3
X	7	0	0	5

Other ways are possible.
(1mark)

b.
Add the smallest uncovered value to any value that is covered by two lines.
Subtract the smallest value from any uncovered value.
This gives

	Athena	**George**	**Makis**	**Yanis**
U	0	3	4	0
V	2	5	0	10
W	1	6	0	0
X	4	0	0	2

	Storeroom
Athena	U
George	X
Markis	V
Yanis	W

4 lines are needed to cover all the zeros so an allocation can be made.
(1 mark)

c.
The table will now be

After row and column reduction it becomes

4 lines are needed to cover all zeros so an allocation can be made.

	Athena	**George**	**Makis**	**Yanis**
U	0	0	1	0
V	5	5	0	13
W	0	6	0	3
X	7	0	0	5

	Storeroom
Athena	W
George	X
Markis	V
Yanis	U

This allocation gives a total of
28 + 29 + 24 + 28 = **109** minutes.

(1mark)

	Athena	**George**	**Makis**	**Yanis**
U	29	24	25	28
V	34	29	24	41
W	**28**	29	23	30
X	41	29	29	38

Module 3: Geometry and measurement

Question 1
a.

$$5\theta = 180° \ \theta = \tfrac{180}{5}$$
$$= 36°$$

(1 mark)

b.
Area of Δ NOP =
$\frac{1}{2}\times 6\times 6 \sin \mathit{sin}\ 36°$
$= 10.588134\ldots$
$\approx 10.58 cm^2$

(1 mark)

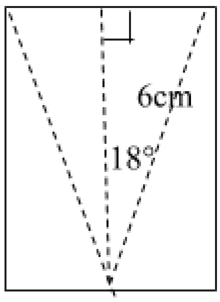
c.
Volume = Area of base × height
$= 5\times \frac{1}{2}\times 6\times 6 \sin \mathit{sin}\ 36° \times 3$

$$= 158.70202 \approx 158.7\ cm^3$$

(1 mark)

d.
Height of rectangle $= 6 \cos \mathit{cos}\ 18°$
Width of rectangle $= 2\times 6\ \sin \mathit{sin}\ 18°$
Area $= 2\times 6\times \sin \mathit{sin}\ 18° \times 6\times \cos \mathit{cos}\ 18°$

$$\approx 21.2\ cm^2$$



(1 mark)

Question 2
a.
Length scale ratio of solid : filter
= 60:270
= 2:9
Volume scale ratio of solid : filter
$= 2^3:9^3$
=**8:729**

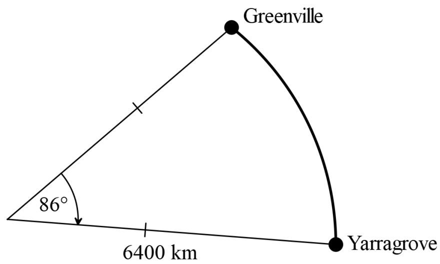
(1 mark)

b.
We have 8 parts solid, so 729 – 8
= 721 parts liquid.
Volume of liquid
$= \frac{721}{8}\times 11520 = 1038240\ cm^3$

(1 mark)

Question 3
a.

Greenville and Yarragrove are on the same longitude. The difference in latitude is



$32 + 54 = 86°$.

Shortest great circle distance between the towns is given by

$$6400 \times \frac{\pi}{180} \times 86 = 9606.2922\ldots$$
$$\approx 9606\ km$$

(1 mark)

b.
Difference in longitude between the towns
$= 146 - 26 = 120°$
Every 15° of longitude equates to 1 hour time difference.
Time difference between Yarragrove and Dairyville = $120 \div 15$ = **8 hours**

(1 mark)

c.
10:23 am Monday + 14 hours 13 minutes brings the time in Yarragrove to 12:36 am Tuesday when Bea lands.

Dairyville is 8 hours **behind** Yarragrove

(Dairyville is closer to the prime meridian)
So, the time in Dairyville when she lands will be
4:36 pm Monday

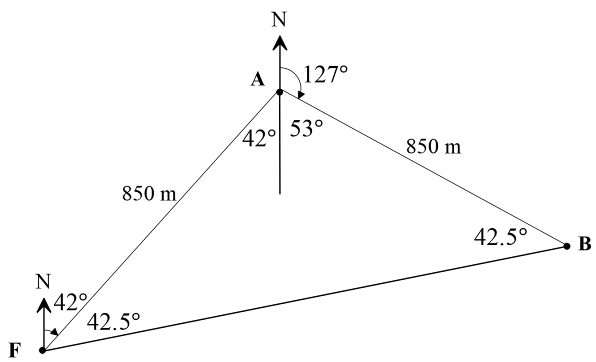
(1 mark)

Module 3: Geometry and measurement

Question 4
a.
Bearing of farmhouse from shed A
= 180 + 42 = **222°**

b.

By alternate and supplementary angles,
angle at A = 42 + 53 = 95°.
Now, FAB is isosceles, so angles at F and B are both 42.5°.
Bearing of shed B from farmhouse
= 42 + 42.5 = **084.5°**.



b.

$distance = \sqrt{850^2 + 850^2 - 2 \times 850 \times 850 \cos cos\ 95°}$
= 1253.37147…
≈ 1253.37 *m*

(1mark)

Module 4: Graphs and relations

Question 1
a.
Charge rate will be the gradient of the graph.

Gradient = $\frac{580-100}{40-0} = \frac{480}{40} = 12$

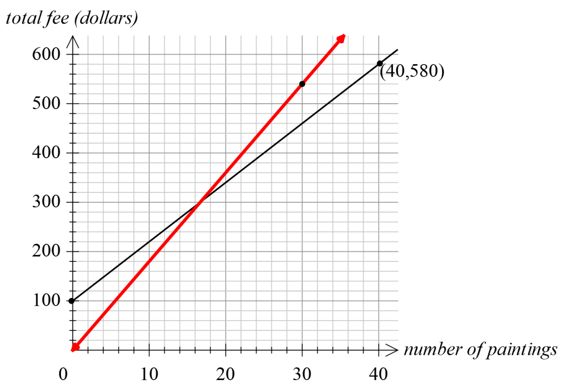
So the charge rate is \$12 per painting.

(1 mark)

b.
David's fee = **100** + **12** × *number of paintings*

(1 mark)

c.
Sue's graph begins at (0,0)
Find another point.
Let number of paintings = 30,
then total fee = 18 × 30 = \$540



(1 mark)

d.
Graphs intersect at $\left(16\frac{2}{3}\ ,\ 300\right)$
For 16 paintings or less, Sue's fees are less.
For 17 paintings or more, David's fees are less.

Minimum number of paintings for David's fee to be less is **17**.

(1 mark)

e.
For 32 paintings, difference in fees =
18×32 – (100 + 12×32) = **\$92**

(1 mark)

Question 2

a.

$\frac{1}{V}$	0.5	**0.25**	0.125	0.1	**0.05**
P	84	42	21	16.8	8.4

(1 mark)

b.

k will be the gradient of the linear graph of P vs $\frac{1}{V}$

Choose two pairs of values from the table from part a.

For example, (0.5, 84) and (0.1, 16.8)

$$k = \frac{16.8-84}{0.1-0.5} = 168$$

(1 mark)

c.

$P = \frac{168}{V}$

$16 = \frac{168}{V}$

$V = \frac{168}{16} = 10.5\ cm^3$

(1 mark)

Question 3

a.

$$20x + 10y \geq 400$$

(1 mark)

b.

Profit = $22x + 20y$

All 4 corner points of the feasible region are integer values, so the solution will be at one of these. Substituting each set of coordinates into the profit function, we find that (40, 80) gives the maximum value.

P = 22×40 + 20×80 = **$2480**

Alternatively, use the sliding rule method.

The profit function P = $22x + 20y$ has a gradient of $\frac{-22}{20} = -1.1$

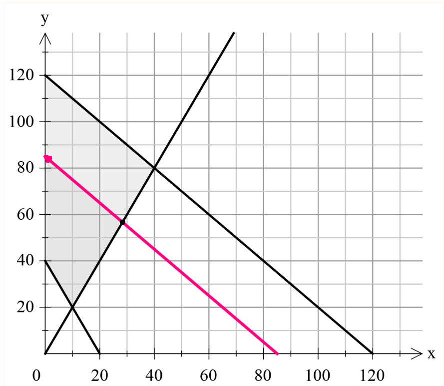
Sliding this line upwards, the last point it touches before leaving the feasible region is (40, 80)

(1 mark)

c.

The lines defined by $x + y = 85$ and $y = 2x$ intersect at $(28\frac{1}{3}, 56\frac{2}{3})$.

This would give the maximum profit, but we must have integer values.



The closest ordered pair containing whole numbers and still within the feasible region is (28, 57).

Maximum profit = 22×28 + 20×57 = **$1756**

(1 mark)

d. (i)
The profit on each bottle of Zesty has gone up to \$30.

The maximum profit occurs at (30, 90). This point is not a corner point and is on the line $x + y = 120$, which has a gradient of -1.
The profit function must also have a gradient of -1 for this to occur.

So, $P = 30x + 30y$
The profit made on a bottle of Dusty is now **\$30**

(1 mark)

d. (ii)
$P = 30 \times 30 + 30 \times 90 =$ **\$3600**

(1 mark)

Data analysis

Question 1

a. (i) Maximum = 23 Minimum = 15 Range = 23 – 15 = **8** (1 mark)	**a. (ii)** Median will be the 16^{th} value. The dot for the 16^{th} value appears in the column of dots above 19. Median = **19** (1 mark)
b. (i) **0 or zero**. There are no days on which exactly 21 eggs were hatched (1 mark)	**b. (ii)** 12 dots are representing values above 19. $\frac{12}{31}\times 100 \approx 38.7\ \%$ (1 mark)

Question 2

a. Bearded Dragon: Negatively skewed with one outlier. Green Iguana: Positively skewed. (1 mark)	**b.** IQR = 42.5 – 37.5 = 5 Lower fence = 37.5 – 1.5 × 5 = **30** (1 mark)
c. The median incubation time of 41 days for the Bearded Dragon is longer than the median incubation time of 34 days for the Green Iguana. (1 mark)	**d.** 34 and 39 days are the median and upper quartile respectively. This represents 25% of the values. $\frac{25}{100}\times 252 = 63$ (1 mark)

Data analysis

Question 3

a. (i) $15 = \bar{x} + 1s$ 16% of values are expected to be above this. **16%**. (1 mark)	**a. (ii)** $8.1 = \bar{x} - 2s$ $15 = \bar{x} + 1s$ 81.5% of values are expected to be between these. **81.5%** (1 mark)
a. (iii) $10.4 = \bar{x} - 1s$ 16% of values are expected to be less than this. $\frac{16}{100} \times 850 = 136$ (1 mark)	**b.** $z = \frac{x - \bar{x}}{s}$ $z = \frac{11.2 - 12.7}{2.3} \approx -0.65$ (1 mark)

Data analysis

Question 4

a.

$gradient = r \times \frac{s_y}{s_x}$
$= 0.87 \times \frac{57.26}{7.28}$
$= 6.842884\ldots$

$intercept = \overline{y} - gradient \times \overline{x} = 339.6983\ldots$

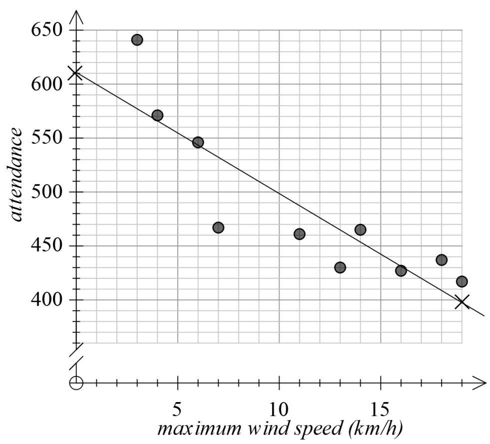
attendance = **340** + **6.84** ×*max temp*.

(2 marks)
(only 1 mark if significant figures not correct)

b. (i)

Plot two points at the extremes of the wind speed values. Join the points.
Let *wind speed* = 0.
Attendance = 610.86

Let *wind speed* = 19
Attendance = 610.86 -11.23 × 19 = 397.49



(1 mark)

b. (ii)

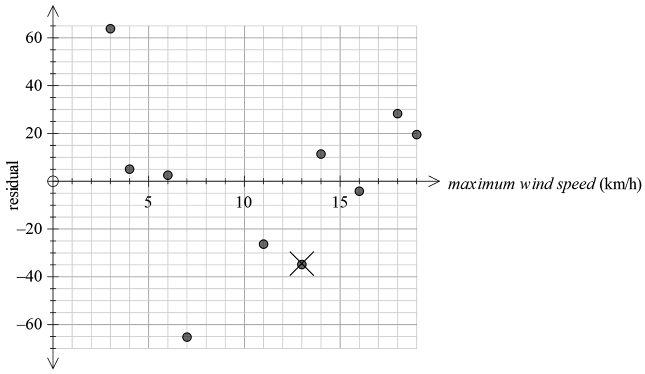
On average, for every 1 km/h increase in *maximum wind speed*, the *attendance* is expected to fall by 11.23 people.

(1 mark)

Data analysis

Question 4 (continued)

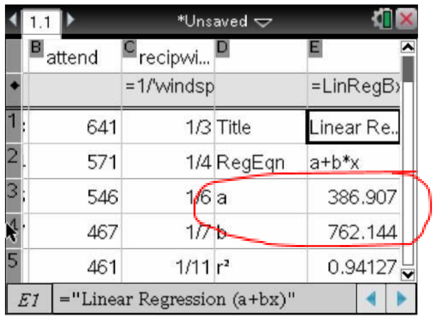
c.



When maximum wind speed = 13 km/h
predicted attendance = 610.86 – 11.23 ×13
= 464.87
Residual = Actual – predicted
= 430 – 464.87 = **-34.87**
(or use technology)

(1 mark)

d.



Apply a reciprocal transformation to the wind speed values and find the least squares regression line equation using technology.

attendance = **387** + **762** × (1/*max wind speed*)

(2 marks)

e.

$$387 + 762\times\frac{1}{26} \approx 416.3$$

(1 mark)

f.

The prediction is an extrapolation and so may not be reliable.

(1 mark)

Question 5

a.

The four seasonal indices must add to 4.
SI for the third quarter
= 4 – (1.5 + 1.1 + 0.8)
= **0.6**

(1 mark)

b.

deseasonalised value = 43516 ÷ 1.1 = **39560**

(1 mark)

c.

$$\textit{deseasonalised attendance} = 61860 - 9748\times4$$

$$= 22868$$

$$\textit{actual attendance}$$

$$= \textit{deseasonalised attendance}\times\textit{seasonal index}$$

$$= 22868\times0.8$$

$$= 18294.4$$

18294

(1 mark)

Recursion and financial modelling

Question 6

a. $B_1 = 48000 - \mathbf{3500} = 44500$ $B_2 = \mathbf{44500 - 3500 = 41000}$ (2 marks)	**b.** **3500** (1 mark)
c. The number of balls of wool remaining at the factory will be the initial amount minus 3500 per week. So, $B_n = \mathbf{48000 - 3500} \times n$ (1 mark)	**d.** After 10 weeks. Solving the equation $48000 - 3500n = 15500$ gives $n = 9.28571\ldots$ So, after the 9^{th} week there will still be over 15500 balls of wool (16500), but after the $\mathbf{10^{th}}$ week there will be less than 15500 (13000) (2 marks)

Question 7

a. $r_{effective} = \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\%$ $= \left[\left(1 + \frac{6.8}{400}\right)^4 - 1\right] \times 100\% = 6.97537\ldots$ (1 mark)	**b.** Depreciation in 10 years = 76000 – 68928 = 7072 Depreciation in 1 year = 7072 ÷ 10 = 707.20 Depreciation per hour = 707.20 ÷ 2080 = 0.34 (1 mark)
c. Value after 20 years $= 76000 \times \left(1 - \frac{8}{100}\right)^{20}$ $= 14340.69302$ **\$14340.69** (1 mark) (**note**: rounding to 70 cents is an incorrect answer)	

Recursion and financial modelling

Question 8

a. To pay the \$1380 each year without changing the investment value (perpetuity), the annual interest earned must be \$1380. $\frac{1380}{32860} \times 100\% = 4.1996\ldots \approx 4.2\%$ (1 mark)	**b.** First find out how long the annuity lasts. $N = ?$ $I = 4.56$ $PV = -32860$ $PMT = 240$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ This gives N = 193.67221 The value of the annuity after the 193rd payment can be found $N = 193$ $I = 4.56$ $PV = -32860$ $PMT = 240$ $FV = ?$ $P/Y = 12$ $C/Y = 12$ This gives FV = 160.82019 The final (194th) payment includes one month's interest. Final payment = $\left(1 + \frac{4.56}{1200}\right) \times 160.82019 \approx \$161.43$ (1 mark)
c. $4.56\%\ pa = \frac{4.56}{12} = 0.38\%\ per\ month$ So, compounding factor is 1.0038, but 240 is being removed each month. $\mathbf{A_{n+1} = 1.0038\ A_n - 240}$ (1 mark)	

Module 1 – Matrices

Question 1

<table>
<tr>
<td>

a.

4 rows and 1 column, so the order is **4×1.**

(1 mark)

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b.

Adding the elements in the column gives **160**.

(1 mark)

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c. (i)

2^{nd} row by 3^{rd} column = 60 × 0.35 = 21
4^{th} row by 2^{nd} column = 40 × 0.25 = 10

$$M = \begin{array}{c} \begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & \boxed{} & 12 \\ 4 & 5 & 7 & 4 \\ 8 & \boxed{} & 14 & 8 \end{bmatrix} \end{array} \begin{array}{l} \\ 21 \\ \\ 10 \end{array}$$

(1 mark)

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c. (ii)

The number of jewelers in class C is shown by the element in row 3, column 3.
This is **7**.

(1mark)

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d. (i)

Matrix F (1×4) multiplied by matrix M (4×4) will give a 1×4 matrix showing the total fees collected for each class, so

F × M

(1 mark)

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d (ii)

Total amount collected in class D =
80×8 + 70×12 + 65×4 + 95×8 = **$2500** as matrix multiplication shows.

[80 70 65 95] [8 10 14 8 12 15 21 12 4 5 7 4 8 1

= [2500 3125 4375 2500]

(1mark)

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</table>

Module 1 – Matrices

Question 2

a.

[2 1 3 1 2 1 1 1 1]

(1 mark)

b.

$[x\ y\ z\ ] = [2\ 1\ 3\ 1\ 2\ 1\ 1\ 1\ 1\ ]^{-1}\ [5640\ 3840\ 2880\ ]$

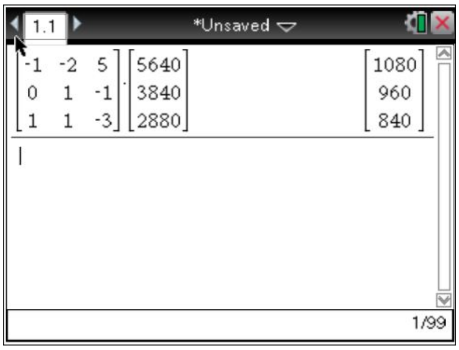
Now,

$[2\ 1\ 3\ 1\ 2\ 1\ 1\ 1\ 1\ ]^{-1} = [-1\ \ -2\ 5\ 0\ 1\ \ -1\ 1\ 1\ \ -$

***a* = - 2**

(1 mark)

c.



$y = 960$ and $z = 840$

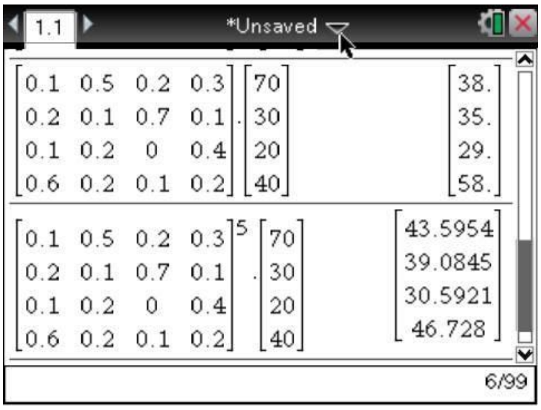
Total cost of accommodation for the three women = 960 × 2 + 840 = **$2760**

(1 mark)

Module 1 – Matrices

Question 3

a.



Pre-multiply the state matrix for the first night by the transition matrix raised to the power of 5 to show the choices for the sixth night.

39 participants are expected to choose Kumara on the sixth night.

(1 mark)

b.

Number choosing Lentil soup on the second night = 29.

Number who had chosen Vegetable soup on the first night who then chose Lentil soup on the second night = $0.4 \times 40 = 16$

$$\frac{16}{29} \times 100 \approx 55\%$$

(1 mark)

c.
Columns of the transition matrix must add to 1.
So, $b = 0.2$
Now the state matrix remains as [120 40]
So [0.8 a 0.2 c] × [120 40] = [120 40]

$0.8 \times 120 + 40a = 120$
Solving this gives $a = 0.6$
So $c = 0.4$

***a* = 0.6   *b* = 0.2 *c* = 0.4**

(1 mark)

Module 2: Networks and decision mathematics

Question 1

<table>
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<td>

a.

Vertex D has 4 edges entering or leaving.
Degree = **4**

(1 mark)

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<td>

b. (i)

His journey must start and finish at an odd vertex. Athens is an odd vertex (3). Island F is the only other one with an odd vertex.

F

(1 mark)

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b. (ii)

The type of path where every edge is used once and only once is a **Eulerian** path.

(1 mark)

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b. (iii)

Islands C, E and G are the only ports visited once on this path. All others are visited twice regardless of which Eulerian path is taken.
So, **5** vertices, including Athens.

(1 mark)

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c.

The path from A to E that will incur the minimum cost is A-B-D-E.
The cost will be $75 + 56 + 98 =$ **\$229**

(1 mark)

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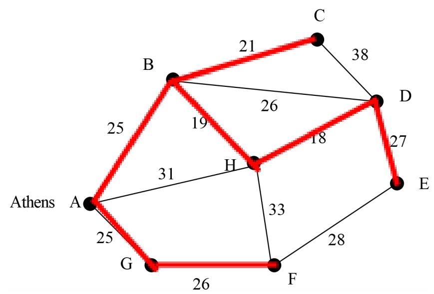
d.

To get the minimum cost, we need to find a minimal spanning tree in the network.



The minimal spanning tree has a value of
$25 + 25 + 26 + 19 + 21 + 18 + 27 = 161$.
The least length of channel needed to be dug is **161** km.

(1 mark)

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</table>

Module 2: Networks and decision mathematics

Question 2

a.
Activity C cannot start until activity A finishes, so earliest starting time is **6 hours.**

(1mark)

b.
If the earliest starting time of activity H is 23 hours, then the ‘longest path’ from the start to the beginning of H must be 23 hours.
A + C + F =22
B + D = 21
So, B + E + G = 23
Duration of activity G = 23 – 3 – 4 = **16 hours**

(1 mark)

b.
The latest start time for H is the same as its earliest start time, i.e. 23 hours
The earliest start time for D is 4 hours
Float time for D = latest start time for H – earliest start time for D – duration of D
= 23 – 4 – 17 = **2 hours.**

(1 mark)

Question 3

a.

	Athena	**George**	**Makis**	**Yanis**
U	0	0	1	0
V	5	5	0	13
W	4	6	0	3
X	7	0	0	5

Other ways are possible.

(1mark)

b.
Add the smallest uncovered value to any value that is covered by two lines.
Subtract the smallest value from any uncovered value.
This gives

	Athena	**George**	**Makis**	**Yanis**
U	0	3	4	0
V	2	5	0	10
W	1	6	0	0
X	4	0	0	2

	Storeroom
Athena	U
Geo	X

rge	
Markis	V
Yanis	W

4 lines are needed to cover all the zeros so an allocation can be made.

(1 mark)

Module 2: Networks and decision mathematics

Question 3 (continued)

c.
The table will now be

After row and column reduction it becomes

4 lines are needed to cover all zeros so an allocation can be made.

	Athena	**George**	**Makis**	**Yanis**
U	0	0	1	0
V	5	5	0	13
W	0	6	0	3
X	7	0	0	5

	Storeroom
Athena	W
George	X
Markis	V
Yanis	U

This allocation gives a total of
28 + 29 + 24 + 28 = **109** minutes.

(1mark)

	Athena	**George**	**Makis**	**Yanis**
U	29	24	25	28
V	34	29	24	41
W	**28**	29	23	30
X	41	29	29	38

Module 3: Geometry and measurement

Question 1

a.

$5\theta = 180°\ \theta = \frac{180}{5}$

$= 36°$

(1 mark)

b.

Area of Δ NOP =

$\frac{1}{2}\times6\times6\sin sin\ 36°$

$= 10.588134\ldots$

$\approx 10.58cm^2$

(1 mark)

c.

Volume = Area of base × height

$= 5\times\frac{1}{2}\times6\times6\sin sin\ 36°\times3$

$= 158.70202\approx158.7\ cm^3$

(1mark)

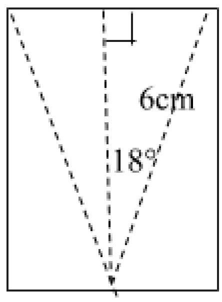
d.

Height of rectangle $= 6\cos cos\ 18°$

Width of rectangle $= 2\times6\ \sin sin\ 18°$

Area $= 2\times6\times\sin sin\ 18°\times6\times\cos cos\ 18°$

$\approx 21.2\ cm^2$



(1mark)

Question 2

a.

Length scale ratio of solid : filter

= 60:270

= 2:9

Volume scale ratio of solid : filter

$= 2^3:9^3$

=**8:729**

(1 mark)

b.

We have 8 parts solid, so 729 – 8

= 721 parts liquid.

Volume of liquid

$= \frac{721}{8}\times11520 = 1038240\ cm^3$

(1 mark)

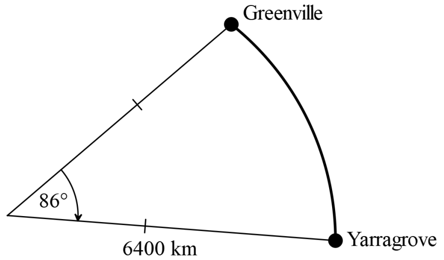
Module 3: Geometry and measurement

Question 3

a.

Greenville and Yarragrove are on the same longitude.

The difference in latitude is



$32 + 54 = 86°$.

Shortest great circle distance between the towns is given by

$$6400 \times \frac{\pi}{180} \times 86 = 9606.2922\ldots$$
$$\approx 9606\ km$$

(1 mark)

b.

Difference in longitude between the towns
$= 146 - 26 = 120°$
Every 15° of longitude equates to 1 hour time difference.
Time difference between Yarragrove and Dairyville = $120 \div 15$ = **8 hours**

(1 mark)

c.

10:23 am Monday + 14 hours 13 minutes brings the time in Yarragrove to 12:36 am Tuesday when Bea lands.

Dairyville is 8 hours **behind** Yarragrove

(Dairyville is closer to the prime meridian)
So, the time in Dairyville when she lands will be **4:36 pm Monday**

(1mark)

Module 3: Geometry and measurement

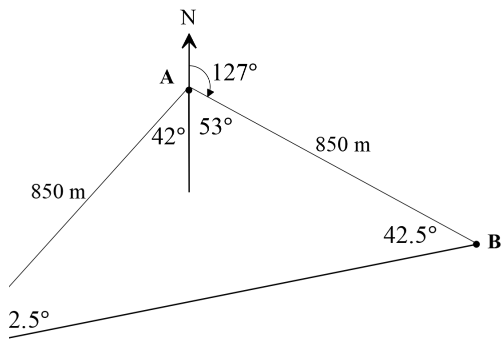
Question 4

a.

Bearing of farmhouse from shed A
= 180 + 42 = **222°**

b.

By alternate and supplementary angles,
angle at A = 42 + 53 = 95°.
Now, FAB is isosceles, so angles at F an B are both 42.5°.
Bearing of shed B from farmhouse
= 42 + 42.5 = **084.5°**.



b.

$distance = \sqrt{850^2 + 850^2 - 2 \times 850 \times 850 \cos cos\, 9}$
= 1253.37147…
≈ 1253.37 *m*

(1mark)

Module 4: Graphs and relations

Question 1

a.
Charge rate will be the gradient of the graph.

Gradient = $\frac{580-100}{40-0} = \frac{480}{40} = 12$

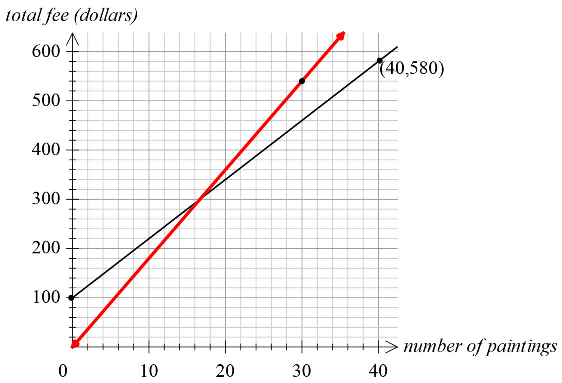
So charge rate is \$12 per painting.

(1 mark)

b.
David's fee = **100** + **12** × *number of paintings*

(1 mark)

c.
Sue's graph begins at (0,0)
Find another point.
Let number of paintings = 30,
then total fee = 18 × 30 = \$540



(1 mark)

d.
Graphs intersect at $\left(16\frac{2}{3}\ ,\ 300\right)$
For 16 paintings or less, Sue's fees are less.
For 17 paintings or more, David's fees are less.

Minimum number of paintings for David's fee to be less is **17**.

(1 mark)

e.
For 32 paintings, difference in fees =
18×32 – (100 + 12×32) = **\$92**

(1mark)

Module 4: Graphs and relations

Question 2

a.

$\frac{1}{V}$	0.5	**0.25**	0.125	0.1	**0.05**
P	84	42	21	16.8	8.4

(1 mark)

b.
k will be the gradient of the linear graph of
$P \ vs \ \frac{1}{V}$
Choose two pairs of values from the table from part a.
For example, (0.5, 84) and (0.1, 16.8)

$$k = \frac{16.8-84}{0.1-0.5} = 168$$

(1 mark)

c.
$P = \frac{168}{V}$
$16 = \frac{168}{V}$
$V = \frac{168}{16} = 10.5 \ cm^3$

(1 mark)

Module 4: Graphs and relations

Question 3

a.

$$20x + 10y \geq 400$$

(1 mark)

b.

Profit = $22x + 20y$

All 4 corner points of the feasible region are integer values, so the solution will be at one of these.

Substituting each set of coordinates into the profit function, we find that (40, 80) gives the maximum value.

P = 22×40 + 20×80 = **$2480**

Alternatively, use the sliding rule method.

The profit function P = $22x + 20y$ has a gradient of $\frac{-22}{20} = -1.1$

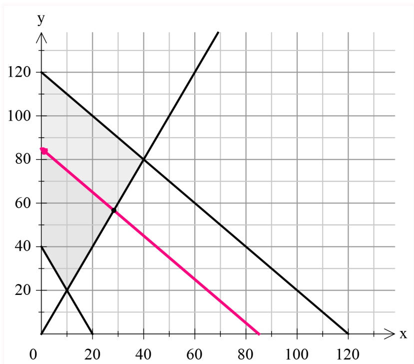
Sliding this line upwards, the last point it touches before leaving the feasible region is (40, 80)

(1 mark)

c.

The lines defined by $x + y = 85$ and $y = 2x$ intersect at $\left(28\frac{1}{3}, 56\frac{2}{3}\right)$.

This would give the maximum profit, but we must have integer values.



The closest ordered pair containing whole numbers and still within the feasible region is (28, 57).

Maximum profit = 22×28 + 20×57 = **$1756**

(1 mark)

Module 4: Graphs and relations

Question 3 (continued)

d. (i)	**d. (ii)**
The profit on each bottle of Zesty has gone up to \$30. The maximum profit occurs at (30, 90). This point is not a corner point and is on the line $x + y = 120$, which has a gradient of -1. The profit function must also have a gradient of -1 for this to occur. So, P = $30x + 30y$ The profit made on a bottle of Dusty is now **\$30** (1 mark)	P = 30×30 + 30×90 = **\$3600** (1 mark)

End of Suggested Solutions 2020 VCE Further Mathematics Trial Examination 2

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