2020 VCE Further Mathematics Trial Examination 2 Suggested Solutions



Quality educational content

Kilbaha Education	Tel: (03) 9018 5376
PO Box 2227	Fax: (03) 9817 4334
Kew Vic 3101	kilbaha@gmail.com
Australia	https://kilbaha.com.au

All publications from Kilbaha Education are digital and are supplied to the purchasing school in both WORD and PDF formats with a school site licence to reproduce for students in both print and electronic formats.



Quality educational content

Kilbaha Education (Est. 1978) (ABN 47 065 111 373)	Tel: +613 9018 5376 Fax: +613 9817 4334
PO Box 2227	Email: <u>kilbaha@gmail.com</u>
Kew Vic 3101	Web: <u>https://kilbaha.com.au</u>
Australia	

IMPORTANT COPYRIGHT NOTICE FOR KILBAHA PUBLICATIONS

(1) The material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Pty Ltd.

(2) The contents of these works are copyrighted. Unauthorised copying of any part of these works is illegal and detrimental to the interests of the author(s).

(3) For authorised copying within Australia please check that your institution has a licence from <u>https://www.copyright.com.au</u> This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

(4) All pages of Kilbaha files must be counted in Copyright Agency Limited (CAL) surveys.

(5) Kilbaha files must not be uploaded to the Internet.

(6) Kilbaha files may be placed on a password protected school Intranet.

Kilbaha educational content has no official status and is not endorsed by any State or Federal Government Education Authority.

While every care has been taken, no guarantee is given that the content is free from error. Please contact us if you believe you have found an error.

CAUTION NEEDED!

All Web Links when created linked to appropriate Web Sites. Teachers and parents must always check links before using them with students to ensure that students are protected from unsuitable Web Content. Kilbaha Education is not responsible for links that have been changed in its publications or links that have been redirected.

Question 1 a. (i) Maximum = 23 Minimum = 15 Range = $23 - 15 = 8$	
a. (ii) Median will be the 16^{th} value. The dot for the 16^{th} value appears in the column of dots above Median = 19	(1 mark) ve 19.
b. (i)	(1 mark)
0 or zero . There are no days on which exactly 21 eggs were hatched	(1 mark)
b. (ii) 12 dots are representing values above 19.	
<u>12</u> ×100≈38.7 %	(1 mark)
Question 2	
a. Bearded Dragon: Negatively skewed with one outlier. Green Iguana: Positively skewed.	(1 mark)
b. $IQR = 42.5 - 37.5 = 5$	(1 mark)
Lower fence = $37.5 - 1.5 \times 5$ = 30	
с.	(1 mark)
The median incubation time of 41 days for the Bearded Dragon is longer than the median incubation time of 34 days for the Green Iguana.	(1 1)
d. 34 and 39 days are the median and upper quartile respectively.	(1 mark)
This represents 25% of the values. $\frac{25}{100} \times 252 = 63$	
100 252 05	(1 mark)

Question 3 a. (i) $15 = \overline{x} + 1s$ 16% of values are expected to be above this. 16%.

a. (ii) $8.1 = \overline{x} - 2s$ $15 = \overline{x} + 1s$ 81.5% of values are expected to be between these. 81.5%

a. (iii)

 $10.4 = \overline{x} - 1s$ 16% of values are expected to be less than this.

$$\frac{16}{100} \times 850 = 136$$
 (1 mark)

b.

 $z = \frac{x - \overline{x}}{z}$ $z = \frac{11.2 - 12.7}{2.3} \approx -0.65$

Question 4

a. $gradient = r \times \frac{s_y}{s_x}$ $= 0.87 \times \frac{57.26}{7.28}$ = 6.842884...

intercept = \overline{y} - gradient $\times \overline{x}$ = 339.6983...

attendance = $340 + 6.84 \times max$ temp.

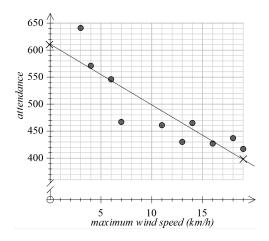
(2 marks) (only 1 mark if significant figures not correct)

(1 mark)

(1 mark)

Plot two points at the extremes of the wind speed values. Join the points. Let *wind speed* = 0. *Attendance* = 610.86

Let *wind speed* = 19 *Attendance* = 610.86 -11.23 × 19 = 397.49



b. (ii)

d.

On average, for every 1 km/h increase in *maximum wind speed*, the *attendance* is expected to fall by 11.23 people.

c. 60 40 20 Ó residual maximum wind speed (km/h) 15 10 -20¢ X -40 -60 When maximum wind speed = 13 km/hpredicted attendance = $610.86 - 11.23 \times 13$ = 464.87 Residual = Actual – predicted = 430 - 464.87 = **-34.87** (or use technology)

(1 mark)

ا	1.1	*Unsav	ed ▽	K 🛛 🛛
	[■] attend	Crecipwi D		E 2
٠		=1/'windsp		=LinRegB>
1	641	1/3 T	ïtle	Linear Re

(1 mark)

(1 mark)

......

Apply a reciprocal transformation to the wind speed values and find the least squares regression line equation using technology.

attendance = $387 + 762 \times (1/max wind speed)$

e.

 $387 + 762 \times \frac{1}{26} \approx 416.3$

f. The prediction is an extrapolation and so may not be reliable.

Question 5 a. The four seasonal indices must add to 4. SI for the third quarter = 4 - (1.5 + 1.1 + 0.8)= 0.6

b. deseasonalised value = $43516 \div 1.1 = 39560$

c.

deseasonalised attendance = $61860 - 9748 \times 4$

= 22868

actual attendance = deseasonalised attendance×seasonal index = 22868×0.8 = 18294.4

18294

Recursion and financial modelling

Question 6

a. $B_1 = 48000 - 3500 = 44500$ $B_2 = 44500 - 3500 = 41000$

(2 marks)

(1 mark)

(1 mark)

(1 mark)

(1 mark)

(1 mark)

(2 marks)

. .

c.

The number of balls of wool remaining at the factory will be the initial amount minus 3500 per week.

So, B_n = **48000** - **3500** \times *n*

 $D_n = 40000 - 3300 \times n$

d. After 10 weeks.

Solving the equation

gives n = 9.28571...

So, after the 9^{th} week there will still be over 15500 balls of wool (16500), but after the 10^{th} week there will be less than 15500 (13000)

48000 - 3500n = 15500

(2 marks)

Question 7

a.

$$r_{effective} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$$
$$= \left[\left(1 + \frac{6.8}{400} \right)^4 - 1 \right] \times 100\% = 6.97537 \dots \approx 7.0\%$$
(1 mark)

b.

Depreciation in 10 years = 76000 - 68928 = 7072Depreciation in 1 year = $7072 \div 10 = 707.20$

Depreciation per hour = $707.20 \div 2080 = 0.34$

(1 mark)

c. Value after 20 years = $76000 \times \left(1 - \frac{8}{100}\right)^{20}$

\$14340.69

= 14340.69302

(1 mark) (note: rounding to 70 cents is an incorrect answer)

(1 mark)

(1 mark)

Recursion and financial modelling

Question 8

a.

To pay the \$1380 each year without changing the investment value (perpetuity), the annual interest earned must be \$1380.

$$\frac{1380}{32860} \times 100\% = 4.1996... \approx 4.2\%$$
(1 mark)

b.

First find out how long the annuity lasts.

N = ? I = 4.56 PV = -32860 PMT = 240 FV = 0 P/Y = 12 C/Y = 12This gives N = 193.67221

The value of the annuity after the 193rd payment can be found

N = 193 I = 4.56 PV = -32860 PMT = 240 FV = ? P/Y = 12 C/Y = 12This gives FV = 160.82019

The final (194th) payment includes one month's interest. Final payment =

$$\left(1 + \frac{4.56}{1200}\right) \times 160.82019 \approx 161.43$$

(1 mark)

c.

4.56% $pa = \frac{4.56}{12} = 0.38\%$ per month So, compounding factor is 1.0038, but 240 is being removed each month.

 $A_{n+1} = 1.0038 A_n - 240$

Module 1 – Matrices

(1 mark)

d. (i)

Matrix F (1×4) multiplied by matrix M (4×4) will give a 1×4 matrix showing the total fees collected for each class, so

d (ii)

Total amount collected in class D = $80 \times 8 + 70 \times 12 + 65 \times 4 + 95 \times 8 =$ **\$2500** as matrix multiplication shows.

$$\begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & 21 & 12 \\ 4 & 5 & 7 & 4 \\ 8 & 10 & 14 & 8 \end{bmatrix}$$
$$= \begin{bmatrix} 2500 & 3125 & 4375 & 2500 \end{bmatrix}$$

Question 2 a.

b.

Question 1

a.

4 rows and 1 column, so the order is 4×1 .

b.

Adding the elements in the column gives 160.

c. (i)

 2^{nd} row by 3^{rd} column = $60 \times 0.35 = 21$ $4^{\text{th}} \text{ row by } 2^{\text{nd}} \text{ column} = 40 \times 0.25 = 10$

 $M = \begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & 12 & 21 \\ 4 & 5 & 7 & 4 \\ 8 & 14 & 8 & 10 \end{bmatrix}$

c. (ii) The number of jewelers in class C is shown by the element in row 3, column 3.

This is **7**.

$\mathbf{F} \times \mathbf{M}$

(1 mark)

1

(1 mark)

(1 mark)

(1 mark)

(1 mark)

2020 Kilbaha Further Mathematics Trial Examination 2 Suggested Solutions

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5640 \\ 3840 \\ 2880 \end{bmatrix}$$

Now,
$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & -2 & 5 \\ 0 & 1 & -1 \\ 1 & 1 & -3 \end{bmatrix}$$

 $a = -2$

c.

₹ 1.1 ▶	*Unsaved 🗢	<[] ×
-1 -2	5 [5640]	[1080]
0 1	-1 3840	960
1 1	-3][2880]	840
1		

y = 960 and z = 840

Total cost of accommodation for the three women = $960 \times 2 + 840 =$ **\$2760**

(1 mark)

Question 3

a.

4	Unsaved	*			1.1
[38.]	[70]	0.3	0.2	0.5	0.1
35.	30	0.1	0.7	0.1	0.2
29.	20	0.4	0	0.2	0.1
_58.]	40	0.2	0.1	0.2	0.6
43.5954	5 [70]	0.3	0.2	0.5	0.1
39.0845	. 30	0.1	0.7	0.1	0.2
30.5921	20	0.4	0	0.2	0.1
46.728	40	0.2	0.1	0.2	0.6

Pre-multiply the state matrix for the first night by the transition matrix raised to the power of 5 to show the choices for the sixth night.

39 participants are expected to choose Kumara on the sixth night.

b.

Number choosing Lentil soup on the second night = 29. Number who had chosen Vegetable soup on the first night who then chose Lentil soup on the second night = $0.4 \times 40 = 16$

(1 mark)

c. Columns of the transition matrix must add to 1. So, b = 0.2Now the state matrix remains as $[120\ 40]$ So $[0.8\ a\ 0.2\ c] \times [120\ 40] = [120\ 40]$ $0.8 \times 120 + 40a = 120$ Solving this gives a = 0.6So c = 0.4a = 0.6 b = 0.2 c = 0.4

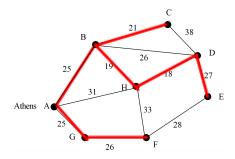
(1 mark)

Module 2: Networks and decision mathematics

Question 1 a. Vertex D has 4 edges entering or leaving. Degree = 4	
b. (i)	(1 mark)
His journey must start and finish at an odd vertex. Athens is an odd vertex (3). Island F is to other one with an odd vertex.	the only
F	(1 mark)
b. (ii) The type of path where every edge is used once and only once is a Eulerian path.	(1
The type of path where every edge is used once and only once is a Edierian path.	(1 mark)
b. (iii)	
Islands C, E and G are the only ports visited once on this path. All others are visited twice regardless of which Eulerian path is taken.	
So, 5 vertices, including Athens.	(1 1)
с.	(1 mark)
The path from A to E that will incur the minimum cost is A-B-D-E. The cost will be $75 + 56 + 98 = $ \$229	
	(1 mark)

d.

To get the minimum cost, we need to find a minimal spanning tree in the network.



The minimal spanning tree has a value of 25 + 25 + 26 + 19 + 21 + 18 + 27 = 161. The least length of channel needed to be dug is **161** km.

(1mark)

(1 mark)

(1 mark)

2020 Kilbaha Further Mathematics Trial Examination 2 Suggested Solutions

Question 2

a.

Activity C cannot start until activity A finishes, so the earliest starting time is 6 hours.

b.

If the earliest starting time of activity H is 23 hours, then the 'longest path' from the start to the beginning of H must be 23 hours.

A + C + F = 22 B + D = 21So, B + E + G = 23Duration of activity G = 23 - 3 - 4 = 16 hours

b.

The latest start time for H is the same as its earliest start time, i.e. 23 hours The earliest start time for D is 4 hours Float time for D = latest start time for H – earliest start time for D – duration of D = 23 - 4 - 17 = 2 hours.

Question 3

a.

	Athena	George	Makis	Yanis
U	0	0	1	0
V	5	5	0	13
W	4	6	0	3
Χ	7	0	0	5

Other ways are possible.

b.

Add the smallest uncovered value to any value that is covered by two lines. Subtract the smallest value from any uncovered value.

This gives

	Athena	George	Makis	Yanis
U	0	3	4	0
V	2	5	0	10
W	1	6	0	0
X	4	0	0	2
	Storeroom			
Athena	U			
George	X			
Markis	V			
Yanis	W			

4 lines are needed to cover all the zeros so an allocation can be made.

c.

The table will now be

After row and column reduction it becomes

4 lines are needed to cover all zeros so an allocation can be made.

	Athena	George	Makis	Yanis
U	0	0	1	0
V	5	5	0	13
W	0	6	0	3
Χ	7	0	0	5

	Storeroom
Athena	W
George	Х
Markis	V
Yanis	U

This allocation gives a total of 28 + 29 + 24 + 28 = 109 minutes.

	Athena	George	Makis	Yanis
U	29	24	25	28
V	34	29	24	41
W	28	29	23	30
Χ	41	29	29	38

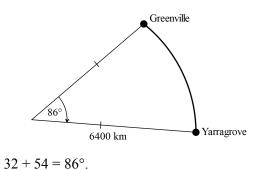
Question 1
a.

$$5\theta = 180^{\circ} \theta = \frac{180}{5}$$
(1 mark)
Area of Δ NOP =
 $\frac{1}{2} \times 6 \times 6 \sin \sin 36^{\circ}$
= 10.588134...
 ≈ 10.588134 ...
 ≈ 10.588134 ...
 ≈ 10.588134 ...
 ≈ 10.588134 ...
 ≈ 10.58817
(1 mark)
Volume = Area of base × height
= $5 \times \frac{1}{2} \times 6 \times 6 \sin \sin 36^{\circ} \times 3$
= $158.70202 \approx 158.7 \text{ cm}^3$
(1 mark)
d.
Height of rectangle = $6 \cos \cos 18^{\circ}$
Width of rectangle = $6 \cos \cos 18^{\circ}$
Width of rectangle = $2 \times 6 \sin \sin 18^{\circ}$
Area = $2 \times 6 \times \sin \sin 18^{\circ} \times 6 \times \cos \cos 18^{\circ}$
 $\approx 21.2 \text{ cm}^2$
Question 2
a.
Length scale ratio of solid : filter
= 60.270
= 2.9
Volume scale ratio of solid : filter
= $2^{\circ}.9^{\circ}$
Volume scale ratio of solid : filter
= $2^{\circ}.9^{\circ}$
We have 8 parts solid, so $729 - 8$
= $721 \text{ parts liquid.}$
We have 8 parts solid, so $729 - 8$
= $721 \text{ parts liquid.}$
Volume of liquid
 $= \frac{78}{4} \times 11520 = 1038240 \text{ cm}^3$

Question 3

a.

Greenville and Yarragrove are on the same longitude. The difference in latitude is



Shortest great circle distance between the towns is given by

$$6400 \times \frac{\pi}{180} \times 86 = 9606.2922...$$

 $\approx 9606 \ km$

(1 mark)

b. Difference in longitude between the towns $= 146 - 26 = 120^{\circ}$ Every 15° of longitude equates to 1 hour time difference. Time difference between Yarragrove and Dairyville = $120 \div 15 = 8$ hours

(1 mark)

c. 10:23 am Monday + 14 hours 13 minutes brings the time in Yarragrove to 12:36 am Tuesday when Bea lands.

Dairyville is 8 hours **behind** Yarragrove

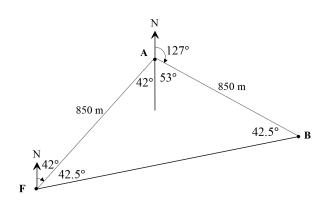
(Dairyville is closer to the prime meridian) So, the time in Dairyville when she lands will be **4:36 pm Monday**

Question 4

a. Bearing of farmhouse from shed A $= 180 + 42 = 222^{\circ}$

b.

By alternate and supplementary angles, angle at $A = 42 + 53 = 95^{\circ}$. Now, FAB is isosceles, so angles at F and B are both 42.5°. Bearing of shed B from farmhouse = $42 + 42.5 = 084.5^{\circ}$.



b.

 $distance = \sqrt{850^2 + 850^2 - 2 \times 850 \times 850 \cos \cos 95^\circ}$ = 1253.37147... \approx 1253.37 m

Module 4: Graphs and relations

Question 1

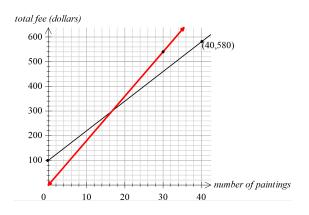
a. Charge rate will be the gradient of the graph.

Gradient = $\frac{580-100}{40-0} = \frac{480}{40} = 12$

So the charge rate is \$12 per painting.

b. David's fee = $100 + 12 \times number of paintings$

c. Sue's graph begins at (0,0)Find another point. Let number of paintings = 30, then total fee = $18 \times 30 = 540



d.

Graphs intersect at $(16\frac{2}{3}, 300)$ For 16 paintings or less, Sue's fees are less. For 17 paintings or more, David's fees are less.

Minimum number of paintings for David's fee to be less is 17.

e. For 32 paintings, difference in fees = $18 \times 32 - (100 + 12 \times 32) =$ **\$92**

(1 mark)

(1 mark)

(1 mark)

2020 Kilbaha Further Mathematics Trial Examination 2 Suggested Solutions

Question 2

a.

$\frac{1}{V}$	0.5	0.25	0.125	0.1	0.05
Р	84	42	21	16.8	8.4

b.

k will be the gradient of the linear graph of *P* vs $\frac{1}{V}$ Choose two pairs of values from the table from part a. For example, (0.5, 84) and (0.1, 16.8)

$$k = \frac{16.8 - 84}{0.1 - 0.5} = 168$$
(1 mark)
$$P = \frac{168}{V}$$

$$16 = \frac{168}{V}$$

$$V = \frac{168}{16} = 10.5 \ cm^3$$

(1 mark)

Question 3

a.

$$20x + 10y \ge 400$$

(1 mark)

b. Profit = 22x + 20y

All 4 corner points of the feasible region are integer values, so the solution will be at one of these. Substituting each set of coordinates into the profit function, we find that (40, 80) gives the maximum value. $P = 22 \times 40 + 20 \times 80 =$ **\$2480**

Alternatively, use the sliding rule method.

The profit function P = 22x + 20y has a gradient of $\frac{-22}{20} = -1.1$ Sliding this line upwards, the last point it touches before leaving the feasible region is (40, 80)

(1 mark)

c.

The lines defined by x + y = 85 and y = 2x intersect at $(28\frac{1}{3}, 56\frac{2}{3})$. This would give the maximum profit, but we must have integer values.

y 120 100 80 60 40 20 20 40 60 40 20 40 60 40 20 40 60 80 100 100 100 120120

The closest ordered pair containing whole numbers and still within the feasible region is (28, 57).

Maximum profit = $22 \times 28 + 20 \times 57 =$ **\$1756**

d. (i)

The profit on each bottle of Zesty has gone up to \$30.

The maximum profit occurs at (30, 90). This point is not a corner point and is on the line x + y = 120, which has a gradient of -1. The profit function must also have a gradient of -1 for this to occur.

So, P = 30x + 30yThe profit made on a bottle of Dusty is now **\$30**

d. (ii) P = $30 \times 30 + 30 \times 90 =$ **\$3600**

(1 mark)

Question 1

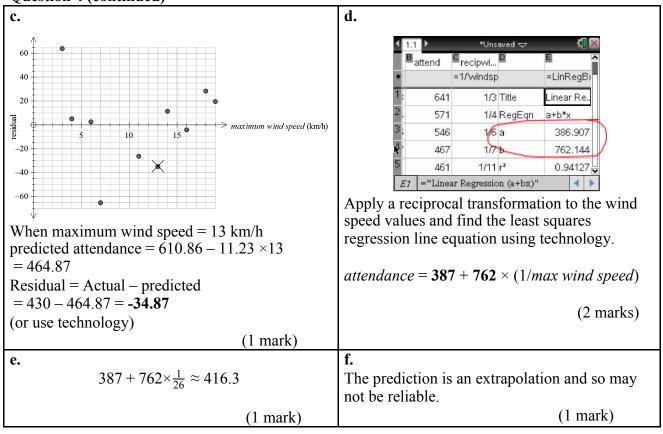
a. (i) Maximum = 23 Minimum = 15 Range = 23 - 15 = 8	a. (ii) Median will be the 16 th value. The dot for the 16 th value appears in the column of dots above 19. Median = 19 (1 mark)
(1 mark)	
b. (i)	b. (ii) 12 dots are representing values above 19.
0 or zero . There are no days on which exactly 21 eggs were hatched	$\frac{12}{31} \times 100 \approx 38.7 \%$ (1 mark)
(1 mark)	

 a.	b.
Bearded Dragon: Negatively skewed with one	IQR = 42.5 - 37.5 = 5
outlier.	Lower fence = $37.5 - 1.5 \times 5$
Green Iguana: Positively skewed. (1 mark)	= 30 (1 mark)
c. The median incubation time of 41 days for the Bearded Dragon is longer than the median incubation time of 34 days for the Green Iguana. (1 mark)	d. 34 and 39 days are the median and upper quartile respectively. This represents 25% of the values. $\frac{25}{100} \times 252 = 63$ (1 mark)

a. (i)	a. (ii)
$15 = \overline{x} + 1s$	$8.1 = \overline{x} - 2s$
	$15 = \overline{x} + 1s$
16% of values are expected to be above this.	
	81.5% of values are expected to be between
16% .	these.
	81.5% (1 mark)
(1 mark)	
a. (iii)	b.
$10.4 = \overline{x} - 1s$ 16% of values are expected to be less than this.	$z = \frac{x - \overline{x}}{z}$ $z = \frac{15 \cdot 2 - 12.7}{2.3} \approx -0.65$
$\frac{16}{100} \times 850 = 136$ (1 mark)	(1 mark)

Question 4

a. gradient = $r \times \frac{s_y}{r}$ $= 0.87 \times \frac{57.26}{7.28}$ = 6.842884... intercept = \overline{y} - gradient $\times \overline{x}$ = 339.6983... attendance = $340 + 6.84 \times max$ temp. (2 marks) (only 1 mark if significant figures not correct) **b.** (i) **b. (ii)** Plot two points at the extremes of the wind speed values. Join the points. On average, for every 1 km/h increase in Let wind speed = 0. maximum wind speed, the attendance is expected to fall by 11.23 people. Attendance = 610.86Let wind speed = 19*Attendance* = $610.86 - 11.23 \times 19 = 397.49$ (1 mark) 650 600 550 attendance 500 Ċ 450 400 2 5 10 15 maximum wind speed (km/h) (1 mark)



Question 4 (continued)

a.	b.	
The four seasonal indices must add to 4. SI for the third quarter = $4 - (1.5 + 1.1 + 0.8)$ = 0.6	deseasonalised value = 43516 ÷ 1.1 = 39560	
(1 mark)	(1 mark)	
c. $deseasonalised attendance = 61860 - 9748 \times 4$		
= 22868		
actual attendance = deseasonalised attendance×seasonal index = 22868×0.8 = 18294.4 18294 (1 mark)		

Recursion and financial modelling

Question 6

	b. 3500
(2 marks)	(1 mark)
c. The number of balls of wool remaining at the factory will be the initial amount minus 3500 per week. So, $B_n = 48000 - 3500 \times n$ (1 mark)	d. After 10 weeks. Solving the equation 48000 - 3500n = 15500 gives $n = 9.28571$ So, after the 9 th week there will still be over 15500 balls of wool (16500), but after the 10 th week there will be less than 15500 (13000) (2 marks)

Question 7

a.	b.
$r_{effective} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$ $= \left[\left(1 + \frac{6.8}{400} \right)^4 - 1 \right] \times 100\% = 6.97537 \dots$ (1 mark)	Depreciation in 10 years = $76000 - 68928 = 7072$ Depreciation in 1 year = $7072 \div 10 = 707.20$ Depreciation per hour = $707.20 \div 2080 = 0.34$ (1 mark)
c. Value after 20 years = $76000 \times (1 - \frac{8}{100})^{20}$ = 14340.69302 \$14340.69 (1 mark) (note: rounding to 70 cents is an incorrect answer)	

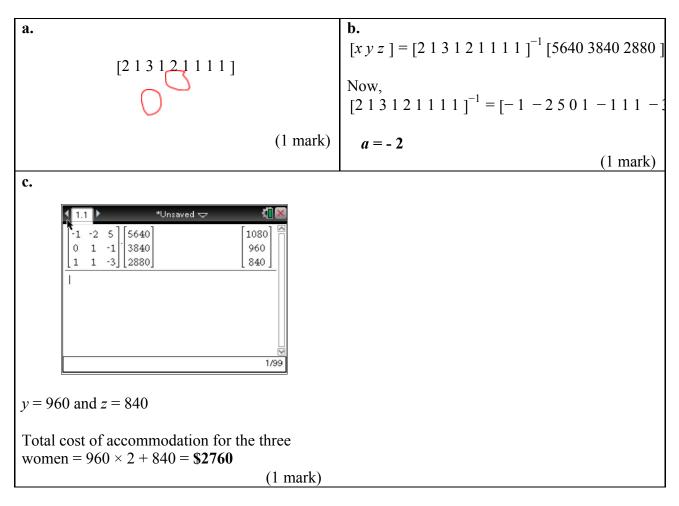
Recursion and financial modelling

a. To pay the \$1380 each year without changing	b. First find out how long the annuity lasts.
the investment value (perpetuity), the annual	This find out now fong the annulty fasts.
interest earned must be \$1380.	N = ?
	I = 4.56
$\frac{1380}{32860} \times 100\% = 4.1996\approx 4.2\%$	PV = -32860
32860 10070 1177011 11270	PMT = 240
	FV = 0
	P/Y = 12
	C/Y = 12
	This gives $N = 193.67221$
	8
	The value of the annuity after the 193 rd payment
	can be found
	<i>N</i> = 193
	<i>I</i> = 4.56
	PV = -32860
	PMT = 240
	FV = ?
	P/Y = 12
	C/Y = 12
	This gives $FV = 160.82019$
(1 mark)	
	The final (194 th) payment includes one month's
	interest.
	Final payment =
	$\left(1+\frac{4.56}{1200}\right) \times 160.82019 \approx 161.43$
	(1200)
	(1 mark)
с.	
$4.56\% pa = \frac{4.56}{12} = 0.38\% per month$	
12	
So, compounding factor is 1.0038, but 240 is	
being removed each month.	
$A_{n+1} = 1.0038 A_n - 240$	
(1 mark)	

Module 1 – Matrices

a.	b.
4 rows and 1 column, so the order is 4 × 1 .	Adding the elements in the column gives 160.
(1 mark)	(1 mark)
c. (i) $2^{nd} \text{ row by } 3^{rd} \text{ column} = 60 \times 0.35 = 21$ $4^{th} \text{ row by } 2^{nd} \text{ column} = 40 \times 0.25 = 10$ $M = \begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & 12 & 21 \\ 4 & 5 & 7 & 4 \\ 8 & 14 & 8 & 10 \end{bmatrix}$	c. (ii) The number of jewelers in class C is shown by the element in row 3, column 3. This is 7.
(1 mark)	(1mark)
d. (i) Matrix F (1×4) multiplied by matrix M (4×4) will give a 1×4 matrix showing the total fees collected for each class, so	d (ii) Total amount collected in class D = $80 \times 8 + 70 \times 12 + 65 \times 4 + 95 \times 8 =$ \$2500 as matrix multiplication shows.
$\mathbf{F} imes \mathbf{M}$	[80 70 65 95][8 10 14 8 12 15 21 12 4 5 7 4 8 10
(1 mark)	= [2500 3125 4375 2500]
	(1mark)

Module 1 – Matrices



Module 1 – Matrices

Image: transition matrix raised to the power of 5 to show the choices for the sixth night. 39 participants are expected to choose Kumara on the sixth night.	Number choosing Lentil soup on the second night = 29. Number who had chosen Vegetable soup on the first night who then chose Lentil soup on the second night = $0.4 \times 40 = 16$ $\frac{16}{29} \times 100 \approx 55\%$ (1 mark)
c. Columns of the transition matrix must add to 1. So, $b = 0.2$ Now the state matrix remains as [120 40] So $[0.8 \ a \ 0.2 \ c] \times [120 \ 40] = [120 \ 40]$ $0.8 \times 120 + 40a = 120$ Solving this gives $a = 0.6$ So $c = 0.4$ $a = 0.6 \ b = 0.2 \ c = 0.4$ (1 mark)	

Module 2: Networks and decision mathematics

a. Vertex D has 4 edges entering or leaving. Degree = 4 (1 mark)	 b. (i) His journey must start and finish at an odd vertex. Athens is an odd vertex (3). Island F is the only other one with an odd vertex. F
	(1 mark)
b. (ii) The type of path where every edge is used once and only once is a Eulerian path.	b. (iii) Islands C, E and G are the only ports visited once on this path. All others are visited twice regardless of which Eulerian path is taken. So, 5 vertices, including Athens.
(1 mark)	(1 mark)
c. The path from A to E that will incur the minimum cost is A-B-D-E. The cost will be 75 + 56 + 98 = \$229 (1 mark)	d. To get the minimum cost, we need to find a minimal spanning tree in the network. $\underbrace{\int_{25}^{26} \underbrace{\int_{26}^{10} \underbrace{\int_{27}^{0} \int_{2$
	(1 mark)

Module 2: Networks and decision mathematics

Question 2

a.	b.	
Activity C cannot start until activity A	If the earliest starting time of activity H is 23 hours,	
finishes, so earliest starting time is 6 hours .	then the 'longest path' from the start to the	
	beginning of H must be 23 hours.	
	A + C + F = 22	
	B + D = 21	
	So, $B + E + G = 23$	
(1mark)	Duration of activity $G = 23 - 3 - 4 = 16$ hours	
	(1 mark)	
b.		
The latest start time for H is the same as its ear.	liest start time, i.e. 23 hours	
The earliest start time for D is 4 hours		
Float time for $D =$ latest start time for $H -$ earliest start time for $D -$ duration of D		
= 23 - 4 - 17 = 2 hours.		
	(1 mark)	

a.					b. Add	the smalle	est uncove	red value	e to any valu
	Athena	George	Makis	Yanis	that	is covered	by two lin	nes.	
							nallest val	ue from a	any uncovere
U 🗖	0	0	1	0	valu				
V	5	5	0	13	This	gives			
W	4	6	0	3]	1	1	1	
Χ	7	0	0	5		Athena	George	Makis	Yani
Other v	ways are po	ossible.	-	-					S
					U	0	3	4	0
					V	2	5	0	10
					W	1	6	0	0
					Χ	4	0	0	2
						Storer			
						oom			
			(1m	ark)	Α	U			
					th				
					en				
					a				
					G	Х			
					eo				

rg e
M V ar
ki
s Y W
a ni
S
4 lines are needed to cover all the zeros so an allocation can be made.
(1 mark)

Module 2: Networks and decision mathematics

Question 3 (continued)

c. The ta	c. The table will now be						
After	After row and column reduction it becomes						
4 lines are needed to cover all zeros so an allocation can be made.							
	Athena	George	Makis	Yanis			
U	0	0	1	0			
V	5	5	0	13			
W	0	6	0	3			
Χ	X 7 0 0 5						
Stavavaam							

	Storeroom
Athena	W
George	Х
Markis	V
Yanis	U

This allocation gives a total of 28 + 29 + 24 + 28 = 109 minutes.

	Athena	George	Makis	Yanis
U	29	24	25	28
V	34	29	24	41
W	28	29	23	30
Χ	41	29	29	38

Question 1

a. $5\theta = 180^{\circ} \theta = \frac{180}{5}$ $= 36^{\circ}$ (1 mark)	b. Area of \triangle NOP = $\frac{1}{2} \times 6 \times 6 \sin \sin 36^{\circ}$ = 10.588134 $\approx 10.58cm^{2}$ (1 mark)
c. Volume = Area of base × height = $5 \times \frac{1}{2} \times 6 \times 6 \sin sin \ 36^{\circ} \times 3$ = $158.70202 \approx 158.7 \ cm^3$	d. Height of rectangle = $6 \cos \cos 18^{\circ}$ Width of rectangle = $2 \times 6 \sin \sin 18^{\circ}$ Area = $2 \times 6 \times \sin \sin 18^{\circ} \times 6 \times \cos \cos 18^{\circ}$ $\approx 21.2 \ cm^2$
(1mark)	(1mark)

a. Length scale ratio of solid : filter = $60:270$ = $2:9$ Volume scale ratio of solid : filter = $2^3:9^3$ = 8:729	b. We have 8 parts solid, so $729 - 8$ = 721 parts liquid. Volume of liquid = $\frac{721}{8} \times 11520 = 1038240 \text{ cm}^3$
(1 mai	rk) (1 mark)

a.	b.
Greenville and Yarragrove are on the same longitude.	Difference in longitude between the towns $= 146 - 26 = 120^{\circ}$
The difference in latitude is	Every 15° of longitude equates to 1 hour time
	difference. Time difference between Yarragrove and
Greenville	Dairyville = $120 \div 15 = 8$ hours
	(1 mark)
6400 km Yarragrove	c. 10:23 am Monday + 14 hours 13 minutes brings the time in Yarragrove to 12:36 am Tuesday when Bea lands.
$32 + 54 = 86^{\circ}$.	when bea lands.
	Dairyville is 8 hours behind Yarragrove
Shortest great circle distance between the towns is given by	(Dairyville is closer to the prime meridian) So, the time in Dairyville when she lands will be 4:36 pm Monday
	(1mark)
$6400 \times \frac{\pi}{180} \times 86 = 9606.2922 \\\approx 9606 \ km$	
(1 mark)	

a.	b.
Bearing of farmhouse from shed A = 180 + 42 = 222°	By alternate and supplementary angles, angle at A = 42 + 53 = 95°. Now, FAB is isosceles, so angles at F an B are both 42.5°. Bearing of shed B from farmhouse = 42 + 42.5 = 084.5 °. Note that the second
b. $distance = \sqrt{850^2 + 850^2 - 2 \times 850 \times 850 \cos \cos 9}$ $= 1253.37147$ $\approx 1253.37 m$	
(1mark)	

Module 4: Graphs and relations

a. Charge rate will be the gradient of the graph.	b. David's fee = $100 + 12 \times number of paintings$
Gradient = $\frac{580-100}{40-0} = \frac{480}{40} = 12$	
So charge rate is \$12 per painting.	(1 mark)
(1 mark)	
c. Sue's graph begins at (0,0) Find another point. Let number of paintings = 30, then total fee = $18 \times 30 = 540 total fee (dollars) 600 500 400 300 200 100 200 100 200 300 400 100 200 300 400 400 100 200 300 400 400 400 400 400 400 400 400 4	 d. Graphs intersect at (16²/₃, 300) For 16 paintings or less, Sue's fees are less. For 17 paintings or more, David's fees are less. Minimum number of paintings for David's fee to be less is 17.
(1 mark)	(1 mark)
e. For 32 paintings, difference in fees = $18 \times 32 - (100 + 12 \times 32) =$ \$92	
(1mark)	

Module 4: Graphs and relations

a.						b. <i>k</i> will be the gradient of the linear graph of
$\frac{1}{V}$	0.5	0.25	0.125	0.1	0.05	$P vs \frac{1}{V}$ Choose two pairs of values from the table from
Р	84	42	21	16.8	8.4	part a. For example, (0.5, 84) and (0.1, 16.8)
					(1 mark)	$k = \frac{16.8 - 84}{0.1 - 0.5} = 168$ (1 mark)
c. $P = \frac{1}{16}$ V =	$\frac{\frac{168}{V}}{\frac{168}{V}}$ $\frac{\frac{168}{V}}{16} = 1$	0.5 cm ³				(1 mark)

а.		b.
$20x + 10y \ge 400$		Profit = 22x + 20y
	(1 mark)	All 4 corner points of the feasible region are
		integer values, so the solution will be at one of
		these.
		Substituting each set of coordinates into the
		profit function, we find that (40, 80) gives the
		maximum value.
		$P = 22 \times 40 + 20 \times 80 = \2480
		Alternatively, use the sliding rule method.
		The profit function $P = 22x + 20y$ has a gradient
		of $\frac{-22}{20} = -1.1$
		Sliding this line upwards, the last point it
		touches before leaving the feasible region
		is (40, 80)
		(1 mark)
с.		
The lines defined by $x + y = 85$	and $v = 2r$ interv	$(28^{\pm}, 56^{\pm})$
This would give the maximum pr		
This would give the maximum p		ered pair containing whole numbers and still
у		ble region is (28, 57).
	within the reasi	ble region is (20, 57).
	Maximum prof	$it = 22 \times 28 + 20 \times 57 = $ \$1756
100	Maximum prof	$11 - 22^{20} + 20^{20} - 91750$
80		
60		
40		

Question 3

20

0

80

40 60

20

100 120

Module 4: Graphs and relations

Question 3 (continued)

d. (i) The profit on each bottle of Zesty has gone up to \$30.	d. (ii) P = $30 \times 30 + 30 \times 90 =$ \$3600
The maximum profit occurs at $(30, 90)$. This point is not a corner point and is on the line $x + y = 120$, which has a gradient of -1. The profit function must also have a gradient of -1 for this to occur.	(1 mark)
So, $P = 30x + 30y$ The profit made on a bottle of Dusty is now \$30	
(1 mark)	

End of Suggested Solutions 2020 VCE Further Mathematics Trial Examination 2

Kilbaha Education	Tel: (03) 9018 5376
PO Box 2227	Fax: (03) 9817 4334
Kew Vic 3101	kilbaha@gmail.com
Australia	https://kilbaha.com.au