2020
VCE
Further Mathematics

Trial Examination 2

Kilbaha Education

Quality educational content

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Letter

STUDENT NUMBER									

VICTORIAN CERTIFICATE OF EDUCATION 2020

FURTHER MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A - Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	8	8	36
Section B - Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	1	12
			Total 48

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 36 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Section A – Core

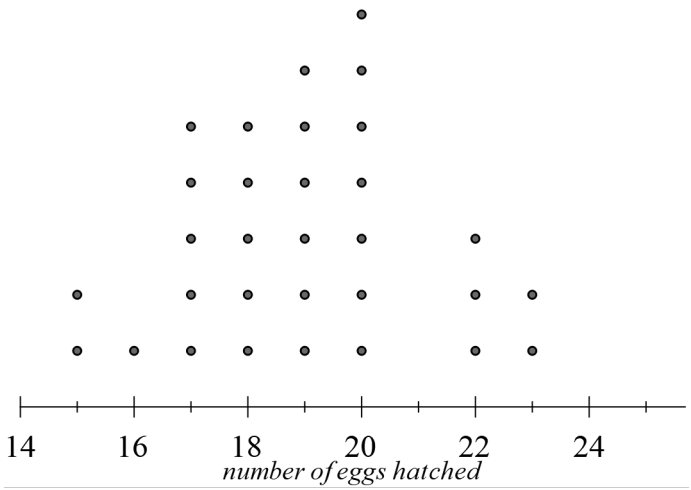
Instructions for Section A

Answer all questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (4 marks)

Water Skink eggs are hatching in the reptile house at the Milville animal sanctuary. The number of eggs hatching each day over a 31 day period is recorded. The results are displayed in the dot plot below.



a. Write down the

i. range.

1 mark

_____

ii. median

1 mark

_____

Data Analysis

Question 1 (continued)

b. Write down

1 mark

i. The number of days on which exactly 21 eggs were hatched.

_____

ii. The percentage of days on which more than 19 eggs were hatched.
Give your answer correct to 1 decimal place.

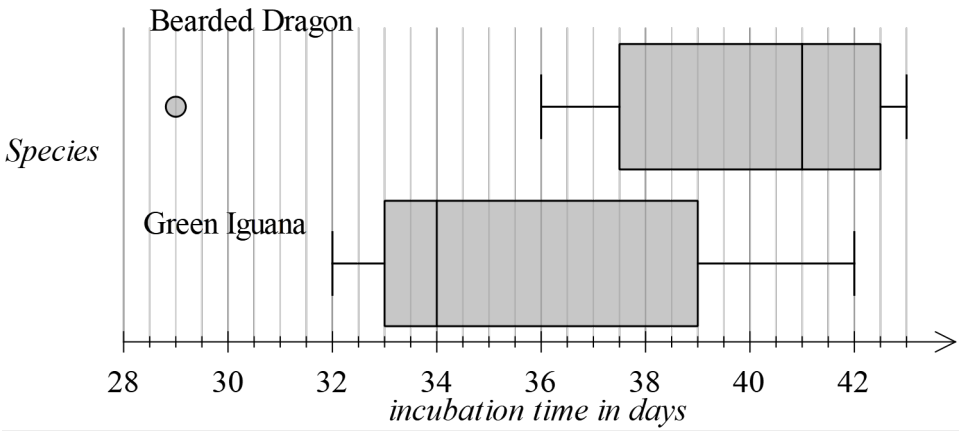
1 mark

_____

Data Analysis

Question 2 (4 marks)

Clutches of eggs are also being hatched in the reptile house. The boxplots below show the distribution of *incubation time* for the eggs of the species Bearded Dragon and Green Iguana.



a. Describe the shapes of the distributions of *incubation time* (including outliers) for the eggs of the Bearded Dragon and the Green Iguana. 1 mark

Bearded Dragon_____

Green Iguana _____

b. Determine the value of the lower fence for the Bearded Dragon boxplot. 1 mark

_____

_____

_____

Data Analysis

Question 2 (continued)

c. Using the information from the boxplots, explain why the *incubation time* is associated with the *species* of lizard. Quote the values of appropriate statistics in your response.

1 mark

_____

_____

_____

_____

d. There is a total of 252 Green Iguana eggs. How many had an *incubation time* between 34 and 39 days?

1 mark

_____

Data Analysis

Question 3 (4 marks)

Lizards in the reptile house are given cricket nymphs to eat at feeding time. The lengths of the cricket nymphs are approximately normally distributed with a mean of 12.7 mm and a standard deviation of 2.3 mm.

a. Use the 68-95-99.7% rule to determine

i. the percentage of cricket nymphs expected to be longer than 15 mm. 1 mark

ii. the percentage of cricket nymphs expected to have a length between 8.1 mm and 15 mm. 1 mark

iii. the number of cricket nymphs expected to be shorter than 10.4 mm in a sample of 850 cricket nymphs. 1 mark

b. A cricket nymph has a length of 11.2 mm. Calculate the standardised length (z score) for this cricket nymph. Round your answer to 2 decimal places. 1 mark

Data Analysis

Question 4 (9 marks)

The Milville animal sanctuary owners want to investigate the effect of the weather on attendance. Each day in March, the *maximum temperature*, in degrees Celsius, and the *attendance*, in numbers of people, were recorded.
The following summary statistics were generated.

	maximum temperature (°C)	*attendance* (numbers of people)
Mean	22.33	492.50
Standard deviation	7.28	57.26
Correlation coefficient (r)	0.87	

a. Determine the equation of the least squares line that can be used to predict attendance from maximum temperature.
Write the values of the intercept and slope of this least squares line in the appropriate boxes provided below.
Round your answers to 3 significant figures. 2 marks

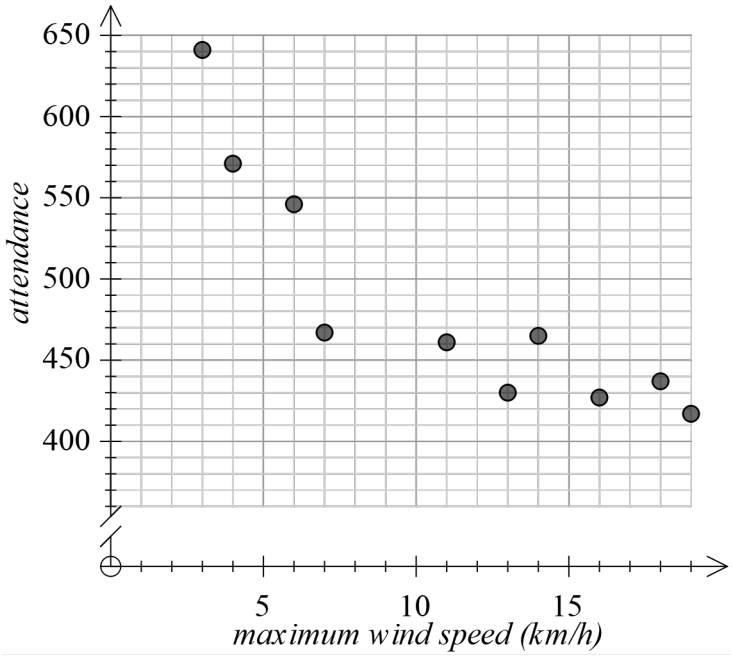
attendance = ☐ + ☐ × *maximum temperature*

Data Analysis

Question 4 (continued)

b. The relationship between *attendance* and *maximum wind speed* is also investigated. Attendance numbers and maximum wind speed (in km per hour) for 10 different days are shown in the table and scatterplot below.

maximum wind speed (km/h)	*attendance*
3	641
4	571
6	546
7	467
11	461
13	430
14	465
16	427
19	417
18	437



On the assumption that the data is linear, a least squares line is to be fitted to the data with the aim of predicting attendance numbers from maximum wind speed.
The equation of this line is

$$attendance = 610.86 - 11.23 \times maximum\ wind\ speed$$

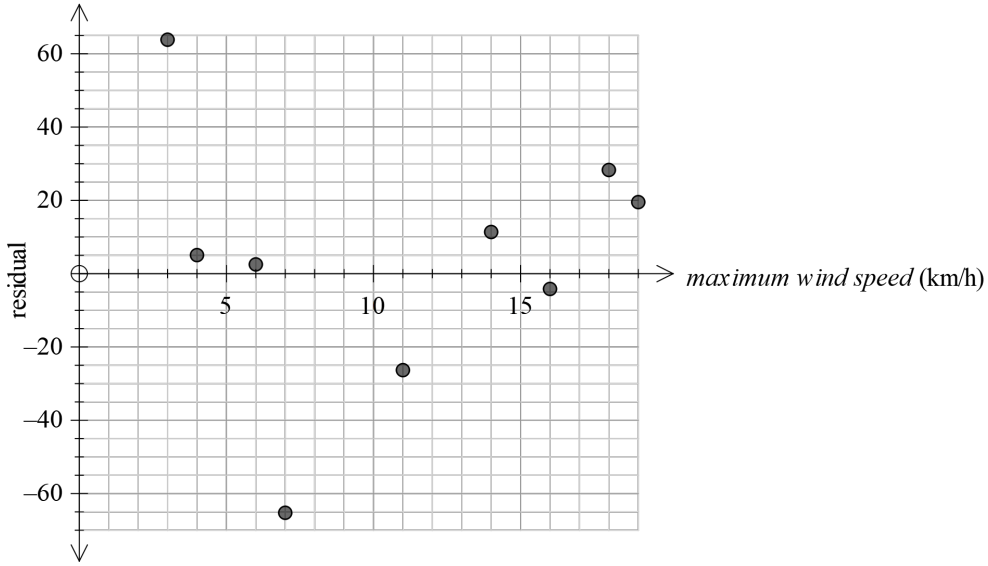
i. Draw the graph of this least squares line on the **scatterplot.** 1 mark

ii. Interpret the slope of the least squares regression line in term of the variables *attendance* and *maximum wind speed*. 1 mark

Data Analysis

Question 4 (continued)

c. A residual plot obtained when the least squares line is fitted to the data is shown below, but the residual value for when *maximum wind speed* is 13 km/h is missing.



Place a cross (X) on the residual plot above to show the residual for when the *maximum wind speed* is 13 km/h. 1 mark

The residual plot suggests that the relationship between attendance and maximum wind speed could be non-linear.
A reciprocal transformation is applied to the variable maximum wind speed and used to linearise the plot.

d. Apply this reciprocal transformation to the data and determine the equation of the least squares regression line that allows *attendance* to be predicted from $\frac{1}{maximum\ wind\ speed}$.

Round your answers to 3 significant figures. 2 marks

$attendance =$ ☐ $+$ ☐ $\times \frac{1}{maximum\ wind\ speed}$

Data Analysis

Question 4 (continued)

e. Use the least squares regression line found in part **d** to predict the value of *attendance* when the *maximum wind speed* is 26 km/h. Round your answer to one decimal place.

1 mark

_____

_____

_____

f. Explain why the prediction made in part **e** might not be reliable. 1 mark

_____

_____

Question 5 (3 marks)

a. The table below shows seasonal indices for attendance at the Milville animal sanctuary for quarters 1, 2 and 4.

Quarter number	1	2	3	4
Seasonal index	1.5	1.1		0.8

Complete the table by calculating the seasonal index for quarter 3. 1 mark

b. In 2019 a total of 43516 people attended the sanctuary in quarter 2.
Determine the deseasonalised value for attendance in that quarter. 1 mark

_____

_____

Data Analysis

Question 5 (continued)

c. A trend line was fitted to the deseasonalised set of quarterly attendance data for 2019.
The equation of the trend line is

$$deseasonalised\ attendance = 61860 - 9748 \times quarter\ number$$

Use this trend line to predict the actual attendance in quarter 4 of 2019.
Give your answer to the nearest whole number.

1 mark

_____

_____

_____

Recursion and financial modelling

Question 6 (6 marks)

The Yarragrove wool factory has 48000 balls of wool to ship out to surrounding retailers at the end of each week.
The number of balls of wool remaining at the factory after *n* weeks, B*n*, can be modelled by the recurrence relation

$$B_0 = 48000, \qquad B_{n+1} = B_n - 3500$$

a. Recursion can be used to calculate the number of balls of wool remaining at the factory after 2 weeks.
Complete the calculations by writing the appropriate numbers in the boxes. 2 marks

$B_0 =$ 48000

$B_1 =$ 48000 - ☐ = 44500

$B_2 =$ ☐ - ☐ = ☐

b. How many balls of wool does the factory ship out each week? 1 mark

_____

c. The number of balls of wool remaining at the factory after *n* weeks, B*n*, can be determined by a rule.
Complete the rule by writing the appropriate numbers in the boxes provided. 1 mark

$B_n =$ ☐ - ☐ $\times n$

Recursion and financial modelling

Question 6 (continued)

d. The factory continues to ship out the balls of wool at the same rate.
After how many weeks does the number of balls of wool remaining at the factory first become less than 15500?

2 marks

_____

_____

_____

_____

Question 7 (3 marks)

The Yarragrove wool factory takes out a loan to buy an Axminster carpet loom.
Interest on the loan is charged at a nominal rate of 6.8% per annum, compounding quarterly.

a. Calculate the effective rate of interest for this loan.
Give your answer correct to one decimal place. 1 mark

_____

_____

_____

b. The value of the loom will depreciate using the unit cost method.
The initial value is $76000.
On average the loom will be used for 2080 hours each year.
After 10 years, the depreciated value of the loom is $68928.

Show that the unit cost depreciation rate is 34 cents per hour of use. 1 mark

_____

_____

_____

_____

Recursion and financial modelling

Question 7 (continued)

c. Suppose the reducing balance method is used to depreciate the loom instead of the unit cost method.
It will depreciate at a rate of 8% per annum of the reducing balance.
Find, correct to the nearest cent, the depreciated value of the loom after 20 years.

1 mark

_____

_____

_____

_____

Question 8 (3 marks)

The Yarragrove wool factory provides a bursary of $1380 each year for a local resident to attend the annual Knitting Forum.
The factory invests $32860 in a perpetuity to provide the $1380 each year.
Interest is calculated yearly.

a. What annual interest rate would be required?
Round your answer to one decimal place.

1 mark

_____

_____

Recursion and financial modelling

Question 8 (continued)

b. The factory decides that a better way to raise funds is to invest the $32860 in an annuity earning interest at the rate of 4.56 % per annum compounding monthly.
A payment of $240 is to be withdrawn every month until the annuity runs out.
The final payment is smaller than the others.
Calculate the value of the final payment, correct to the nearest cent. 1 mark

_____

_____

_____

_____

c. Let A_n be the amount left in the annuity after n months.

Write down the recurrence relation, in terms of A_0, A_{n+1} and A_n , that could be used to represent the amount remaining in the annuity after n months.

1 mark

_____

_____

END OF SECTION A

Section B – Modules

Instructions for Section B

Select one modules and answer all questions within the selected module. Write using blue or black pen. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Contents **Page**

Module 1 - Matrices

Question 1 (6 marks)

Participants at Crafter's Forum are classified into four different interest groups: Quilters (Q), Painters (P), Jewelers (J) and Sculptors (S).
Matrix I below shows the number of participants belonging to each interest group.

$$I = \begin{bmatrix} 40 \\ 60 \\ 20 \\ 40 \end{bmatrix} \begin{matrix} \text{Q} \\ \text{P} \\ \text{J} \\ \text{S} \end{matrix}$$

a. What is the order of matrix I ? 1 mark

_____

b. How many participants are there at the Crafter's Forum?

1 mark

_____

Crafter's Forum offers four different classes, A, B, C, and D.
Each class contains a proportion of participants from each interest group as shown in the matrix R below.

$$R = \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \end{matrix} \\ \begin{bmatrix} 0.2 & 0.25 & 0.35 & 0.2 \end{bmatrix}$$

The matrix product $M = I \times R$ can be used to show the number of participants from each interest group in each class.

c. **i.** Complete matrix M, shown below, by writing in the missing elements. 1 mark

$$M = \begin{bmatrix} 8 & 10 & 14 & 8 \\ 12 & 15 & \square & 12 \\ 4 & 5 & 7 & 4 \\ 8 & \square & 14 & 8 \end{bmatrix}$$

ii. How many jewelers are in class C? 1 mark

_____

Module 1 - Matrices

Question 1 (continued)

The fees charged to participate in Crafter's Forum are different for each group. Quilters pay $80 each, Painters $70 each, Jewelers $65 each and Sculptors $95 each.
The matrix below shows these amounts.

$$F = \begin{matrix} Q & P & J & S \end{matrix} \\ \begin{bmatrix} 80 & 70 & 65 & 95 \end{bmatrix}$$

The fees collected in each class can be found using a matrix calculation.

d. **i.** Write down a matrix calculation in terms of M and F that results in a matrix that lists the fees collected in each class.

_____

1 mark

ii. What is the total amount of fees collected in class D?

1 mark

_____

_____

_____

_____

Question 2 (3 marks)

The Crafter's Forum runs for 7 days.
Participants can choose from three types of accommodation; single, twin-share or dormitory.

Let x = the cost, in dollars, of single accommodation.
Let y = the cost, in dollars, of twin-share accommodation.
Let z = the cost, in dollars, of dormitory accommodation.

Three linear equations can be used to calculate the cost, in dollars, of each type of accommodation.

$$\begin{aligned} 2x + y + 3z &= 5640 \\ x + 2y + z &= 3840 \\ x + y + z &= 2880 \end{aligned}$$

a. These equations can be written equivalently in matrix form. Complete the information in the matrix below by filling in the missing two elements.

1 mark

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & \_\_ & 1 \\ \_\_ & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5640 \\ 3840 \\ 2880 \end{bmatrix}$$

This matrix equation can be solved in the following way.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & a & 5 \\ 0 & 1 & -1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 5640 \\ 3840 \\ 2880 \end{bmatrix}$$

b. Determine the value of a shown in the matrix equation above. 1 mark

_____

c. Claudia and Aukie each pay for twin-share accommodation and Trang pays for dormitory accommodation.
How much, in dollars, do the three women pay for accommodation altogether? 1 mark

_____

_____

_____

Module 1 - Matrices

Question 3 (3 marks)

Participants at Crafter's Forum are given four options for the soup accompanying the evening meal, Eggplant (E), Kumara (K), Lentil (L) and Vegetable (V).
The caterers note that the participants change their preference from one evening to the next according to the transition matrix shown below.

	this evening				
	E	K	L	V	
	0.1	0.5	0.2	0.3	E
	0.2	0.1	0.7	0.1	K
	0.1	0.2	0.0	0.4	L
	0.6	0.2	0.1	0.2	V

next evening

The number of participants choosing each option on the first night is shown in the matrix below.

70	E
30	K
20	L
40	V

a. How many participants are expected to choose Kumara on the sixth night?
Round your answer to the nearest whole number.

1 mark

_____

b. Of the participants expected to choose Lentil soup on the second night, what percentage had chosen Vegetable soup on the first night?
Write your answer to the nearest whole percentage.

1 mark

_____

_____

_____

Module 1 - Matrices

Question 3 (continued)

c. Dessert is offered every evening.

The caterers find that, on any evening, 120 participants want dessert (D) and the remaining 40 participants do not want dessert (N).
The transition matrix below shows how the 160 participants are expected to change their choice each evening between wanting dessert and not wanting dessert.

$$T = \begin{array}{c} \text{this evening} \\ \begin{array}{cc} \text{D} & \text{N} \end{array} \\ \begin{bmatrix} 0.8 & a \\ b & c \end{bmatrix} \end{array} \begin{array}{l} \text{D} \\ \text{N} \end{array} \quad \text{next evening}$$

Write down the values of a, b and c in the boxes provided. 1 mark

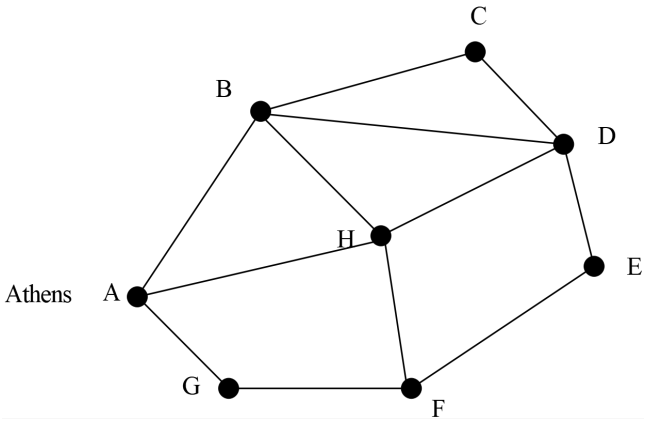
$a =$ $b =$ $c =$

End of Module 1: Matrices

Module 2: Networks and decision mathematics

Question 1 (6 marks)

Jim and Viv are visiting some Greek islands. There are seven islands, B, C, D , E, F, G and H under consideration. They are to start their journey from the port of Athens (A).
In the network diagram below, the ports of Athens and the seven islands are shown as nodes, with the edges representing the ferry routes between them.



a. What is the degree of vertex D?

_____

1 mark

b. Jim wants to visit each island but use every ferry route only once. He will begin his journey in Athens.

i. At which island will his journey finish?

_____

1 mark

ii. What is the mathematical name given to this type of path?

_____

1 mark

iii. How many of the ports (including those at the start and finish) will he see on exactly two occasions?

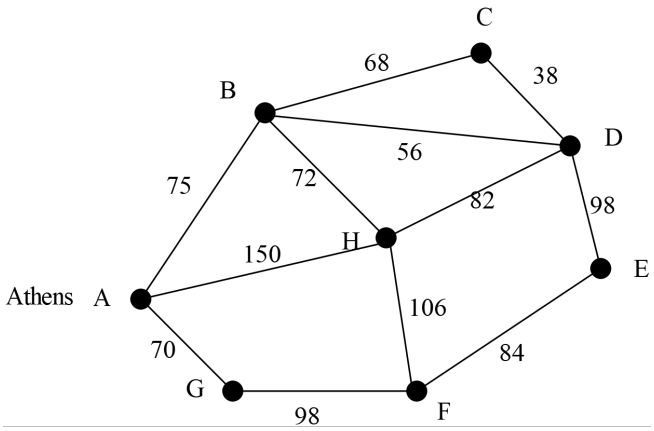
_____

1 mark

Module 2: Networks and decision mathematics

Question 1 (continued)

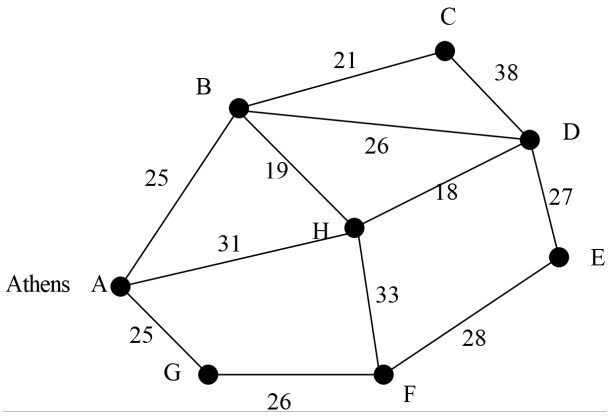
The cost, in dollars, of a one-way ticket for each of the ferry routes is shown on the diagram below.



Viv also begins her journey at Athens, but only wants to visit island E.

c. What is the least cost, in dollars, for her to get from Athens to island E? 1 mark

The distances, in kilometres, of the ferry routes between each port are shown on the diagram below.



A new ferry with a deeper hull is to be built. This will require a channel to be dug along any route the ferry needs to travel.

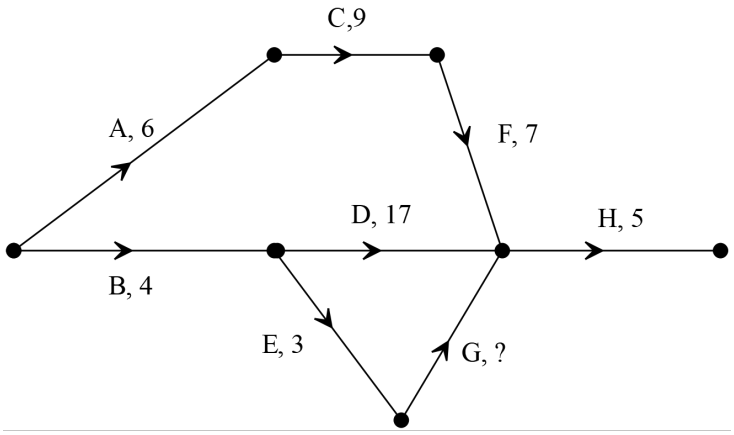
d. What is the least length, in kilometres, of channel that needs to be dug so that all the islands are reachable from Athens?

1 mark

Module 2: Networks and decision mathematics

Question 2 (3 marks)

For maintenance, eight activities must be carried out on the ferry when it docks in Athens.
The directed network below shows these activities and their completion time in hours.
The minimum completion time for the maintenance is 28 hours.



a. What is the earliest starting time for activity C?

1 mark

_____

b. Given that the earliest starting time for activity H is 23 hours, what is the duration of activity G?

1 mark

_____

c. What is the float time for activity D? 1 mark

_____

Module 2: Networks and decision mathematics

Question 3 (3 marks)

Athena, George, Makis and Yanis work on the ferry unloading the luggage.
The luggage is stored in four storerooms, U, V, W and X.
The time taken, in minutes, for each of these workers to unload the luggage from each of these storerooms is shown in the table below.

	Athena	**George**	**Makis**	**Yanis**
U	29	24	25	28
V	34	29	24	41
W	32	29	23	30
X	41	29	29	38

Each worker will be allocated a storeroom so that the total time taken to unload is a minimum.
The Hungarian algorithm is to be used to find the optimal allocation.
The first two steps of **row reduction** and **column reduction** result in the following table.

	Athena	**George**	**Makis**	**Yanis**
U	0	0	1	0
V	5	5	0	13
W	4	6	0	3
X	7	0	0	5

a. Only 3 horizontal or vertical lines are needed to cover all zero elements.
Draw 3 such lines on the table **above**.

1 mark

(answer on table above)

Module 2: Networks and decision mathematics

Question 3 (continued)

An optimal allocation cannot be made yet.

b. Continue the steps of the Hungarian algorithm to find the optimal allocation.
Write down the storeroom that each worker should unload to minimise the total time taken.

1 mark

	Storeroom
Athena	
George	
Markis	
Yanis	

c. Athena finds that she can reduce her time unloading storeroom W to 28 minutes.
The overall minimum time to unload is now reduced by reallocating the storerooms.
How long, in minutes, will the overall minimum time now be?

_____

_____

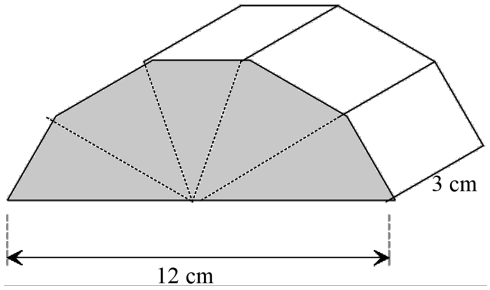
1 mark

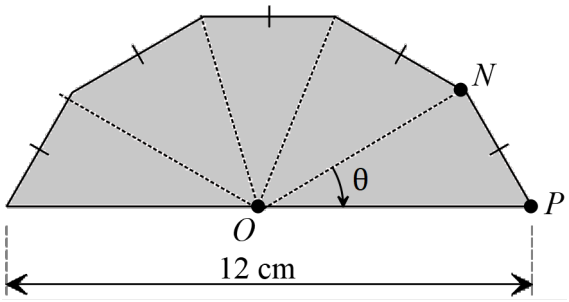
End of Module 2: Networks and decision mathematics

Module 3: Geometry and measurement

Question 1 (4 marks)

The Yarragrove artisan cheese factory sells blue cheese in a cardboard packet.
The packet is a prism. The base is half of a regular decagon. The longest straight edge of the base is 12 cm as shown.
Angle NOP = θ.





a. Show that the size of angle θ is 36°.

1 mark

_____

_____

b. Determine the area, in cm^2, of triangle NOP.
Give your answer correct to 2 decimal places.

1 mark

_____

_____

_____

Module 3: Geometry and measurement

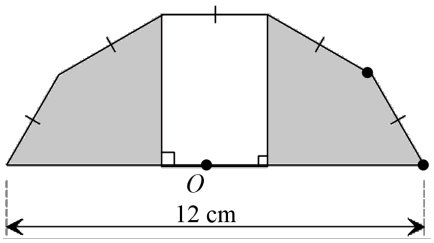
Question 1 (continued)

The height of the packet is 3 cm.

c. Calculate the volume, in cm^3, of the packet correct to one decimal place.

1 mark

A label is to be stuck on the base of the packet.
The width of the label is equal to the width of one of the equal sides of the base. The label extends from that side to the longest side of the base as shown.

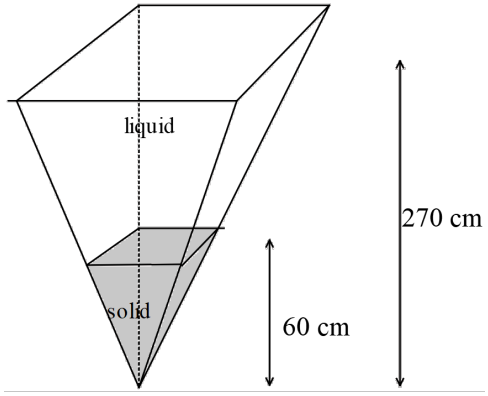


d. Calculate the area, in cm^2, of the label.
Give your answer correct to one decimal place. 1 mark

Module 3: Geometry and measurement

Question 2 (2 marks)

At the cheese factory, a filter is filled to the top with a mixture of milk solids and liquids.
The filter is an inverted right pyramid as shown. It has a vertical height of 270 cm.
After separating, the solids come to a vertical height of 60 cm. The remaining space in the filter is liquid.



a. Calculate the ratio of the volume of solid to the volume of the filter. 1 mark

_____

_____

b. The volume of solid in the filter is 11520 cm^3.
Calculate the volume, in cm^3, of liquid in the filter. 1 mark

_____

_____

_____

Module 3: Geometry and measurement

Question 3 (3 marks)

The Yarragrove artisan cheese factory exports cheese to Greenville.

Yarragrove is located at latitude 32° S and longitude 146° E.

Greenville is located at latitude 54° N and longitude 146° E.

Assume that the radius of Earth is 6400 km.

a. Find the shortest great circle distance between Yarragrove and Greenville.
Give your answer to the nearest kilometre. 1 mark

_____

_____

_____

The factory also exports cheese to Dairyville.

Dairyville is located at latitude 20° S and longitude 26° E.

b. Show that the time difference between Yarragrove and Dairyville is 8 hours. 1 mark

_____

_____

Bea is a sales representative at the factory. She will fly from Yarragrove to Dairyville.

The flight will take 14 hours and 13 minutes.

The flight leaves Yarragrove at 10:23 am on Monday.

c. On what day and at what time does Bea arrive in Dairyville? 1 mark

_____

_____

_____

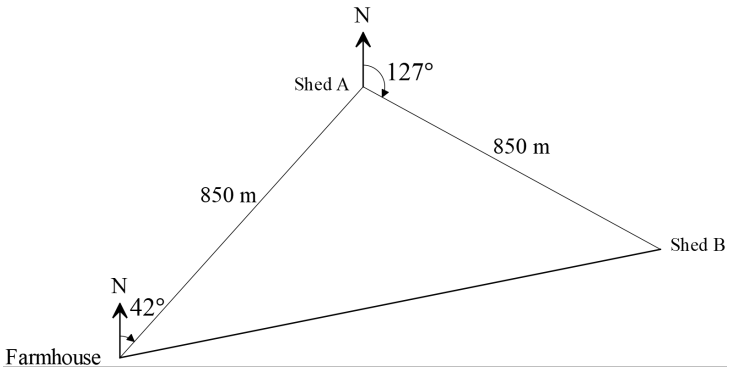
Module 3: Geometry and measurement

Question 4 (3 marks)

The Yarragrove dairy farm has two milking sheds and a farmhouse.
Shed A and shed B can both be seen from the farmhouse.

Shed A is 850 metres from the farmhouse on a bearing of 042°.

Shed B is 850 metres from shed A on a bearing of 127°.



a. Determine the bearing of the farmhouse from shed A. 1 mark

_____

_____

b. Determine the bearing of shed B from the farmhouse. 1 mark

_____

_____

c. Calculate the direct distance, in metres, from the farmhouse to shed B.
Give your answer correct to two decimal places. 1 mark

_____

_____

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_____

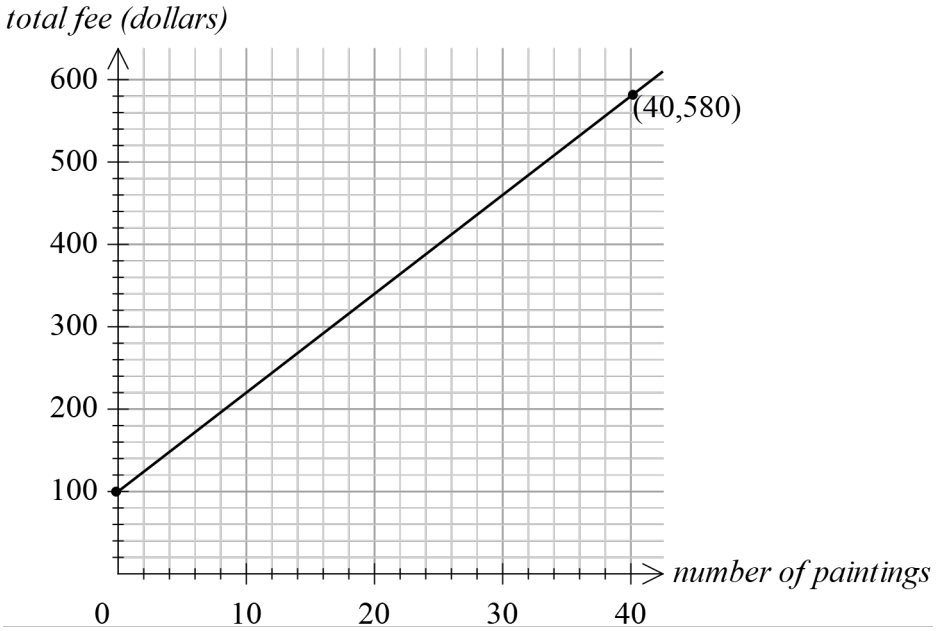
End of Module 3: Geometry and measurement

Module 4 : Graphs and relations

Question 1 (5 marks)

Milville Estate is a winery and art gallery.
The gallery is to hold an exhibition and needs a curator to hang the paintings.
One curator, David, charges a flat fee plus an amount per painting.

The graph below shows David's total fee in dollars.



a. Show that the amount David charges per painting is \$12. 1 mark

_____

_____

b. Complete the following linear relation for David's fee in dollars. 1 mark

David's fee = [ ] + [ ] x *number of paintings*

Another curator, Sue, has no flat fee but charges at a rate of \$18 per painting.

c. Draw the graph of the line representing Sue's fee on the **graph above**. 1 mark

(Answer on graph above)

Module 4 : Graphs and relations

Question 1 (continued)

d. What is the minimum number of paintings required to be hung for David's fee to be less than Sue's fee? 1 mark

_____

_____

e. If a total of 32 paintings are to be hung for the exhibition, what is the difference, in dollars, between David's fee and Sue's fee?

1 mark

_____

_____

Question 2 (3 marks)

A tank of Helium gas is required to fill balloons for the opening night of the exhibition.
Gas pressure is measured in pascal (Pa).
At room temperature, the pressure, P, of the gas for various volumes, V, in cm^3 is shown in the table below.

$\boldsymbol{V}$ (cm3)	2	4	8	10	20
$\boldsymbol{P}$ (Pa)	84	42	21	16.8	8.4

It is found that pressure and volume of the gas are related according to the rule

$$P = \frac{k}{V}$$

a. Complete the table below by filling in the two missing values. 1 mark

$\frac{\mathbf{1}}{\boldsymbol{V}}$	0.5		0.125	0.1	
$\boldsymbol{P}$	84	42	21	16.8	8.4

A graph of P against $\frac{1}{V}$ produces a straight line.

b. **Show** that the value of $\boldsymbol{k}$ is 168. 1 mark

_____

_____

c. Calculate the volume V, in cm^3, of the gas when the pressure, P, is 16 Pa. 1 mark

_____

_____

Module 4 : Graphs and relations

Question 3 (4 marks)

Milville Estate winery produces two types of wine, Zesty and Dusty.

- Let x be the number of bottles of Zesty produced daily.
- Let y be the number of bottles of Dusty produced daily.
- A maximum of 120 bottles in total can be produced each day.
- The number of bottles of Dusty produced must be at least twice the number of bottles of Zesty.

Inequalities 1 to 4 represent these constraints.

Inequality 1	$\boldsymbol{x \geq 0}$
Inequality 2	$\boldsymbol{y \geq 0}$
Inequality 3	$\boldsymbol{x + y \leq 120}$
Inequality 4	$\boldsymbol{y \geq 2x}$

a. Each bottle of Zesty contains 20 grams of local grapes and each bottle of Dusty contains 10 grams of local grapes.
Each day a minimum of 400 grams of local grapes must be used.

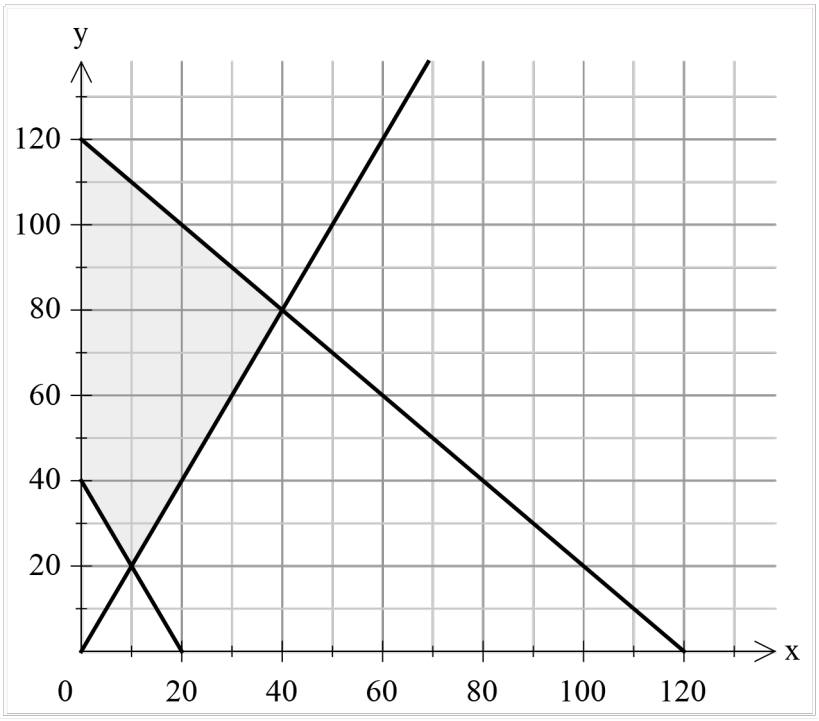
Use this information to write inequality 5 1 mark

Inequality 5

Module 4 : Graphs and relations

Question 3 (continued)

The graph below shows the feasible region (shaded) defined by these constraints.



A profit of $22 is made on each bottle of Zesty.
A profit of $20 is made on each bottle of Dusty.

b. Determine the maximum daily profit that can be made on sales of the wines.

1 mark

_____

_____

c. Find the maximum profit to be made in a day when only a total of 85 bottles are sold.

_____

_____

Module 4 : Graphs and relations

Question 3 (continued)

After a time, Milville Estate wines become very popular.

The profit on a bottle of Zesty increases by $8. The profit on a bottle of Dusty also increases. A new maximum profit can now be made.

On one particular day, this maximum profit is made by selling 30 bottles of Zesty and 90 bottles of Dusty.

d. **i.** What is the profit now made on a bottle of Dusty? 1 mark

_____

_____

_____

ii. Calculate the new maximum daily profit to be made on sales of the wines. 1 mark

_____

_____

End of Module 4 : Graphs and relations

End of 2020 VCE Further Mathematics Written Examination 2
Question and Answer Book

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FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index$= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core: Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1: Matrices

determinant of a 2×2 matrix:	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix:	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$
recurrence relation:	$S_0 =$ initial state, $\quad S_{n+1} = TS_n + B$

Module 2: Networks and decision mathematics

Euler's formula:	$v + f = e + 2$

Module 3: Geometry and measurement

area of a triangle:	$A = \frac{1}{2}bc\sin(\theta^0)$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle:	$2\pi r$
length of an arc:	$r \times \frac{\pi}{180} \times \theta^0$
area of a circle:	πr^2
area of sector	$\pi r^2 \times \frac{\theta^0}{360}$
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a prism:	area of base $\times$ height
volume of a pyramid:	$\frac{1}{3}$ $\times$ area of base $\times$ height

Module 4: Graphs and relations

gradient (slope) of a straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line:	$y = mx + c$

END OF FORMULA SHEET