



THE SCHOOL FOR EXCELLENCE (TSFX)

VCE FURTHER MATHEMATICS UNIT 3 & 4

WRITTEN EXAMINATION 1 – 2020

Reading Time: 15 minutes
Writing Time: 1 hour 30 minutes

MULTIPLE CHOICE QUESTION BOOK

Letter

**Student
Number:**

Structure of Book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of Modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A – Core	30	30			30
B – Modules	40	10	4	1	10
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials Supplied

- Question and answer book of 38 pages.
- Formula sheet.
- Answer sheet for multiple choice questions.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

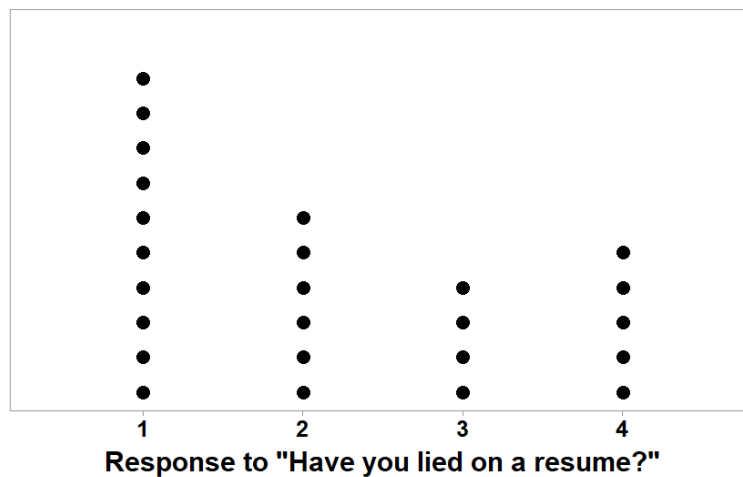
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Use the following information to answer Questions 1 and 2.

The dotplot below shows the responses of 25 people to the question “Have you lied on a resume?” The responses were recorded as:

- 1 = “No”.
- 2 = “Yes, but no-one found out”.
- 3 = “Yes, and it was found out with no consequences”.
- 4 = “Yes, and it was found out with negative consequences”.



QUESTION 1

The percentage of people who lied on their resumes was

- A. 15%
- B. 20%
- C. 40%
- D. 60%
- E. 80%

QUESTION 2

The type of data recorded for the response to “Have you lied on a resume?” is

- A. nominal categorical data.
- B. ordinal categorical data.
- C. discrete numerical data.
- D. continuous numerical data.
- E. discrete categorical data.

QUESTION 3

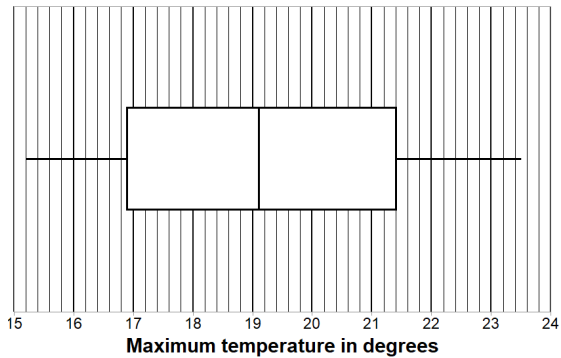
The stem and leaf plot below shows the maximum temperature, in degrees, recorded at Melbourne Olympic Park for each of the 30 days in April, 2020.

15	3	5	7		
16	4	4	5	6	9
17	7				
18	1	2	3	4	8
19	0	4	6	7	8
20	7	8			
21	2	5	7		
22	0	3			
23	1	2	4	5	

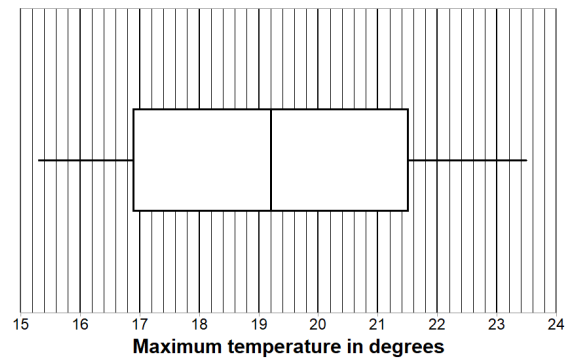
key: 21|2 = 21.2°

A boxplot that represents the same information is

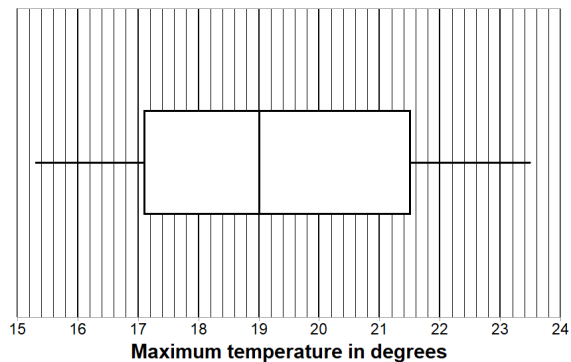
A.



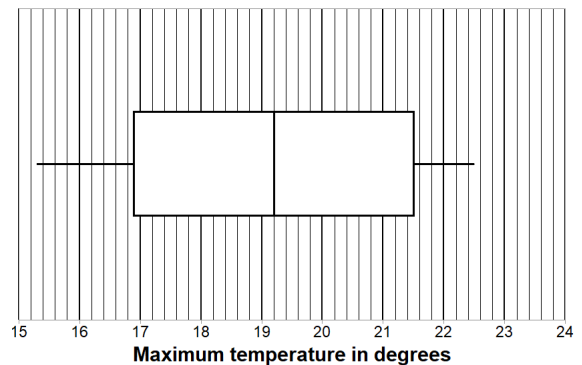
B.



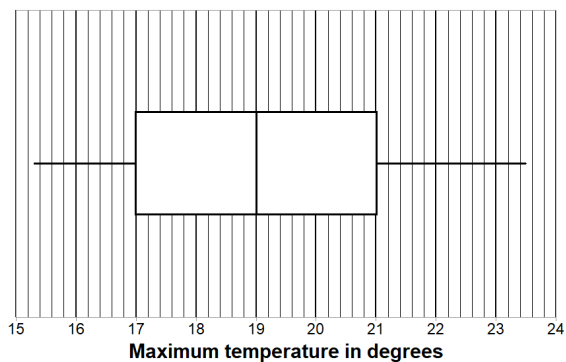
C.



D.

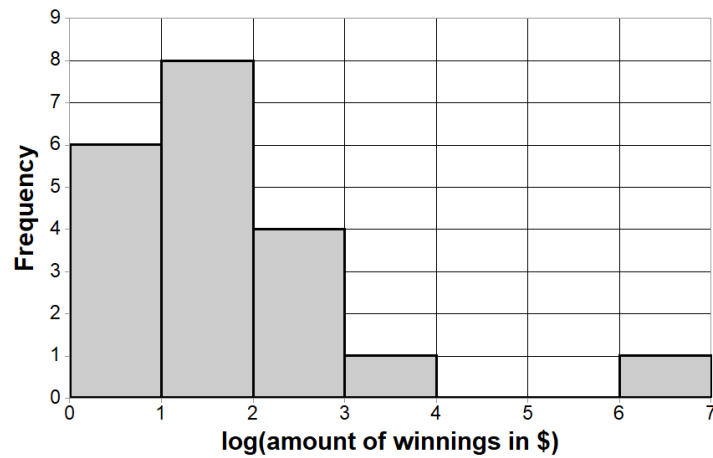


E.



QUESTION 4

The histogram below shows the $\log_{10}(\text{amount of winnings in \$})$ of 20 people who won money in a recent tattslotto draw and collected their winnings on the same day from a particular tattslotto outlet.



The number of people who won between \$100 and \$1000 in this group was

- A. 6
- B. 8
- C. 4
- D. 1
- E. 5

Use the following information to answer Questions 5 and 6.

Eleven students attend a school camp. The students' weights in kg are listed below in ascending order:

44 51 52 55 55 56 57 57 a 62 64

QUESTION 5

There is one outlier for weight in this group of students. The value of a could be

- A. 57
- B. 58
- C. 59
- D. 60
- E. 61

QUESTION 6

While on a walking trail, each student is required to carry a 6 kg backpack.

Which of the following statements is true about the combined weight of each student and their pack compared to the weight of each student?

- A. The mean weight and the standard deviation of weight will both increase by 6.
- B. The mean weight and the standard deviation of weight will both decrease by 6.
- C. The mean weight will increase by 6 and the standard deviation will decrease by 6.
- D. The mean weight will increase by 6 and the standard deviation will increase by $\sqrt{6}$.
- E. The mean weight will increase by 6 and the standard deviation will stay the same.

QUESTION 7

A pasta manufacturer produces packets of pasta with a mean weight of 500 grams. The weights of the packets vary and are normally distributed.

In a box of 250 packets of pasta, 40 packets will weigh more 512 grams.

The standard deviation of weights of pasta packets in grams will be

- A. two
- B. three
- C. four
- D. six
- E. twelve

QUESTION 8

At an athletic club the mean time taken by athletes to run 100 m is 11.1 seconds with a standard deviation of 0.7 seconds. The times for members of this club to run 100 m is normally distributed.

Five members of the club run 100 m in the following times; Scarlett in 10.8 seconds, Martin in 11.5 seconds, Louisa in 10.7 seconds, Joe in 10.1 seconds and William in 12.1 seconds.

The runner with the highest standard score for the 100 m race is

- A. William
- B. Louisa
- C. Scarlett
- D. Martin
- E. Joe

QUESTION 9

53% of the variation in a student's exam scores can be explained by the variation in the number of days absent from class.

The correlation coefficient for the association between a student's exam scores and the number of days absent from school would be closest to

- A. 0.28
- B. -0.28
- C. -0.53
- D. 0.73
- E. -0.73

QUESTION 10

The sale price of a number of houses in two different locations, Newtown and Oldsville, is recorded in dollars.

Which of the following statements could be used to justify an association between the sale prices of houses and their location?

- A. The distribution of the sale prices of the houses in both locations is positively skewed.
- B. The maximum sale price in Newtown is \$650 000, which is greater than the maximum sale price of \$575 000 in Oldsville.
- C. The median sale price in Newtown is \$520 000 which is greater than the median sale price of \$480 000 in Oldsville.
- D. The distribution of sale prices in Newtown has an outlier whereas there are no outliers in the distribution of sale prices in Oldsville.
- E. The range of sale prices in Newtown is \$280 000 which is similar to the range of sale prices of \$270 000 in Oldsville.

QUESTION 11

At a small rural school, a total of ten students study both English and Further Mathematics. The ten students achieved scores of 47, 25, 36, 32, 39, 31, 22, 15, 29 and 27 for Further Mathematics.

The association between the students' Further Mathematics scores (*FM score*) and their English scores (*Eng score*) has a correlation coefficient of 0.66 .

The least squares regression equation that can be used to predict a student's *FM score* from the *Eng score* is $FM\ score = 12.84 + 0.58 \times Eng\ score$.

The mean of the English scores for these ten students would be closest to

- A. 26.4
- B. 26.3
- C. 30.1
- D. 30.3
- E. 10.4

QUESTION 12

A least squares line has been fitted to a scatterplot showing points obtained from asking twelve people, who recently constructed new homes, the *size* of their houses in metres squared and the *cost* in dollars of the construction.

The least squares line aims to predict the *cost* of the construction from the *size* of the houses.

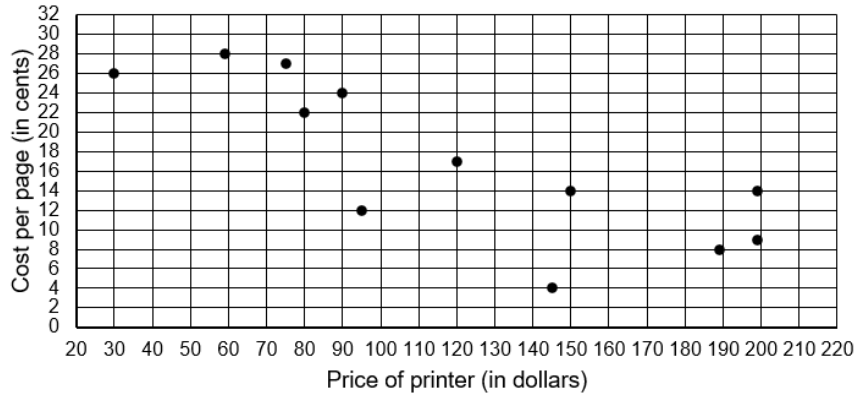
Which of the following statements will not be true?

- A. The sum of the squares of the vertical distances of the points from the line will be the least possible amount.
- B. There must be an equal number of positive and negative residuals.
- C. The gradient of the least squares line must have the same sign as the correlation coefficient.
- D. The gradient of the line will represent the increase in *cost* of construction for every additional one metre squared in *size*.
- E. The least squares line will pass through a point with the coordinates (mean of *size*, mean of *cost*).

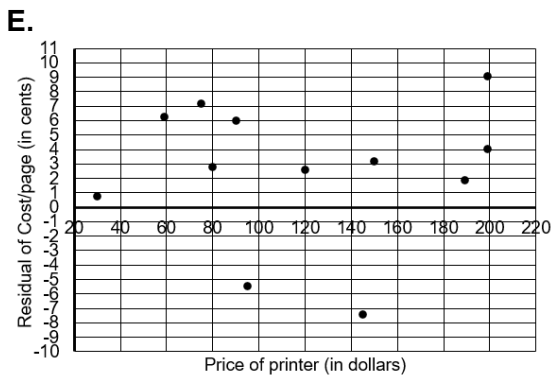
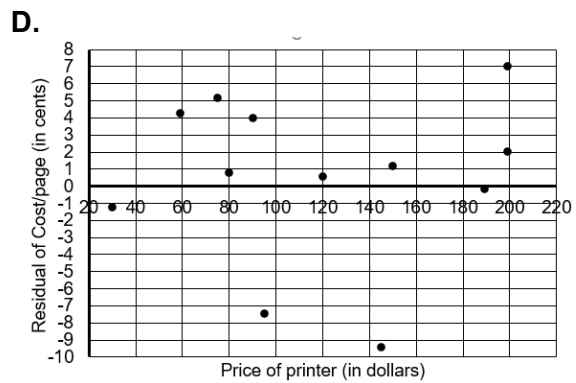
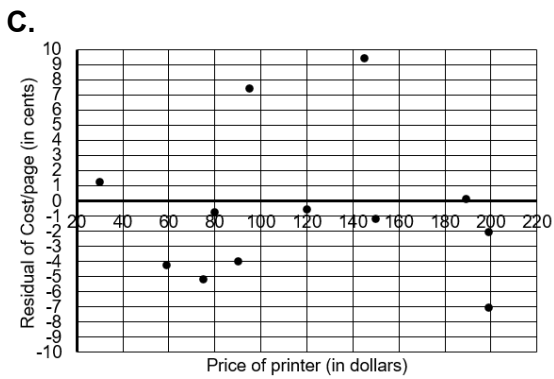
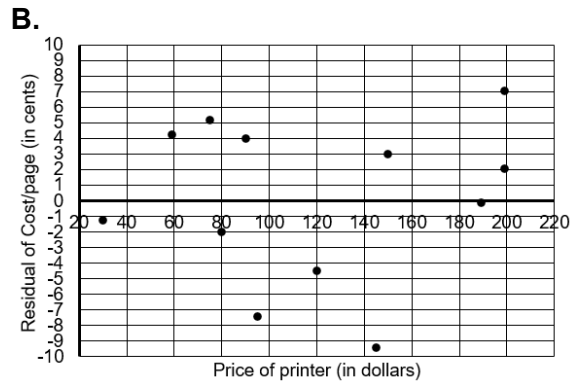
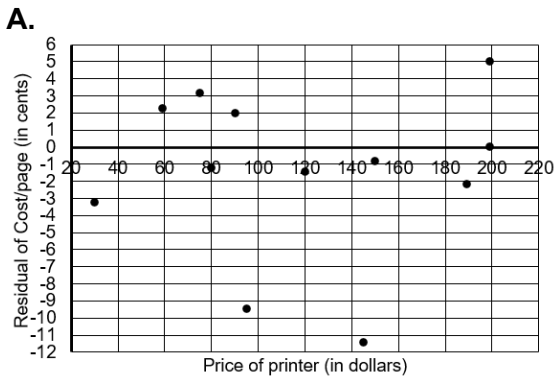
QUESTION 13

A scatterplot is shown below. The least squares line to predict the *Cost per page (in cents)* from the *Price of printer (in dollars)* for this data has the equation

$$\text{Cost per page (in cents)} = 30.83 - 0.12 \times \text{Price of printer (in dollars)}.$$

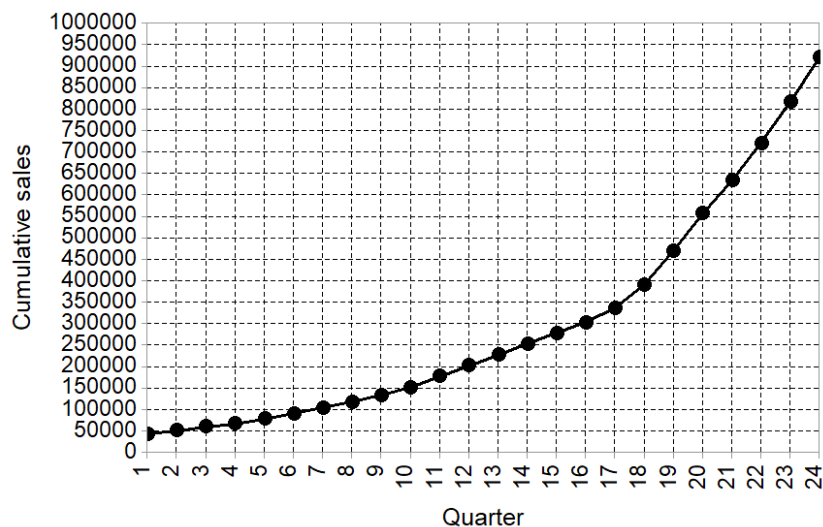


The residual plot produced from this scatterplot would be



QUESTION 14

The time series plot below shows the *cumulative sales* of Tesla cars worldwide for the period Quarter 1, 2014 (quarter = 1) through to Quarter 4, 2019 (quarter = 24).



The relationship between *cumulative sales* and *quarter* is not linear.

An appropriate transformation that could be used to linearise this association would be a

- A. log transformation of the *quarter* variable.
- B. square transformation of the *cumulative sales* variable.
- C. reciprocal transformation of the *quarter* variable.
- D. reciprocal transformation of the *cumulative sales* variable.
- E. square transformation of both *cumulative sales* and *quarter* variables.

QUESTION 15

The number of digital cameras sold worldwide (in millions) has been recorded from 2010 to 2019. The *sales (in millions)* are shown in the table below:

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Sales (in millions)	121.5	115.5	98.1	62.8	43.4	35.4	24.2	25.0	19.4	15.2

Data Source: CIPA

A logarithmic transformation of the *sales (in millions)* variable would enable the logarithm of *sales (in millions)* to be predicted from the *year*.

The relationship between the logarithm of *sales (in millions)* and *year*, correct to two significant figures would be

- A. $\text{sales}(\text{in millions}) = 220 - 0.11 \times \text{year}$
- B. $\log_{10}(\text{sales}(\text{in millions})) = 220 - 0.11 \times \text{year}$
- C. $\log_{10}(\text{sales}(\text{in millions})) = 26000 - 13 \times \text{year}$
- D. $\log_{10}(\text{sales}(\text{in millions})) = 2030 - 9.0 \times \text{year}$
- E. $\log_{10}(\text{sales}(\text{in millions})) = 1700 - 502 \times \text{year}$

QUESTION 16

Beyond Meat, a food manufacturing company, produce vegan meat substitute products. The quarterly *Revenue (in \$millions)* for Beyond Meat, during 2017 and 2018, where Quarter 1, 2017 is $t = 1$, Quarter 2, 2017 is $t = 2$, etc can be predicted using the least squares equation:

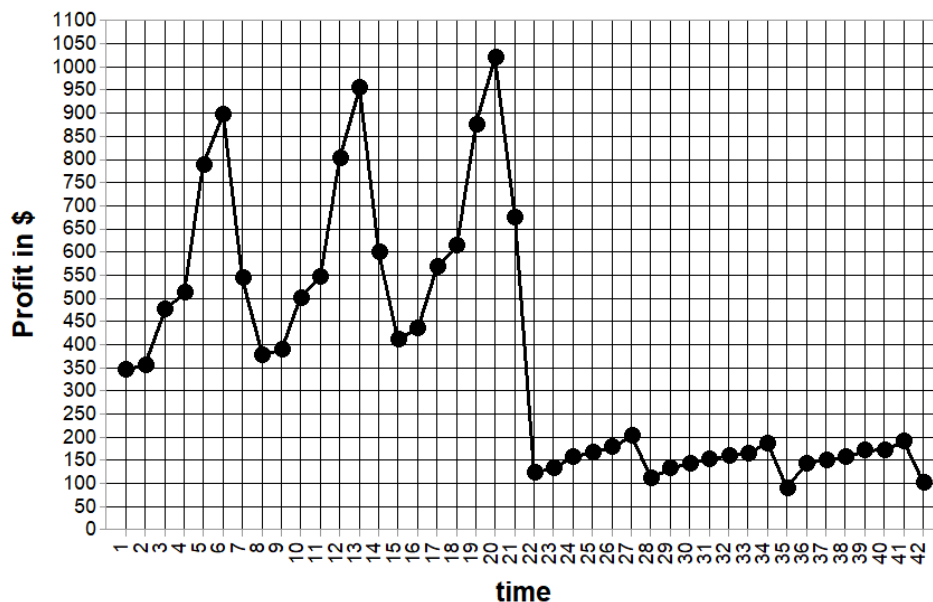
$$\frac{1}{\text{Revenue(in \$millions)}} = 0.17 - 0.016 \times t$$

In Quarter 1, 2018 the *Revenue (in \$millions)* was \$12.8 million.
Which of the following statements is **not** true?

- A. The predicted *Revenue (in \$millions)* during Quarter 3, 2017 was \$8.2 million correct to two significant figures.
- B. There would be a positive linear relationship between *Revenue (in \$millions)* and t .
- C. The *Revenue* is predicted to decrease by \$16 000 per year.
- D. The residual value for *Revenue (in \$millions)* for Quarter 1, 2018 is \$1.7 million correct to two significant figures.
- E. The least squares equation will not give a reliable prediction for the *Revenue (in \$millions)* during 2020.

QUESTION 17

The time series graph below shows the daily profit made by a coffee shop during a six week period early in 2020. For this graph, the time = 1 is Monday 2nd March, time = 2 is Tuesday 3rd March, etc.



Which of the following best describes the time series plot?

- A. A seasonal pattern with a structural change.
- B. A cyclic pattern with a structural change.
- C. A seasonal pattern with an outlier.
- D. A cyclic pattern with random fluctuations.
- E. A seasonal pattern with random fluctuations.

QUESTION 18

The maximum temperatures, in degrees celcius, below were recorded over a week long period in December:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temperature	25°	35°	32°	31°	22°	26°	29°

The two-point moving mean and four-point moving mean, both with centring, are calculated for Thursday.

Which of the following statements is true?

- A. The four-point moving mean is less than one degree more than the two-point moving median.
- B. The four-point moving mean is more than one degree more than the two-point moving median.
- C. The two-point moving mean is less than one degree more than the four-point moving median.
- D. The two-point moving mean is more than one degree more than the four-point moving median.
- E. The two-point moving mean is the same as the four-point moving median.

QUESTION 19

A tourism operator records the income (in dollars) he receives each quarter during 2019. The results are shown below:

Quarter	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Income (in dollars)	\$98 700	\$78 600		\$86 400

Unfortunately, some of the tourism operator's records have been lost and he does not know the total income made during Quarter 3. He does know that the seasonal index for Quarter 3 is 0.71.

The income for Quarter 3 would be closest to

- A. \$56 900
- B. \$654 600
- C. \$46 800
- D. \$62 400
- E. \$61 300

QUESTION 20

A small business sells sunglasses. Sales are seasonal and the seasonal indices for each quarter are shown below:

Quarter	1	2	3	4
Seasonal Index	1.4	0.6	0.5	1.5

The least squares line that predicts the deseasonalised sales of sunglasses for this business is

$$\text{deseasonalised sales} = a + b \times t$$

where $t = 1$ is Quarter 1, 2018, $t = 2$ is Quarter 2, 2018, etc.

The actual sales in Quarter 3, 2018 were 3964 and the actual sales in Quarter 4, 2019 were 12 837.

The residuals of the deseasonalised values for Quarter 3, 2018 and Quarter 4, 2019 are both zero.

The values of a and b will be closest to

- A. $a = -1360, b = 1770$
- B. $a = 21\,945, b = -2030$
- C. $a = 126, b = 7550$
- D. $a = 1770, b = -1360$
- E. $a = 7550, b = 126$

Recursion and Financial Modelling

QUESTION 21

The first four terms of a financial sequence are \$10 000, \$10 800, \$11 664, \$12 597.12.

This sequence would be best described as

- A. arithmetic growth.
- B. arithmetic decay.
- C. geometric growth.
- D. geometric decay.
- E. neither arithmetic nor geometric growth.

QUESTION 22

The balance of an account after n years can be determined using the recurrence relation

$$A_0 = 12000, A_{n+1} = A_n + 888$$

The account could be

- A. a simple interest investment at 8.88% per annum.
- B. a simple interest investment at 7.4% per annum.
- C. a compound interest investment at 8.88% per annum.
- D. a compound interest investment at 7.4% per annum.
- E. an annuity investment at 7.4% per annum with annual deposits of \$888.

QUESTION 23

Which of the following interest rates is not equivalent to 4.8% per annum?

- A. 2.4% per half year
- B. 1.2% per quarter
- C. 0.2% per month
- D. $\frac{6}{65}\%$ per week
- E. $\frac{24}{1825}\%$ per day

QUESTION 24

A carpenter has a utility that he uses for his work. He purchases the utility for \$30 000.

The utility is depreciated using the unit cost method based on the number of kilometres travelled. On average, each year he travels 40 000 km and, after 3 years, the utility will have a book value of \$12 000.

A recurrence relation that would give the book value, in dollars, after n kilometres could be

- A. $U_0 = 30000, U_{n+1} = U_n - 15$
- B. $U_0 = 30000, U_{n+1} = U_n - 6000$
- C. $U_0 = 30000, U_{n+1} = U_n - 0.1$
- D. $U_0 = 30000, U_{n+1} = U_n - 0.15$
- E. $U_0 = 30000, U_{n+1} = 0.85 \times U_n$

Use the following amortisation table to answer Question 25.

Repayment number	Repayment	Interest	Principal Reduction	Balance of Loan
-	-	-	-	\$2280
1	\$578	\$12.54	\$565.46	\$1714.54
2	\$578	\$9.43	\$568.47	\$1145.97
3	\$578	\$6.30	\$571.70	\$574.27
4	a	b	c	\$0

QUESTION 25

The amortisation table records the payments made on a loan that is finalised over a four month period.

The last line of the table is incomplete. The values of a , b and c in this table would be

- A. $a = \$3.16, b = \$577.43, c = \$574.27$
- B. $a = \$578.00, b = \$3.16, c = \$574.27$
- C. $a = \$571.70, b = \$3.16, c = \$574.27$
- D. $a = \$574.27, b = \$3.16, c = \$571.70$
- E. $a = \$577.43, b = \$3.16, c = \$574.27$

QUESTION 26

A recurrence relation that calculates the balance of an annuity investment after n months is

$$V_0 = 10000, \quad V_{n+1} = 1.00325 \times V_n + 270.$$

The effective rate of interest per annum for this annuity investment is closest to

- A. 3.25%
- B. 3.30%
- C. 3.90%
- D. 3.97%
- E. 4.03%

QUESTION 27

Daniel has a perpetuity at 5.1% per annum that pays him an amount of \$348.50 per month. He needs \$500 per month, so he changes the perpetuity to an annuity also at 5.1% compounding monthly.

After he makes this change, how long will the annuity last before it reaches a zero balance?

- A. 275 months
- B. 278 months
- C. 281 months
- D. 282 months
- E. 285 months

QUESTION 28

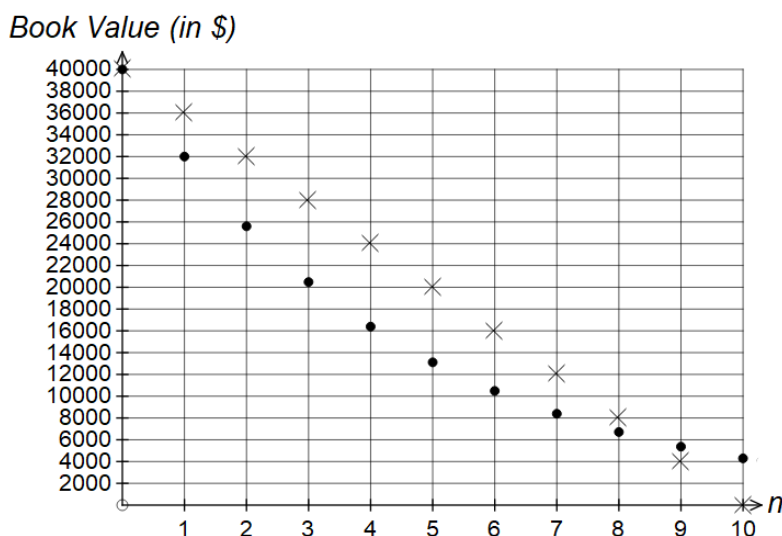
Manuel has an annuity that is earning 8.5% per annum compounding monthly. Every month he receives \$4000 from the account. Twelve months after the annuity started Manuel has \$460 268.37 remaining in the account.

Three years after the annuity started, the total amount of interest that Manuel has earned in his annuity account is closest to

- A. \$47 760
- B. \$116 240
- C. \$115 520
- D. \$124 630
- E. \$144 000

QUESTION 29

The graph below shows the book value after n years for two different alternatives for depreciating a \$40 000 delivery van. Option A is represented by crosses, whereas Option B is represented by dots.



Which of the following statements about this graph would be true?

- A. Option B represents reducing balance depreciation at 15% per annum.
- B. Option A represents flat rate depreciation at 20% per annum.
- C. If the van is to be replaced after three years, Option B would be more tax effective.
- D. If the van is to be replaced after eight years, Option A would be more tax effective.
- E. The annual decrease in book value is constant for Option B.

QUESTION 30

Gordon and Samantha are buying a new house. They need to borrow \$480 000 from the bank to make the purchase.

The bank has offered them a loan at 3.85% per annum compounding monthly. They have the option of paying interest only on their loan for the first two years, followed by equal monthly payments of both interest and principal. The loan is to be finalised over a 30 year period in total.

Gordon and Samantha decide that they would rather pay both principal and interest from the start of the loan.

Given the same interest rate and equal monthly payments over a 30 years period, the total amount that they would save over the entire loan period would be closest to

- A. \$9 000
- B. \$12 000
- C. \$14 000
- D. \$25 000
- E. \$31 000

END OF SECTION A

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

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Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1: Matrices

Before answering these questions, you must select the “Matrices” module on the answer sheet for multiple choice questions and write the name of the module in the space provided.

QUESTION 1

$3 \begin{bmatrix} 5 & 4 \\ -6 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 \\ -4 & -1 \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 17 & 6 \\ 10 & 11 \end{bmatrix}$

B. $\begin{bmatrix} 13 & 6 \\ -10 & 11 \end{bmatrix}$

C. $\begin{bmatrix} -13 & 18 \\ -26 & 7 \end{bmatrix}$

D. $\begin{bmatrix} -126 & -114 \\ -108 & -126 \end{bmatrix}$

E. $\begin{bmatrix} 13 & 6 \\ -26 & 9 \end{bmatrix}$

QUESTION 2

The determinant of the matrix $\begin{bmatrix} 5 & -4 \\ 6 & -7 \end{bmatrix}$ is

A. 2

B. 11

C. -11

D. -59

E. 59

QUESTION 3

The matrix $\begin{bmatrix} T \\ H \\ E \\ E \\ Y \\ E \\ S \end{bmatrix}$ is to be changed to the matrix $\begin{bmatrix} T \\ H \\ E \\ Y \\ S \\ E \\ E \end{bmatrix}$ using a permutation matrix.

The number of different permutation matrices that could achieve this change is

- A. one
- B. two
- C. three
- D. six
- E. nine

QUESTION 4

Matrix A is a 3×4 matrix, Matrix B is a 2×3 matrix, Matrix C is a 4×2 matrix and Matrix D is a 3×2 matrix.

Which of the following matrices may not be defined?

- A. ACB
- B. $(BD)^{-1}$
- C. $(DB)^3$
- D. $(AC)^2$
- E. $AC - D$

QUESTION 5

The element in row i and column j of matrix A is a_{ij} .

Matrix A is an $m \times n$ matrix, where elements of the matrix are defined by the rule $3i - j$.

Which of the following statements would not be true for matrix A ?

- A. If $m = n$, the elements on the leading diagonal could also be calculated using $i + j$.
- B. If $m < n$, there could be negative elements in Matrix A .
- C. If $m > n$, there will be no negative elements in Matrix A .
- D. The transpose of Matrix A would have the rule $3j - i$.
- E. If $n = 4$ there could be two zero elements in Matrix A .

QUESTION 6

Kyah buys 4 packets of paper and 12 manilla folders and this costs him \$33.80. On another day he buys 5 packets of paper and 15 manilla folders costing \$42.25.

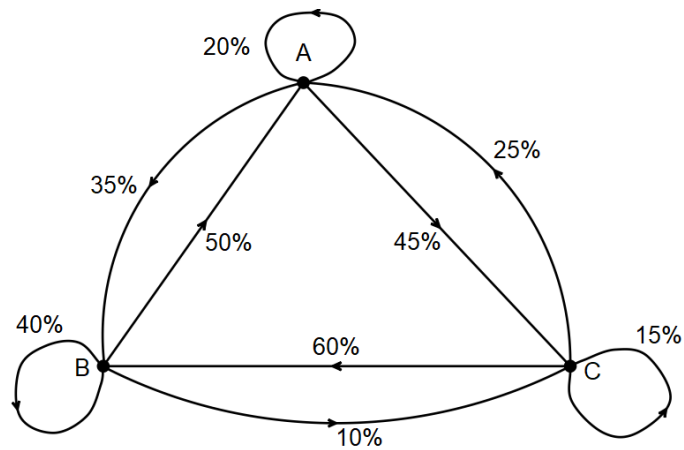
He wants to use matrices to solve two simultaneous equations to determine the cost of each packet of paper and each manilla folder.

Which of the following statements is true?

- A. The packets of paper must cost \$5.90 each and the manilla folders are 85 cents each.
- B. The packets of paper must cost \$5.15 each and the manilla folders are \$1.10 each.
- C. The simultaneous equations cannot be solved as the equations are dependent.
- D. The simultaneous equations cannot be solved as the equations are inconsistent.
- E. The simultaneous equations cannot be solved as the matrix is regular.

QUESTION 7

A transition diagram is shown below:



There are 1000 people who transition according to the diagram above. In the long run the number of people at B will be closest to

- A. 517
- B. 429
- C. 340
- D. 276
- E. 231

QUESTION 8

A tourism operator runs tours seven days a week. The number of adults (A) and children (C) who take the tour on each day one particular week is given by matrix

$$W = \begin{array}{cccccc} & M & Tu & W & Th & F & Sa & Su \\ \begin{bmatrix} 23 & 32 & 45 & 65 & 107 & 125 & 154 \\ 2 & 4 & 7 & 4 & 11 & 23 & 35 \end{bmatrix} & A & & & & & & C \end{array}$$

Each adult who takes the tour will pay \$50 and each child will pay \$30. 25% of each tour price is used for entry into an attraction.

Which of the following matrix expressions will not calculate the total amount paid to the attraction this week by the tourism operator?

A. $[12.5 \ 7.5] \times W \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

B. $[50 \ 30] \times W \times \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$

C. $[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \times W^T \times \begin{bmatrix} 50 \\ 30 \end{bmatrix} - [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \times W^T \times \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix} \times \begin{bmatrix} 50 \\ 30 \end{bmatrix}$

D. $\begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \times \begin{bmatrix} 50 \\ 30 \end{bmatrix} \times W \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

E. $[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \times W^T \times \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \times \begin{bmatrix} 50 \\ 30 \end{bmatrix}$

QUESTION 9

Due to extended periods at home many people have been watching streaming services to fill in time. A group of 1000 people were asked which streaming service or services, Netflix (N), Stan (S) or Amazon Prime (A) they were watching in March.

Their answers are contained in the matrix $V_{Mar} = \begin{bmatrix} 820 \\ 110 \\ 235 \end{bmatrix} \begin{matrix} N \\ S \\ A \end{matrix}$.

The numbers in this matrix do not add up to 1000 because some people had multiple streaming services.

For all three services, 80% of the people who had each streaming service in March also had the same streaming service in April. Additionally:

- for those customers who had Netflix in March, $P\%$ took up Stan and $P\%$ took up Amazon Prime in April.
- for those customers who had Stan in March, $Q\%$ took up Netflix and $Q\%$ took up Amazon Prime in April.
- for those customers who had Amazon Prime in March, $R\%$ took up Stan and $R\%$ took up Netflix in April.

The number of services in April is given by the matrix $V_{Apr} = \begin{bmatrix} 747 \\ 381 \\ 478 \end{bmatrix} \begin{matrix} N \\ S \\ A \end{matrix}$.

The values of P , Q and R respectively are

- A.** $P = 30$, $Q = 60$, $R = 20$
- B.** $P = 30$, $Q = 40$, $R = 20$
- C.** $P = 50$, $Q = 40$, $R = 20$
- D.** $P = 30$, $Q = 40$, $R = 30$
- E.** $P = 0.3$, $Q = 0.4$, $R = 0.2$

QUESTION 10

Tim (T), Uriah (U), Vincenzo (V), Wahid (W) and Xavier (X) are playing a round robin chess tournament where every player plays every other player exactly once.

The matrix that shows the two-step dominance after the tournament has been completed is

$$M = \begin{matrix} & & & \text{loser} \\ & & & T & U & V & W & X \\ \text{winner} & T & \begin{bmatrix} 0 & 0 & 1 & 3 & 2 \end{bmatrix} \\ & U & \begin{bmatrix} 0 & 0 & 0 & 2 & 1 \end{bmatrix} \\ & V & \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\ & W & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ & X & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

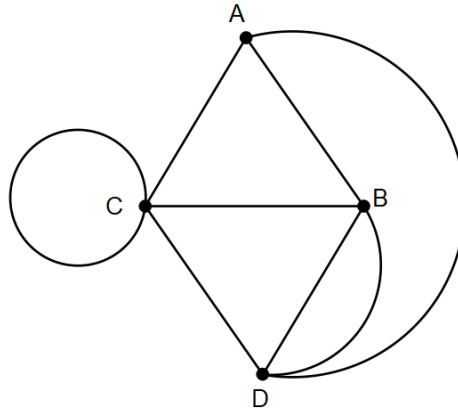
A result that must have happened in the tournament is

- A. Xavier defeated Vincenzo.
- B. Wahid defeated Uriah.
- C. Uriah defeated Vincenzo.
- D. Vincenzo defeated Tim.
- E. Xavier defeated Uriah.

Module 2: Networks and Decision Mathematics

Before answering these questions, you must select the “Networks and decision mathematics” module on the answer sheet for multiple choice questions and write the name of the module in the space provided.

Use the network diagram shown below to answer Questions 1 and 2.



QUESTION 1

The sum of the degrees of the vertices for the network is

- A. 14
- B. 15
- C. 16
- D. 17
- E. 18

QUESTION 2

The network contains

- A. an Eulerian circuit, a Hamiltonian path and a Hamiltonian cycle only.
- B. an Eulerian trail, a Hamiltonian path and a Hamiltonian cycle only.
- C. an Eulerian trail and a Hamiltonian cycle only.
- D. an Eulerian trail and a Hamiltonian path only.
- E. an Eulerian circuit, an Eulerian trail, a Hamiltonian path and a Hamiltonian cycle.

QUESTION 3

A network is a connected, planar graph.

If the number of vertices and the number of edges are the same, which of the following is true?

- A. The number of faces will be two more than the number of vertices.
- B. The number of faces will be two less than the number of vertices.
- C. The number of faces will be two more than double the number of vertices.
- D. The number of faces will be two less than double the number of vertices.
- E. The number of faces will be two.

QUESTION 4

An adjacency matrix for a network is shown below:

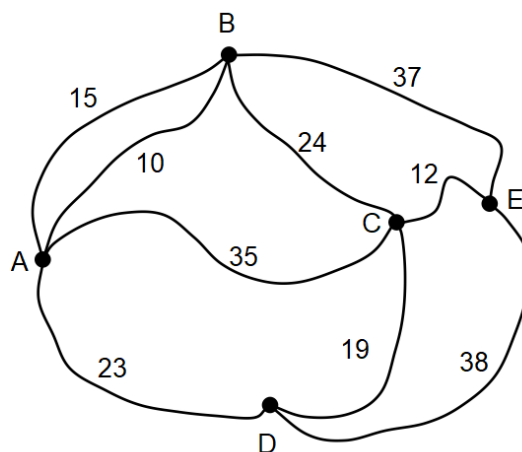
		to				
		A	B	C	D	E
from	A	1	1	2	0	0
	B	1	0	0	1	0
	C	2	0	0	0	0
	D	0	1	0	0	1
	E	0	0	0	1	0

Which of the following is **not** true about this network?

- A. The network could be undirected
- B. The network has a loop
- C. The sum of the degrees of the vertices in the network is 12
- D. The network has a bridge
- E. The graph is a simple graph

QUESTION 5

A weighted network is shown below with distances in kilometres:



The minimum spanning tree for this network is determined using Prim's algorithm.

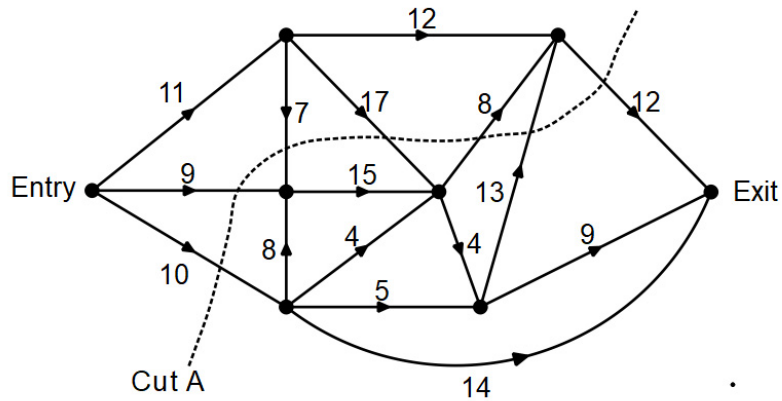
The shortest path from A to E is also determined using Dijkstra's algorithm.

The number of edges the minimum spanning tree and the shortest path from A to E have in common is

- A. none
- B. one
- C. two
- D. three
- E. four

Use the following diagram to answer Questions 6 and 7.

The flow network below shows the paths through exhibits in a museum. Due to social distancing patrons may only enter in one place, move in one direction and exit through one place. The maximum number of people who are allowed on each path per hour is shown on the diagram, along with a line representing one possible cut, Cut A, through the network:



QUESTION 6

The capacity of Cut A is

- A. 40
- B. 55
- C. 63
- D. 68
- E. 76

QUESTION 7

The maximum number of people per hour who could visit the museum would be

- A. 26
- B. 30
- C. 35
- D. 43
- E. 55

QUESTION 8

Four people Petra, Quinton, Ryan and Sienna who all work in the same office are to be allocated to complete four separate tasks, filing, rosters, collecting mail from the post office and cleaning the staff kitchen. Each person will complete one task. The allocation should ensure the minimum time in total for completion of all tasks.

The time, in minutes, that each person would take to complete each allocated task is given in the table below:

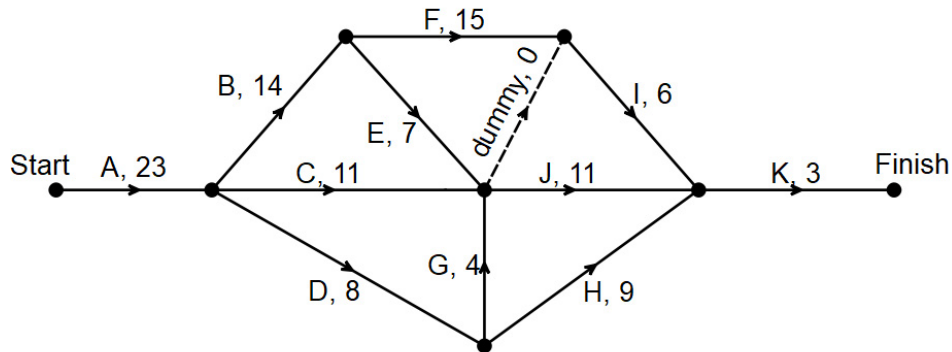
	Filing	Rosters	Mail	Kitchen
Petra	15	10	21	13
Quinton	16	9	22	13
Ryan	18	11	24	14
Sienna	17	10	23	13

An allocation that would not be minimal in this situation is

- A. Petra – Filing, Quinton – Rosters, Ryan – Mail and Sienna – Kitchen.
- B. Petra – Mail, Quinton – Rosters, Ryan – Filing and Sienna – Kitchen.
- C. Petra – Filing, Quinton – Kitchen, Ryan – Rosters and Sienna – Mail.
- D. Petra – Mail, Quinton – Filing, Ryan – Kitchen and Sienna – Mail.
- E. Petra – Filing, Quinton – Mail, Ryan – Kitchen and Sienna – Rosters.

Use the network and information below to answer Questions 9 and 10.

The activity network below shows the activities along with their duration in minutes that make up a larger project.



QUESTION 9

The largest number of immediate predecessors for any single activity in this network is

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

QUESTION 10

Which of the following statements is **not** true about this network?

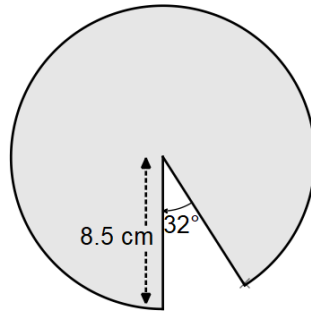
- A. The critical path in this network is ABFIK.
- B. The smallest float time for a non-critical activity is 3 minutes.
- C. The latest start time for activity C is 36 minutes.
- D. A reduction in time of activity F by 2 minutes would result in a second critical path.
- E. The largest activity float time is 18 minutes.

Module 3: Geometry and Measurement

Before answering these questions, you must select the “Geometry and measurement” module on the answer sheet for multiple choice questions and write the name of the module in the space provided.

QUESTION 1

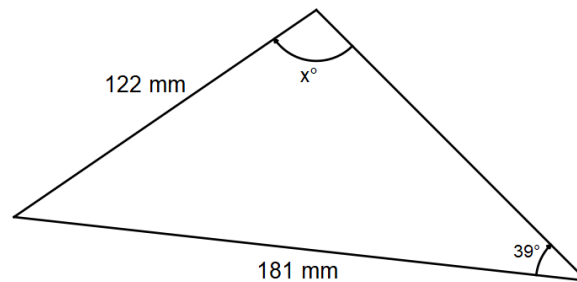
Consider the major sector shaded below:



The radius of the sector is 8.5 cm. The area of the major sector shown is closest to

- A. 4.7 cm²
- B. 20.2 cm²
- C. 48.7 cm²
- D. 93.3 cm²
- E. 206.8 cm²

QUESTION 2



The size of the obtuse angle x° is closest to

- A. 61°
- B. 69°
- C. 111°
- D. 119°
- E. 132°

QUESTION 3

A hemispherical bowl of radius 8 cm is carved out of the top of a wooden cube of side length 20 cm to form a feeding bowl for birds.

The total surface area of the feeding bowl in cm^2 is closest to

- A. 1797 cm^2
- B. 2199 cm^2
- C. 2601 cm^2
- D. 2802 cm^2
- E. 6928 cm^2

QUESTION 4

Six points are located in the Pacific Ocean. The points are A at $(0^\circ\text{N}, 150^\circ\text{W})$, B at $(0^\circ\text{N}, 120^\circ\text{W})$, C at $(30^\circ\text{S}, 150^\circ\text{W})$, D at $(30^\circ\text{S}, 120^\circ\text{W})$, E at $(60^\circ\text{S}, 150^\circ\text{W})$ and F at $(60^\circ\text{S}, 120^\circ\text{W})$.

The shortest direct distance between any two of these points would be the arc

- A. AB
- B. EF
- C. CD
- D. AC
- E. DF

QUESTION 5

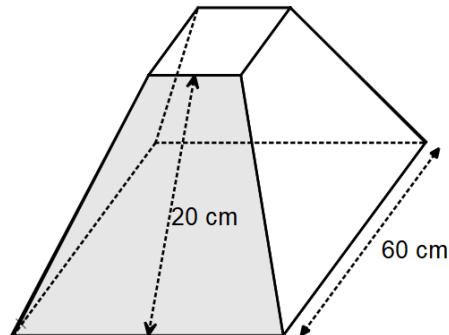
A plane flies from Hokkaido, Japan, $(45^\circ\text{N}, 141^\circ\text{E})$ to Minneapolis, USA, $(45^\circ\text{N}, 124^\circ\text{W})$.

Assuming the radius of the Earth to be 6400 km, the shortest distance between these two places is given by

- A. $6400 \times \cos(45) \times \frac{\pi}{180} \times 17$
- B. $6400 \times \cos(17) \times \frac{\pi}{180} \times 45$
- C. $6400 \times \cos(45) \times \frac{\pi}{180} \times 265$
- D. $6400 \times \cos(45) \times \frac{\pi}{180} \times 95$
- E. $6400 \times \cos(95) \times \frac{\pi}{180} \times 45$

QUESTION 6

A square based pyramid of height 40 cm is cut so that a smaller similar square based pyramid is taken off the top. The length of the sides of the square base of the original pyramid is 60 cm. After the top has been removed the perpendicular height of the slanted side is 20 cm. This is shown in the diagram below:



The vertical height of the truncated pyramid would be

- A. 16 cm
- B. 24 cm
- C. 36 cm
- D. 12 cm
- E. 30 cm

QUESTION 7

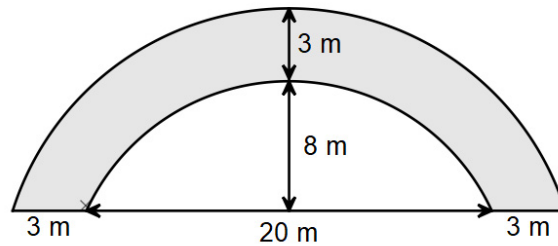
Tom is looking at two vertical towers, Tower A which is 100 metres high and Tower B which is 20 metres high. Tower A is 200 m away from Tom on a bearing of 021° , whilst Tower B is 300 m away from Tom on a bearing of 078° .

Tom and both towers are all on flat ground so that they are on the same horizontal plane. The angle of elevation from Tower B to Tower A is closest to

- A. 9.1°
- B. 17.5°
- C. 38.7°
- D. 57.0°
- E. 72.5°

QUESTION 8

At the end of a car race, the vehicles pass under an arch as shown shaded in the diagram below:



The arch is formed from two arcs from concentric circles.

The radius of the larger circle is 15 m.

The area of the front face of the arch, shaded in the diagram, is closest to

- A. 101 m²
- B. 108 m²
- C. 122 m²
- D. 127 m²
- E. 223 m²

QUESTION 9

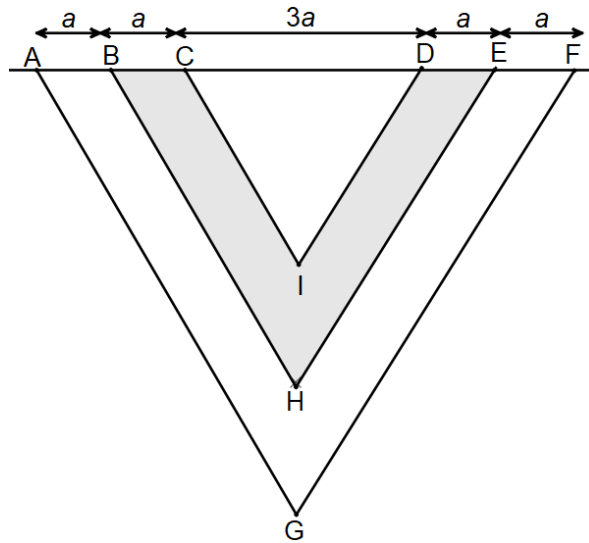
Geraldine walks 4 km on a bearing of 042° and Hugo walks 5 km on a bearing of 343°.

An expression that could be used to determine the distance between Geraldine and Hugo is

- A. $\sqrt{4^2 + 5^2}$
- B. $4 \sin(42^\circ) + 5 \sin(17^\circ)$
- C. $\sqrt{4^2 + 5^2 - 2 \times 4 \times 5 \times \cos(59^\circ)}$
- D. $4 \cos(42^\circ) + 5 \cos(17^\circ)$
- E. $5 \cos(42^\circ) + 4 \cos(17^\circ)$

QUESTION 10

Flags are made to form a line of bunting. Each triangular flag is made up of white and grey material as shown below. On each flag the triangles AGF, BHE and CID are all similar.



The ratio of grey area to white area on each flag is

- A. 16:33
- B. 1:2
- C. 1:4
- D. 25:58
- E. 24:25

Module 4: Graphs and Relations

Before answering these questions, you must select the “Graphs and relations” module on the answer sheet for multiple choice questions and write the name of the module in the space provided.

QUESTION 1

A horizontal line passes through the point (3, 5).

Which of the following would be the equation of this line?

- A. $x = 3$
- B. $y = 3$
- C. $x = 5$
- D. $y = 5$
- E. $3x + 5y = 15$

QUESTION 2

A line with a gradient of -3 passes through the point $(-4, -5)$. The equation of the line would be

- A. $3x - y = 17$
- B. $3x + y = -17$
- C. $3x + y = -7$
- D. $3x - y = -17$
- E. $y - 3x = -17$

QUESTION 3

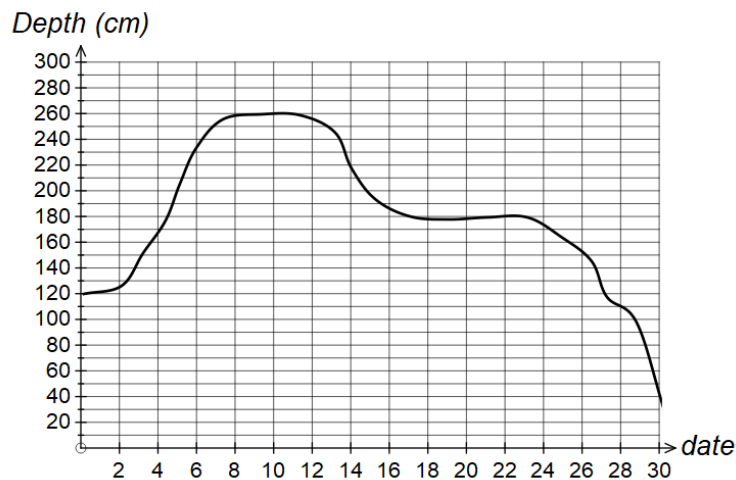
A coffee van proprietor makes a total of 800 coffees and hot chocolates at a particular event. Coffees sell for \$4.10 whereas hot chocolates sell for \$4.80. The proprietor takes a total revenue of \$3476.

If the number of hot chocolates sold is h , an equation that could be used to determine h would be

- A. $4.1h + 4.8(800 + h) = 3476$
- B. $4.1h + 4.8(800 - h) = 3476$
- C. $4.1(800 + h) + 4.8h = 3476$
- D. $4.1(800 - h) + 4.8h = 3476$
- E. $4.1(h - 800) + 4.8h = 3476$

QUESTION 4

The depth of water, in cm, in a dam is recorded during April, 2019. The graph that shows the changes in depth of water in the dam during April is shown below:



The average decrease in depth from the 8th of April to the 28th of April is closest to

- A. 7.5 cm/day
- B. 13 cm/day
- C. 7 cm/day
- D. 9.3 cm/day
- E. 2.7 cm/day

QUESTION 5

A company has been losing business at a steady rate. The company started trading at the start of 2018.

At the end of January 2018 ($n = 1$), they had 3000 regular customers.

At the end of August 2018 ($n = 8$), they only had 2419 regular customers.

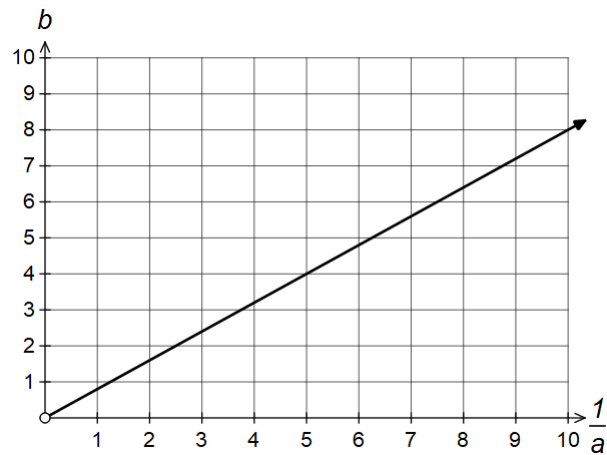
The relationship between the number of customers, c , and the number of months after the start of trading, n , is linear and continues to follow the same pattern.

The number of customers that the company will have at the end of February, 2019, is expected to be

- A. 1755
- B. 1838
- C. 1921
- D. 1983
- E. 2056

QUESTION 6

The graph below shows a relationship between two variables a and b .

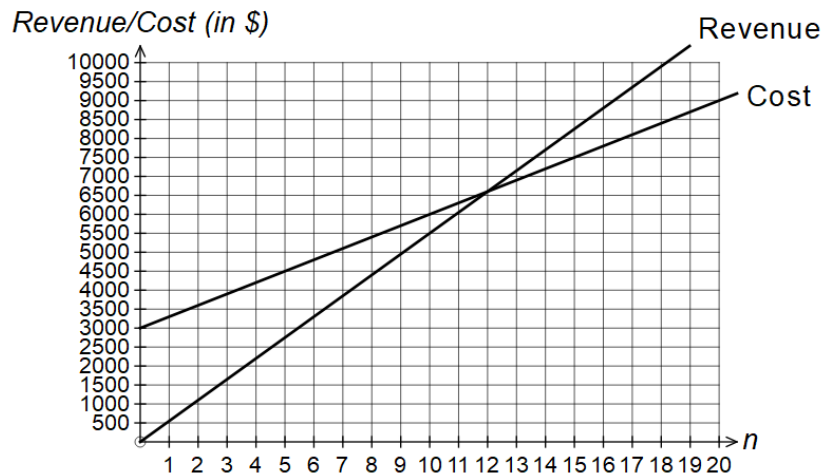


The rule that represents the relationship between a and b is

- A. $b = \frac{4}{5}a$
- B. $b = \frac{5}{4}a$
- C. $b = \frac{4}{5a}$
- D. $b = \frac{5}{4a}$
- E. $a = \frac{4}{5}b$

QUESTION 7

The graph below shows the relationship between the number of items sold by a company, n , and the revenue generated by those sales as well as the costs of manufacturing:



The rule that gives the profit, P , made by this company is

- A. $P = 250n + 3000$
- B. $P = 250n - 3000$
- C. $P = 850n + 3000$
- D. $P = 850n - 3000$
- E. $P = 550n - 3000$

QUESTION 8

Two service stations, Bloo Roo and Pink Galah, hire out lawn mowers.

Bloo Roo hires the lawn mowers to customers for \$2 per hour.

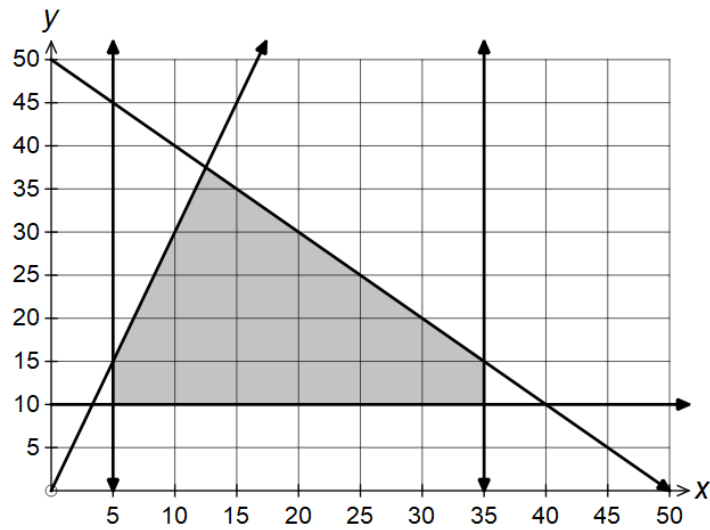
Pink Galah charges \$6 up-front fee with an hourly charge of \$1 for the first 12 hours, increasing to \$4 per hour after 12 hours.

Which of the following statements is true about this situation?

- A. Pink Galah is the cheapest option only for up to six hours of hire.
- B. Bloo Roo would be cheapest only for more than 15 hours of use.
- C. Pink Galah would be cheapest for more than four hours and less than 12 hours of use.
- D. Pink Galah would be cheapest for more than six hours and less than 15 hours of use.
- E. Bloo Roo would be cheapest for more than six hours and less than 15 hours of use.

QUESTION 9

A feasible region is shown shaded below:



The inequalities that form the boundaries of the feasible region are:

- A. $5 \leq y \leq 35, x \geq 10, x + y \leq 50, y \leq 3x$
- B. $5 \leq x \leq 35, y \geq 10, x + y \leq 50, y \leq 3x$
- C. $5 \leq x \leq 35, y \geq 10, x + y \geq 50, y \leq \frac{x}{3}$
- D. $5 \leq x \leq 35, y \geq 10, x + y \leq 50, y \leq \frac{x}{3}$
- E. $5 \leq x \leq 35, y \leq 10, x + y \geq 50, y \leq 3x$

QUESTION 10

A linear programming problem has the following constraint inequalities:

$$x \leq 70, y \leq 80, x + y \leq 100, 4x + 6y \geq 240, 6x + 3y \geq 180$$

Which of the following objective functions would minimise only at the point (15, 30)?

- A. $Z = 4x + 2y$
- B. $Z = 2x + 3y$
- C. $Z = 7x + 2y$
- D. $Z = 5x + 4y$
- E. $Z = 3x + 7y$

END OF MULTIPLE-CHOICE QUESTION BOO