THE SCHOOL FOR EXCELLENCE (TSFX)

VCE FURTHER MATHEMATICS UNIT 3 & 4

WRITTEN EXAMINATION 2 – 2020

Reading Time: 15 minutes
Writing Time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

									Letter
Student Number:									

Structure of Book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	8	8	45
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	1	15
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials Supplied

- Question and answer book of 36 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

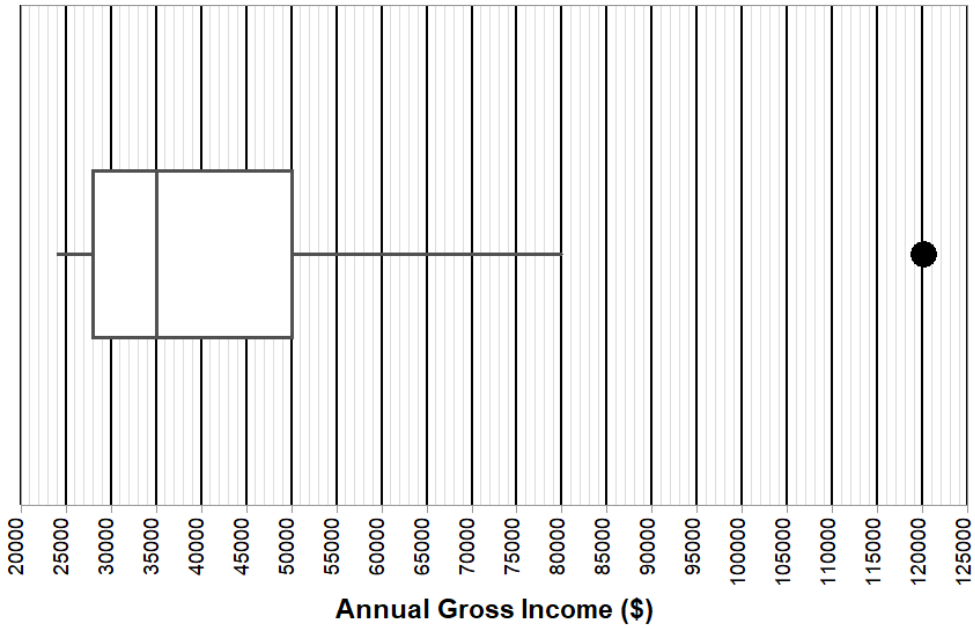
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

DATA ANALYSIS

QUESTION 1 (6 marks)
The boxplot below shows the distribution of *Annual Gross Income* ($) of 200 randomly selected families in a particular suburb.



a. Describe the shape of the distribution of *Annual Gross Income* shown by these 200 families. 1 mark

_____

b. How many of these families have an *Annual Gross Income* of more than $50 000? 1 mark

_____

c. One of these families has an *Annual Gross Income* of $120 000. 2 marks

Show why this income is an outlier in this data set. Show an appropriate calculation to support your answer.

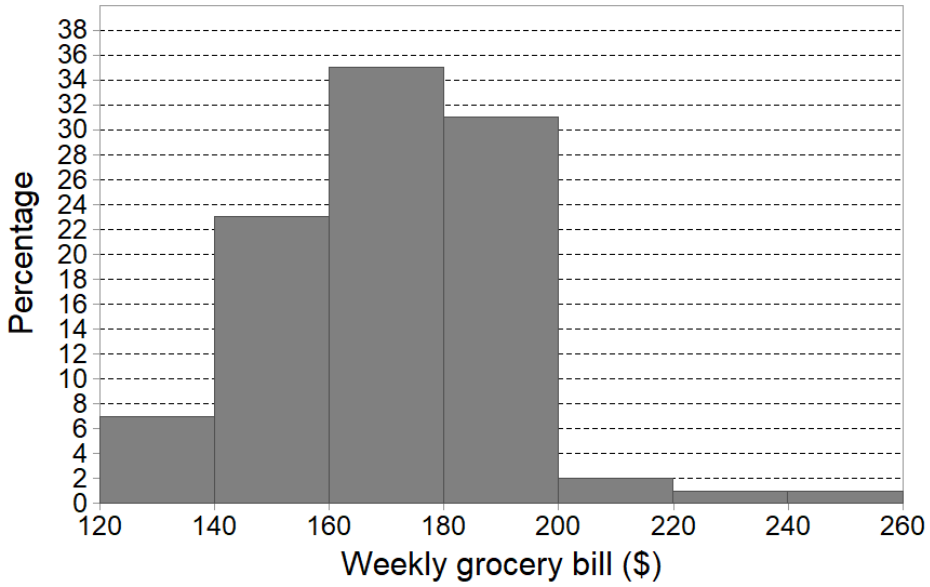
_____

_____

_____

_____

The same 200 families were asked what their weekly grocery bill was. The results are shown in the histogram below:



d. Complete the following statements: 2 marks

i. The modal class interval for the weekly grocery bill is _____.

ii. The median class interval for the weekly grocery bill is _____.

QUESTION 2 (4 marks)
A survey of 5000 Australian people was conducted. They were asked how they felt about their financial situation. Each respondent was asked to classify themselves as “Struggling”, “Just getting by”, “Doing OK” or “Doing well”. An incomplete table of responses is shown below:

		Number of Responses
Financial Situation	Struggling	1405
	Just getting by	706
	Doing OK	
	Doing well	195
	Total	5000

a. The number of people who were “Doing OK” is missing.

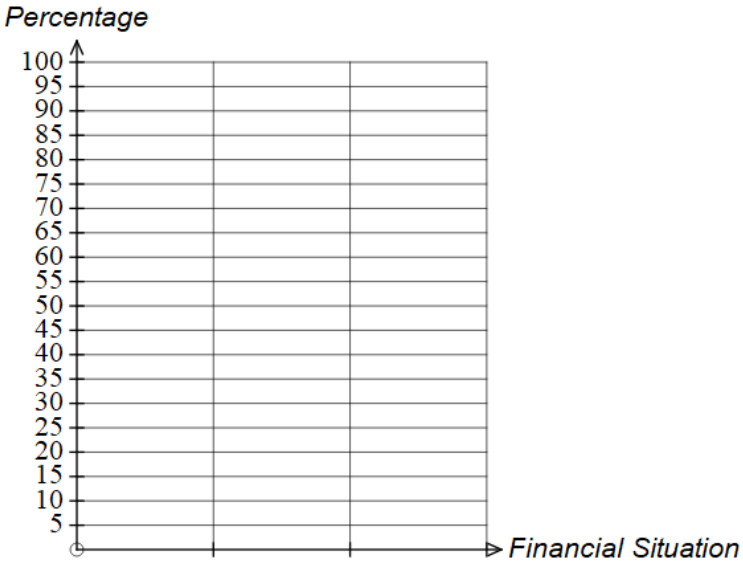
How many people were “Doing OK”? 1 mark

_____

b. What percentage of people, correct to the nearest whole number, in this survey were “Struggling”? 1 mark

_____

c. Construct a percentaged segmented bar chart on the grid below showing this data. Include an appropriate key for the display. 2 marks



QUESTION 3 (4 marks)
The Australian Council of Social Services (ACOSS) has recorded the percentage of children and young people living below the poverty line in Australia in 2020. ACOSS considers two different *Levels of Poverty*, below 50% of the median income (Below 50%) and below 60% of the median income (Below 60%).

The percentaged two way table below shows the percentages in each *Age Group* of children under 15 years and young people 15 to 24 years who are living below the poverty line:

		Age Group	
		Children under 15 Years	*Young people 15 to 24 Years*
Level of Poverty	*Below 50%*	17.7%	13.9%
	Below 60%	25.7%	19.3%

a. What type of categorical data is *Level of Poverty*? 1 mark

b. Which of the two variables, *Level of Poverty* or *Age Group,* is the explanatory variable? 1 mark

c. The percentaged two way table confirms that, in Australia, the *Level of Poverty* is associated with the *Age Group*. 2 marks

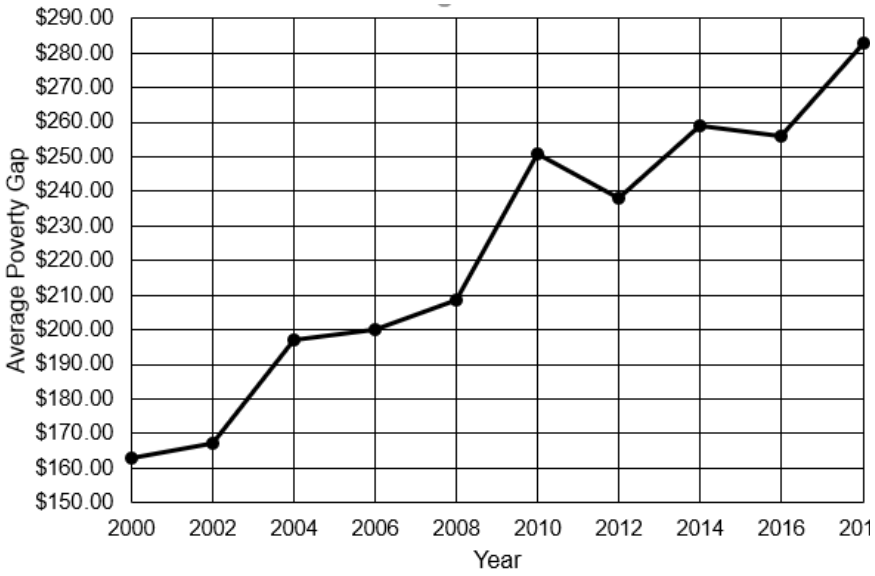
Use the information in the table to explain why the *Level of Poverty* is associated with the *Age Group*, quoting appropriate statistics.

QUESTION 4 (11 marks)
ACOSS have recorded the *average poverty gap* in dollars per week every two years for the years 2000 to 2018. The data is recorded at the start of each two year period.

The *average poverty gap* is determined from all households in poverty in Australia using 50% of the median after housing costs.

The data is shown below, along with a time series plot of the data:

Year	Average Poverty Gap (in $)
2000	$162.96
2002	$167.03
2004	$197.08
2006	$200.07
2008	$208.59
2010	$250.69
2012	$237.84
2014	$259.13
2016	$256.00
2018	$282.97



The association between A*verage Poverty Gap* and *Year* has a correlation coefficient, correct to three significant figures, of r = 0.971.

a. Describe the association between A*verage Poverty Gap* and *Year* during this period. 1 mark

_____

_____

b. A least squares line for the association between A*verage Poverty Gap* and *Year* can be calculated. 2 marks

Write the equation for this least squares line below. Round the slope and intercept correct to four significant figures.

Average Poverty Gap = [ ] + [ ] × *Year*

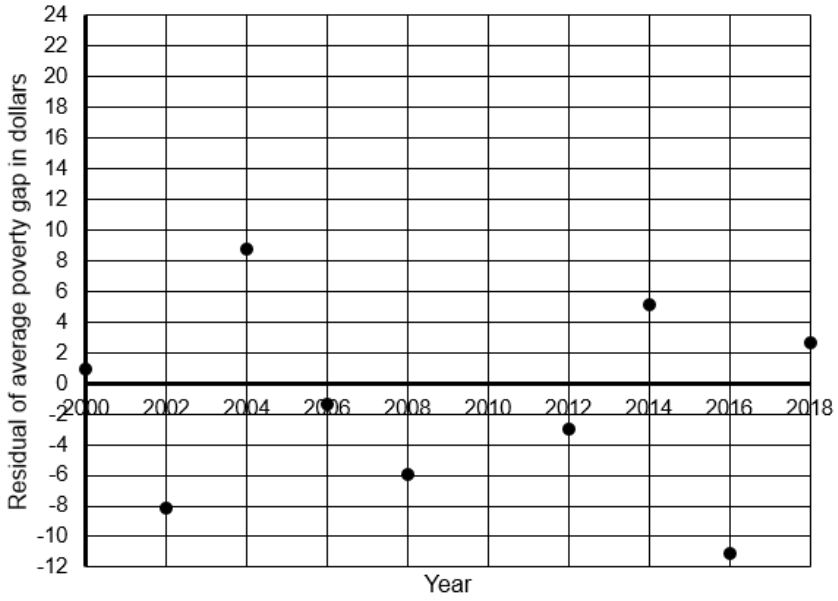
c. Add the least squares line to the time series plot above. 1 mark

ANSWER ON GRAPH

d. Interpret the slope of the least squares regression line in this context. 1 mark

e. Interpret the coefficient of determination for the association between *Average Poverty Gap* and *Year*. State appropriate values correct to one decimal place. 1 mark

A residual plot for the association between *Average Poverty Gap* and *Year* is shown below. The residual value for the year 2010 is missing.



f. **i.** Calculate the residual for the year 2010. Give your answer correct to the nearest dollar. 1 mark

ii. Add the residual point for 2010 to the residual plot above. 1 mark

ANSWER ON GRAPH

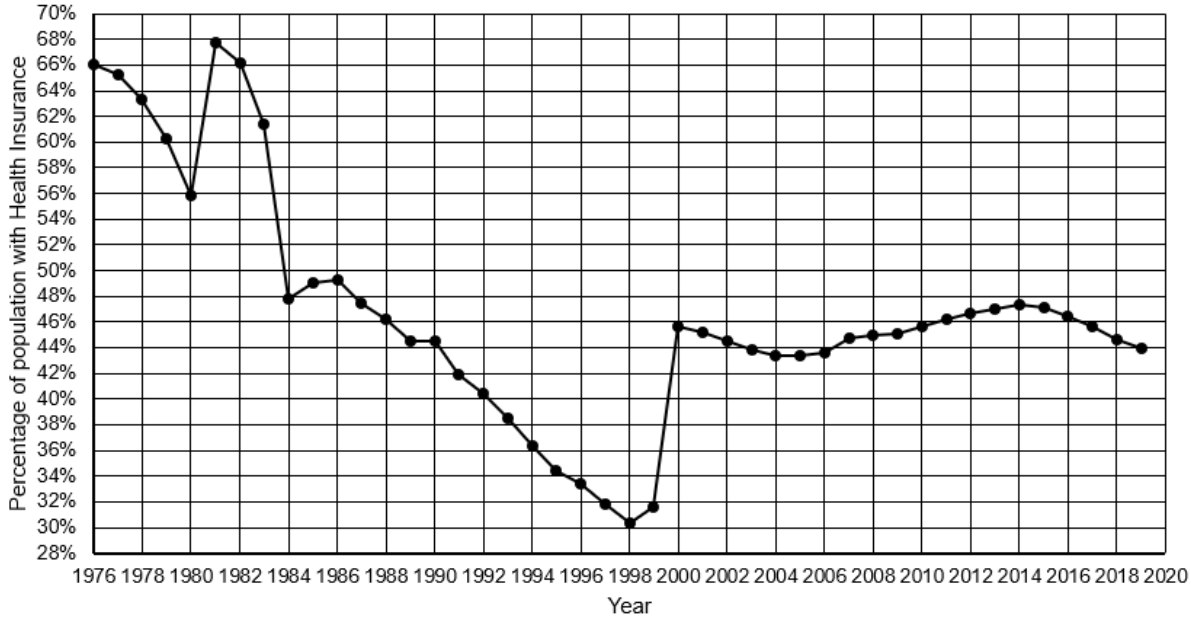
g. What does the residual plot indicate about the appropriateness of the linear model? Explain your answer. 1 mark

At some stage in the future the least squares line predicts that the average poverty gap will exceed $350.

h. During which year is the *average poverty gap* predicted to exceed $350? 1 mark

i. Explain why the prediction made in part **h.** is unlikely to be reliable. 1 mark

QUESTION 5 (5 marks)
One result of poverty is an inability to access health insurance. A time series plot is shown below of the *percentage of population with health insurance* in Australia over the years 1976 to 2019:



a. Describe the trend(s) in the *percentage of population with health insurance* over the years 1976 to 2019. 1 mark

_____

_____

One way of understanding the trends in this data is to smooth the data using median smoothing.

b. Smooth the graph below showing the data from 1976 through to 1990 using the five point median smoothing method. Mark each smoothed point with a cross (×). 2 marks

The data could also be smoothed using moving means. A table of the original data for the period 1977 to 1987 is shown below:

Year										
Percentage										

c. Use this table to show that the six-point moving mean with centring for 1982 is 61.2% correct to one decimal place. 2 marks

RECURSION AND FINANCIAL MODELLING

QUESTION 6 (4 marks)
During a recent financial crisis, Tracey's superannuation account reduced in value. The balance *n* days after the start of the year is modelled by the recurrence relation below:

$$S_0 = 470\,000,\ S_{n+1} = 0.99 \times S_n$$

a. What was Tracey's superannuation balance at the start of the year? 1 mark

_____

b. What is the percentage reduction in Tracey's superannuation account each day according to the recurrence relation? 1 mark

_____

_____

_____

c. Use the recurrence relation to show that Tracey's balance would be $460 647 after **two** days. 1 mark

_____

_____

_____

d. What is the total percentage decrease on Tracey's account after 30 days? 1 mark
Write your answer correct to the nearest percentage.

_____

_____

_____

_____

QUESTION 7 (4 marks)
Tracey is panicked about losing all of her superannuation, so she takes out $20 000 of her balance and invests this into an annuity investment.

Her investment accrues 4.2% pa compound interest compounding monthly. She also deposits a further $250 every month into the account.

a. What is monthly interest rate for Tracey's investment? 1 mark

b. Write a recurrence relation, in terms of T_0, T_{n+1} and T_n, that would give the balance of Tracey's account after n months. 1 mark

c. What is the balance in Tracey's account after 12 months? 1 mark

d. How much interest would Tracey have earned, in total, in her annuity investment 12 months after the account began? 1 mark

QUESTION 8 (7 marks)
Tracey wants to purchase a car for her work. She decides to buy a car that will cost $45 000 in total and she will borrow the entire purchase cost from the bank.

The bank offers Tracey a loan of $45 000 at 7.1% per annum compounding weekly, with equal weekly repayments. The loan will be paid off over a four year period.

a. What is the weekly payment that Tracey would need to make on this loan, if she is pay the loan off entirely after four years of equal weekly payments? 1 mark

b. How much interest would Tracey pay, in total, if she takes out this loan? 1 mark
Give your answer correct to the nearest dollar.

Tracey takes out the loan of $45 000 at 7.1% per annum compounding weekly, but she decides to pay $300 per week for the first year and then she plans to pay $400 per week until she makes a final smaller payment to reduce the balance to zero.

c. How many weeks would it take Tracey pay off her loan under these circumstances? 2 marks

Unfortunately, Tracey has a car accident exactly two years after she starts her loan. The car is a “write off” meaning that she cannot use the car any more.

She has insurance on her car and the insurance company work out the book value of the car using reducing balance depreciation of 22% per annum.

The insurance company will use their book value to pay off the remainder of her loan and give the rest of the balance of the book value to Tracey.

d. How much money will Tracey receive from the insurance company? 2 marks

The settlement after the accident and the fact that she now has no car, have meant that Tracey’s car has cost a significant amount of money over the two years that she owned it.

e. Show a calculation that gives the amount of money that Tracey’s car cost her in total over the two year period, taking into account all payments made and received. 1 mark

END OF SECTION A

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1: Matrices

QUESTION 1 (3 marks)
A tour company organise tours in Victoria (*V*) and New South Wales (*N*). Some tour groups stay in tents (*T*) and other tour groups stay in motels (*M*).

The numbers of tours in Victoria and New South Wales that stay in tents or motels during January 2017 is shown in the matrix, *J*, below:

$$J = \begin{matrix} \\ T \\ M \end{matrix} \begin{matrix} \begin{matrix} V & N \end{matrix} \\ \begin{bmatrix} 28 & 16 \\ 12 & 20 \end{bmatrix} \end{matrix}$$

a. How many tours went to Victoria in January 2017? 1 mark

_____

In February 2017, after an advertising campaign, the number of all tours increased by 25%.

The number of tours in February 2017, represented by the matrix *F*, can be calculated by muliplying matrix *J* by a scalar number, *x*.

b. What is the scalar number, *x*, so that $F = xJ$? 1 mark

_____

The tour company will use two more matrices *A* and *B*, to obtain a 1×1 matrix, *K*, that shows the total number of tours in January 2017.

The multiplication will be $AJB = K$.

c. Write down matrices *A* and *B*. 1 mark

A = *B* =

QUESTION 2 (3 marks)
The tour company organise a number of weekend tours to Echuca. An equal profit is made from each person staying in a tent and a different equal profit is made from each person staying in a motel.

In March 2017,120 people who went to Echuca stayed in tents (T) and 85 people stayed in motels (M). The company made a profit of $40 950 from these tours.

In April 2017, 94 people who went to Echuca stayed in tents (T) and 72 people stayed in motels (M). The company made a profit of $33 540 from these tours.

a. Complete the matrix equation below that represents these simultaneous equations. 1 mark

$$\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} T \\ M \end{bmatrix} = \begin{bmatrix} 40\,950 \\ 33\,540 \end{bmatrix}$$

The simultaneous equations can be solved using the inverse of one of the matrices above.
The inverse used can be written as $a\begin{bmatrix} 72 & -85 \\ -94 & 120 \end{bmatrix}$.

b. What is the value of the scalar multiple, *a,* in the inverse shown above? 1 mark

_____

_____

_____

In May 2017, 70 people who went to Echuca stayed in tents (T) and 50 people stayed in motels (M).

c. What profit would be made by the tour company from these people who went to Echuca in May? 1 mark

_____

_____

_____

QUESTION 3 (4 marks)
The tour company has a manager Aiden (*A*), two team leaders, Binod (*B*) and Charis (*C*) and three group leaders Dino (*D*), Etienne (*E*) and Farina(*F*).

If the tourists have a problem, they are encouraged to speak first to their group leader. The group leaders can discuss the issue amongst themselves or speak to their team leader. The team leaders can discuss the issue amongst themselves or speak to the manager.

A matrix, *M*, is shown below that shows who can communicate with each other:

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} \multicolumn{6}{c}{\textit{from}} \\ A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \textit{ to}$$

a. Who is Dino's team leader?

1 mark

_____

b. Explain, in this context, why this communication matrix is symmetric around the leading diagonal. 1 mark

_____

_____

_____

The manager, Aiden, believes that there is good communication in the tour company because he says that everyone in the company can speak either directly or indirectly to every other person.

c. With reference to the matrix $M + M^2$, explain why Aiden is correct. 2 marks

_____

_____

_____

_____

QUESTION 4 (5 marks)
In 2018 the tour company had 80 000 people who went on their tours. 50 000 of these people went on tours in Victoria (*V*) and 30 000 of the people went on tours in New South Wales (*N*).

They decide to increase their business to include tours of Queensland (*Q*) from 2019 onwards. In order to encourage their customers to take up their Queensland tours, they offer incentives for these tours.

Some people do not return for another tour (*R*) next year, although they may return in the future.

The matrix that gives the number of people taking tours in 2018 is $G_{2018} = \begin{bmatrix} 50\,000 \\ 30\,000 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} V \\ N \\ Q \\ R \end{matrix}$

They find that 50% of people who toured Victoria in 2018 returned to Victoria in 2019, 10% toured NSW, 10% went to Queensland and 30% of people did not do another tour in 2019.

Furthermore, 40% of people who toured New South Wales in 2018 returned to New South Wales in 2019, 20% toured Victoria, 30% went to Queensland and 10% of people did not do another tour in 2019.

The number of people who took tours in 2019 is given by the matrix, G_{2019}. This matrix can be calculated using the following formula:

$$G_{2019} = \begin{bmatrix} 0.5 & 0.2 & 0 & 0 \\ 0.1 & 0.4 & 0 & 0 \\ 0.1 & 0.3 & 1 & 0 \\ 0.3 & 0.1 & 0 & 1 \end{bmatrix} \times G_{2018}$$

a. Write down the matrix G_{2019} below. 1 mark

$$G_{2019} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix} \begin{matrix} V \\ N \\ Q \\ R \end{matrix}$$

From 2019 onwards the transition matrix includes the people who have been to Queensland or who did not return for one year or more. The transition matrix, T, for this situation is

$$T = \begin{array}{c} \\ V \\ N \\ Q \\ R \end{array} \begin{array}{c} \begin{array}{cccc} \multicolumn{4}{c}{\textit{this year}} \\ V & N & Q & R \end{array} \\ \begin{bmatrix} 0.5 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.2 & 0.1 \\ 0.1 & 0.3 & 0.4 & 0.1 \\ 0.3 & 0.1 & 0.2 & 0.7 \end{bmatrix} \end{array} \begin{array}{l} V \\ N \\ Q \\ R \end{array} \quad \textit{next year}$$

Using this transition matrix, $G_{2020} = T \times G_{2019}$ and in general terms, $G_{n+1} = T \times G_n$.

b. Show a calculation that demonstrates that 14 500 people are predicted to tour New South Wales with this company in 2020. 1 mark

c. If this pattern was to continue, in which year would the largest number of people go to Queensland with this company? 1 mark

After 2021, the company offer vouchers to encourage people who have not returned each year to do another tour.

Their aim is to retain the numbers on each tour to Victoria, New South Wales and Queensland that they have in 2021.

The matrix equation that will give the numbers from 2022 onwards will be given by:

$$G_{n+1} = T \times G_n + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

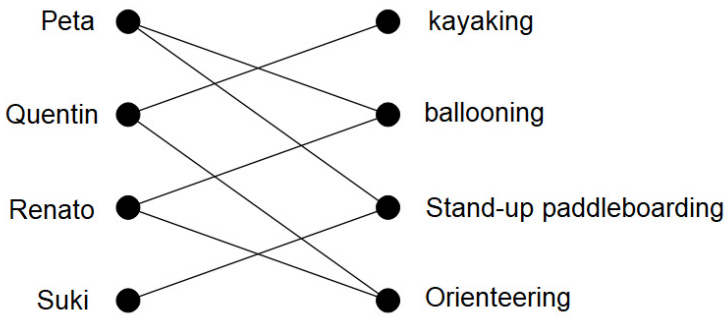
d. State the values *a, b, c* and *d* in the matrix above that would ensure the number of people touring to Victoria, New South Wales and Queensland remain the same as they were in 2021. 2 marks

MODULE 2: NETWORKS AND DECISION MATHEMATICS

QUESTION 1 (8 marks)
At a holiday camp there are a number of staff Peta, Quentin, Renato and Suki.

The camp run activites such as kayaking, ballooning, stand-up paddleboarding and orienteering.

Staff run activites according to their skills. The activities that each staff member can take is shown below in a bipartite graph:



a. Given that one staff member should do each activity and all staff members must have a job, complete the table below showing which staff member should do each activity. 1 mark

Staff Member	Activity
Peta	
Quentin	
Renato	
Suki	

The holiday camp want more flexibility in their staff allocations. They will train one staff member in another activity to ensure that they could have another allocation where each of the staff members does a different activity to the allocation listed above.

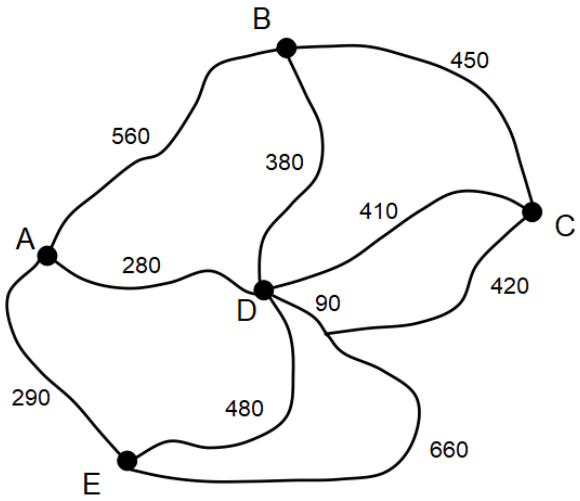
b. Which staff member should be trained and what activity should they train to do? 1 mark

_____

_____

_____

The orienteering activity is conducted in a local park. The tracks in this park, along with checkpoints A, B, C, D and E are shown below, along with the track lengths in metres:



The participants must collect a flag from each of the checkpoints as part of the activity. The staff member organising this activity must take a box of flags to each checkpoint before the activity, starting and finishing at A. Ideally the staff member will do this efficiently by not repeating any checkpoints.

c. **i.** What is the mathematical name of the route the staff member should take to distribute the flags? 1 mark

_____

_____

ii. State one route that the staff member could take to go to each checkpoint exactly once starting and finishing at A. 1 mark

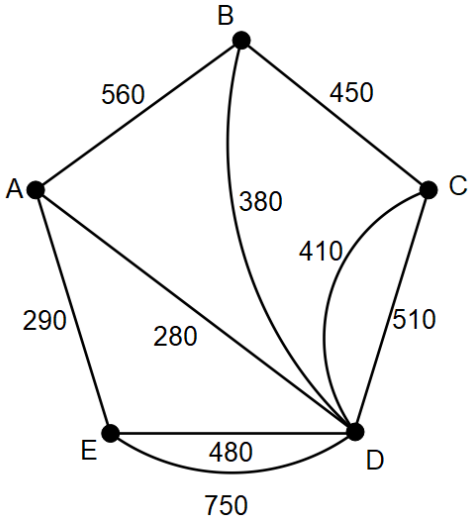
_____

_____

_____

A network is to be drawn showing the connections and distances between the checkpoints.

In this network the checkpoints are considered to be vertices and the connections between vertices are the edges. The partially completed network is shown below:



One edge is missing in this diagram.

d. Add the missing edge and its weight to the network above. 1 mark

ANSWER ON GRAPH

Obstructions on the network must be cleared before the orienteering activity. The staff member wants to start at A, check all network edges for obstructions without repeating any edge, and then return to A.

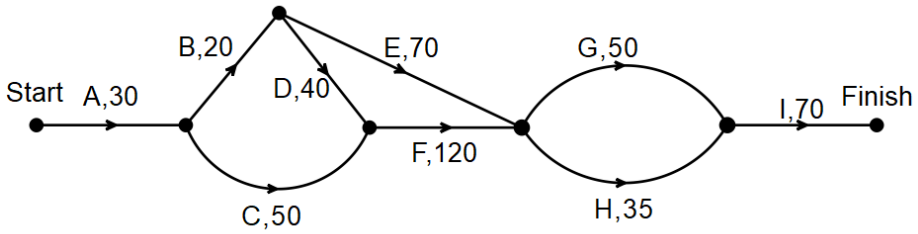
e. Use the information in the completed network to explain fully, in mathematical terms, why the staff member would not be able to clear the obstructions in the way described. 2 marks

The staff member still must clear the obstructions in the minimum distance, starting and finishing at A, but they will need to repeat one edge.

f. Which edge would need to be repeated to achieve this purpose? 1 mark

QUESTION 2 (7 marks)
The ballooning program requires a number of preliminary activites and clear up activities after the flight.

The activity network for this program, along with the time in minutes for each activity, is shown below. The actual flight is activity F:



a. What is the minimum number of minutes that the whole ballooning project will take? 1 mark

_____

b. State the critical path for this network. 1 mark

_____

c. What is the latest time after the start that Activity C can start without delaying the project? 1 mark

_____

d. Which activity has the largest float time? 1 mark

_____

The camp organisers are exploring options to reduce the overall time for the project.

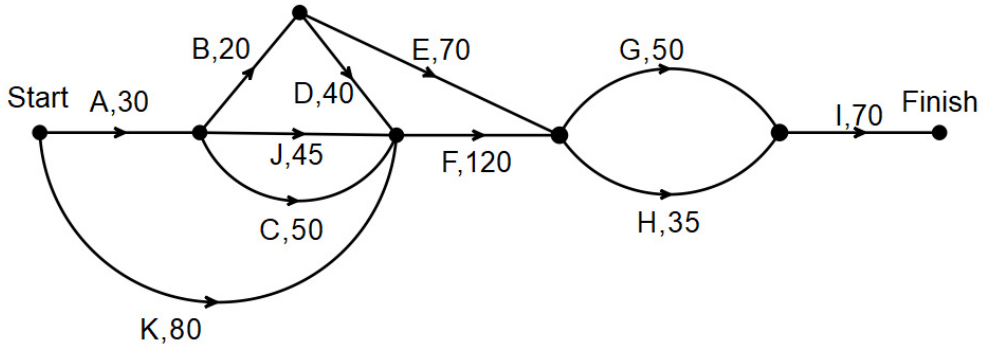
e. What is the minimum time reduction in any one activity that would result in two critical paths? 1 mark

_____

_____

_____

New safety regulations mean that the ballooning project must have two additional activities, J and K. These are shown in the adjusted network below:



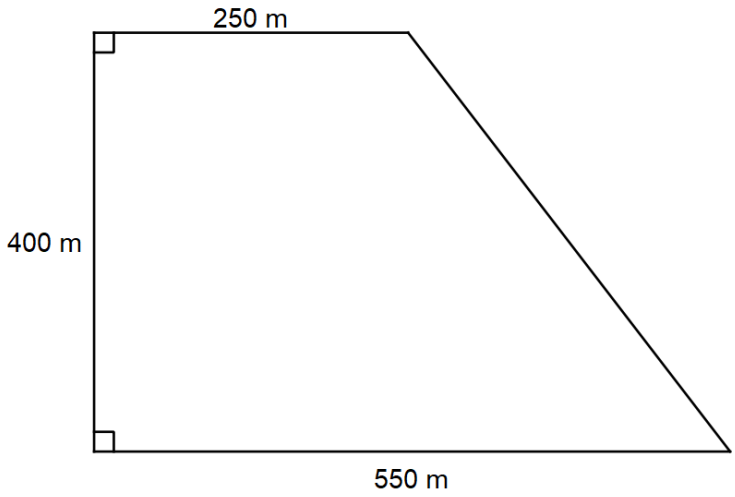
The following activites could be reduced in time:

Activity	Maximum Reduction in Minutes	Cost per Minute of Reduction
A	12	$5
B	10	$5
D	10	$4
E	20	$2
G	20	$4
H	15	$5
K	20	$2

f. What is the minimum cost that would achieve the greatest possible reduction in overall time? 2 marks

MODULE 3: GEOMETRY AND MEASUREMENT

QUESTION 1 (4 marks)
A holiday camp takes in school groups for activity camps. A map of the camp is shown below:

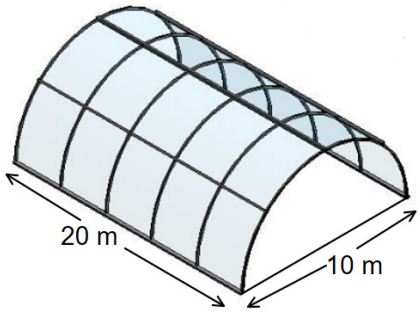


a. Show that the area of the camp, in metres squared, is 160 000 m^2. 1 mark

When the camp was set up, the managers had to fence the perimeter of the property for security.

b. What length of fencing, in metres, was required to fence the perimeter of the property? 1 mark

The camp has a number of shelters each constructed from a 20 metre long half-cylinder of diameter 10 metres. The ends of the shelters are open as shown below:



The shelters are covered with canvas, shown by the shaded region of the diagram.

c. What area of canvas would be used to cover each shelter?

Give your answer in metres squared, correct to one decimal place. 1 mark

The school groups have been finding it cold in the shelters during winter, so the management have covered in the open ends and will now heat the shelters.

To do this they must determine the volume of each shelter.

d. Calculate the volume of each shelter, in cubic metres, correct to the nearest metre. 1 mark

QUESTION 2 (3 marks)
The camp is located near Echuca, Victoria (36ºS, 145ºE). The camp is part of a multi-national group with headquarters in Johannesburg, South Africa (26ºS, 28ºE).

a. Explain, with reference to the information provided, how we know that Johannesburg is closer to the equator than Echuca. 1 mark

The managers of the camp in Victoria must telephone their headquarters in Johannesburg at 2.00 pm Johannesburg time.

b. Assuming 15º of latitude equates to one hour difference in time, what local time in Victoria would the camp manager need to call Johannesburg? 1 mark

There is another camp at Blackall, Queensland (24ºS, 145ºE) that is also part of the multi-national group. Some equipment must be transported from the Victorian camp to the Queensland camp by plane.

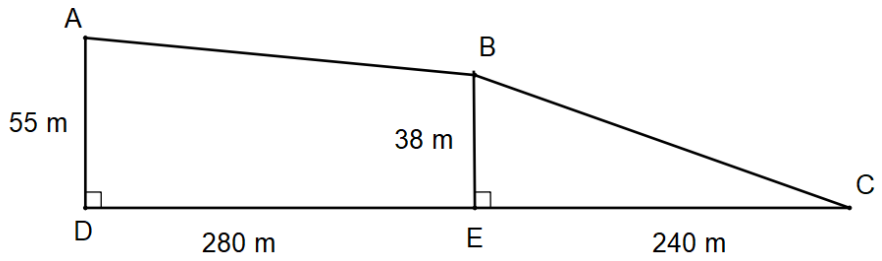
c. Assuming the radius of the Earth to be 6400 km, what is the minimum distance between these two camps, correct to the nearest kilometre? 1 mark

QUESTION 3 (5 marks)
The camp has two flying foxes that are favourites with their guests.

One flying fox has two stages on horizontal ground. One direct line is from A to B, followed by a second straight line from B to C. AD is a vertical tower at the start of the flying fox and it is 55 metres high. BE is also a vertical tower and is 38 metres high. Point C is on the ground.

The length DE is 280 metres and the length EC is 240 metres.

This information is shown in the diagram below:



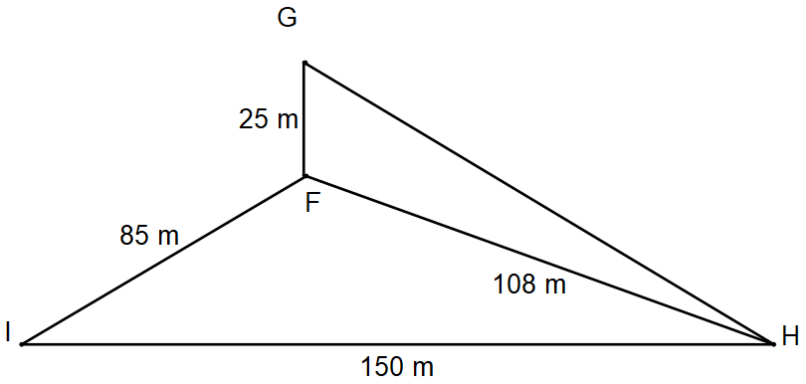
a. What is the angle of depression from A to C?

Give your answer in degrees, correct to one decimal place. 1 mark

b. What is the size of angle of ABC? 1 mark

Give your answer in degrees, correct to one decimal place.

The other flying fox, GH, starts from the top of a hill (represented by the triangle FHI below). It is started from the top of a 25 metre high tower (FG) on top of the hill.



c. Show that the size of angle FHI is 33.8º correct to one decimal place. 1 mark

d. What is the length of the second flying fox GH? 2 marks
Give your answer in metres correct to the nearest metre.

QUESTION 4 (3 marks)
The camp organisers have a cross country running course around a nearby national park.

The national park is triangular and the course requires the participants to start in one corner and run around the perimeter of the park, returning to the starting point.

To follow this path the participants run 850 m on a bearing of 068º, then to turn at the first corner of the path to follow a bearing of 170º for 580 m, before turning at the next corner to return on a direct line to the starting point.

a. What is the area of the national park, in metres squared? 1 mark

Give your answer correct to the nearest whole metre squared.

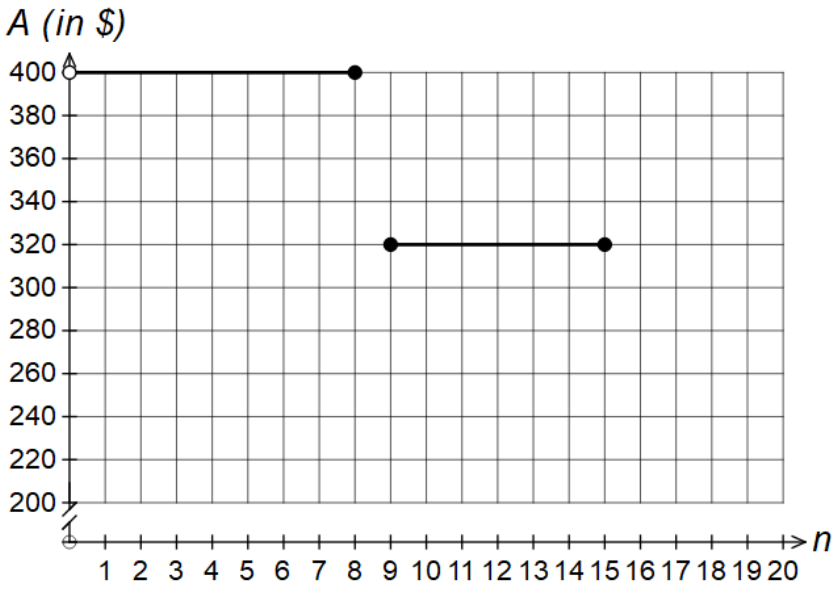
b. What bearing will the runners take on the final leg of this race? 2 marks

Give your answer correct to the nearest degree.

MODULE 4: GRAPHS AND RELATIONS

QUESTION 1 (6 marks)
A tour company runs group tours in Western Victoria. The company charge different amounts in dollars per person, A, depending on the size of the group for each tour, n.

A graph showing some of the charges is shown below:



a. How much would it cost, in total, for five people to go on a group tour? 1 mark

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b. The incomplete inequality below shows the size of the group required for an amount per person on dollars of $320. 1 mark

Complete the inequality by writing the appropriate symbol and number in the box provided.

$$9 \leq n \square$$

The tour company charge $300 per person, if there are 16 people or more in a group.

c. Add the line that would represent this information to the graph above. 1 mark

ANSWER ON GRAPH

One group have examined these charges and they realise that, if they add two extra people to their original group, the overall cost would not change.

d. How many people were in the original group? 1 mark

The tour company propose to change their charging structure to $A = 400 - 6.25n$, where A is the amount in dollars per person and n is the size of the group for each tour.

e. Add the line $A = 400 - 6.25n$ to the graph. 1 mark

ANSWER ON GRAPH

f. What group sizes would pay more per person for their tours under this new proposal? 1 mark

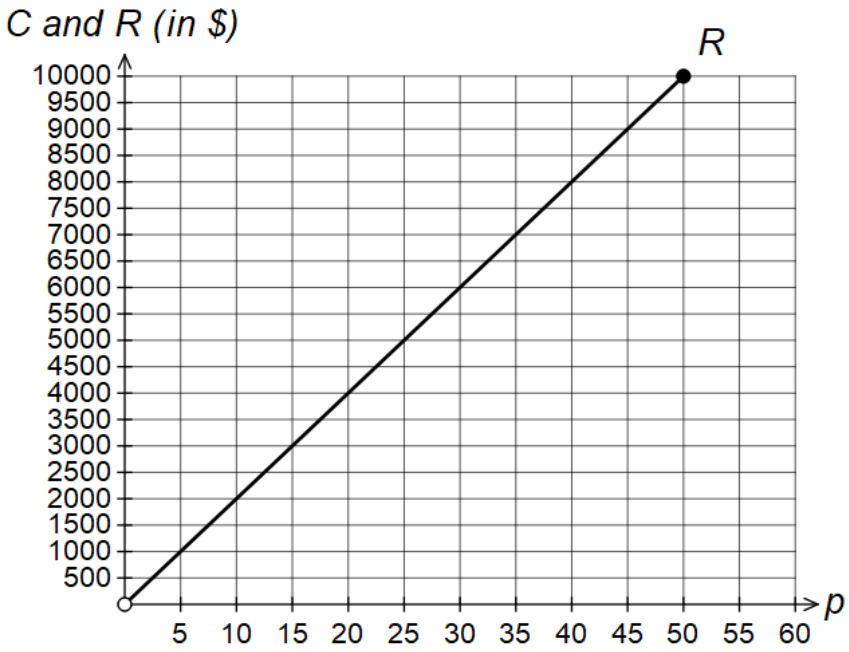
QUESTION 2 (4 marks)
The tour company also run bus tours. One of the buses that they use can hold up to 50 passengers in addition to the driver and a tour guide.

One particular day tour costs the company $1200 for the driver, tour guide and fuel. In addition they need to pay $150 per passenger for entry to attractions.

The equation that gives the costs for this tour is $C = 1200 + 150p,\ 0 < p \leq 50$, where C is the costs in dollars and p is the number of passengers.

a. Construct the graph of the line $C = 1200 + 150p,\ 0 < p \leq 50$ on the axes below. 1 mark

ANSWER ON GRAPH



The line that shows the relationship between the number of people, p, and the revenue in dollars, R, is on the graph above.

b. Write the equation of this line. 1 mark

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c. Show that the tour company will break even if they have 24 passegers. 1 mark

Often the bus is not full on day tours. They adjust their price per passenger so that they make a profit of $2500 when there are 40 passengers.

d. How much will the tour company charge per passenger on the day tour? 1 mark

QUESTION 3 (5 marks)
The tour company also run camping tours. The camping tours require staff to supervise and run activities for their guests.

Let x be the number of guests on each tour and y be the number of staff on the tour.

The bus used for the tour can hold a maximum of 50 people, including both staff and guests.

a. Write an inequality that would represent the information that the total number of people cannot exceed 50. 1 mark

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b. Another inequality for the camping tours is $y \geq \frac{x}{5}$. Explain the meaning of this in this context. 1 mark

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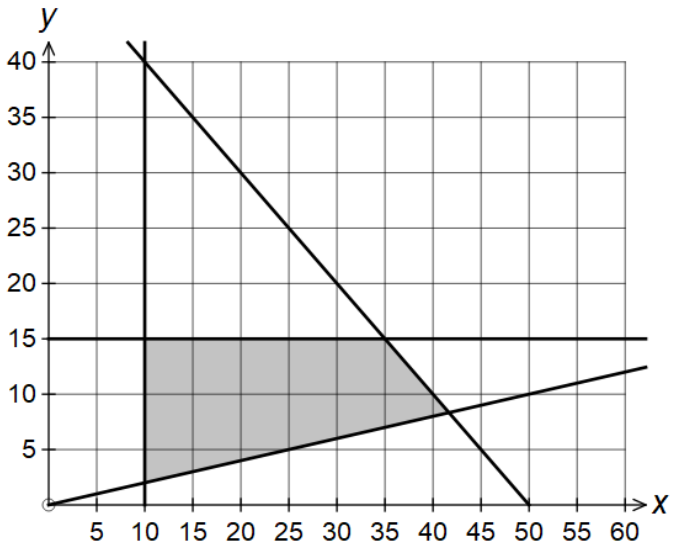
A shaded feasible region showing all of the lines for the inequalities for the camping tour has been constructed below:



The tour company charge $550 per guest and have to pay $1000 for every staff member per tour.

c. Assuming there are no other revenues or costs, write an equation for the profit made by the company for each tour. 1 mark

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d. What is the maximum profit that the company could make under these circumstances? 2 marks

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End of Answer and Question Booklet