2021
VCE
Further Mathematics

Trial Examination 1

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VICTORIAN CERTIFICATE OF EDUCATION
2021

FURTHER MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A - Core	24	24			24
B - Modules	32	8	4	2	16
					Total 40

- Students are permitted to bring into the exam room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer - based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question book of 39 pages.
- Formula sheet
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions

- Check that your **name and student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book and formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A - Core

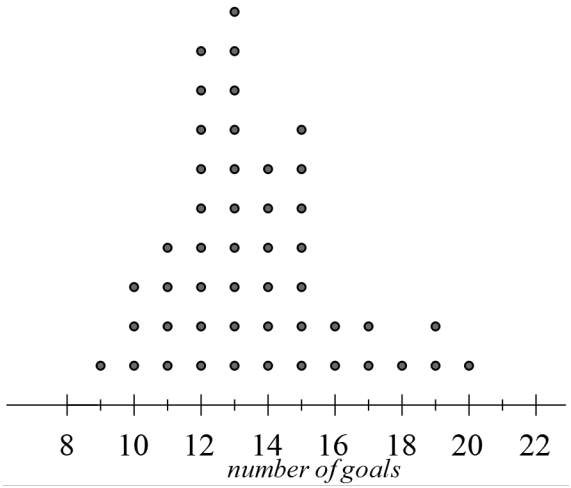
Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Use the following information to answer questions 1 - 3

The dot plot below shows the number of goals kicked by the Warburn football team each game over a series of 48 games.



Question 1

The number of games in which Warburn scored more than 17 goals is

A. 3

B. 1

C. 5

D. 4

E. 6

Question 2

The median number of goals scored in a game is

A. 12

B. 13

C. 14

D. 15

E. 16

Question 3

The interquartile range for the number of goals scored is

A. 1

B. 2

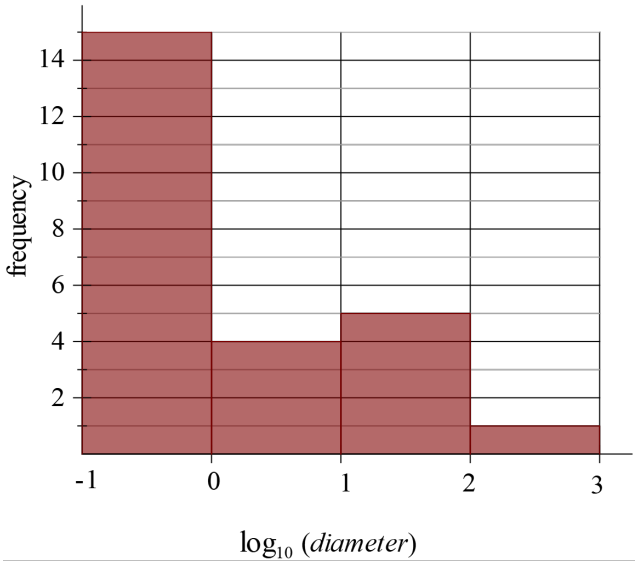
C. 2.5

D. 3

E. 3.5

Question 4

The histogram below shows the distribution of the *diameter*, in millimetres, for an assortment of drill bits. The histogram has been plotted on a log 10 scale.



The median *diameter* of these drill bits, in millimetres is between

A. 0 and 1

B. 1 and 2

C. 1 and 10

D. 10 and 20

E. 100 and 1000

Use the following information to answer questions 5 and 6

The length, in centimetres, of a sample of 1600 trout were approximately normally distributed with a mean of 25.6 cm and a standard deviation of 1.7 cm.

Question 5

Trout with a length less than 22.2 cm are undersized. The number of undersized trout in the sample is expected to be

A. 12

B. 37

C. 40

D. 216

E. 544

Question 6

The standardized length of one of these trout is $z = -1.4$
The actual length , in centimetres, is closest to

A. 24.2

B. 27.0

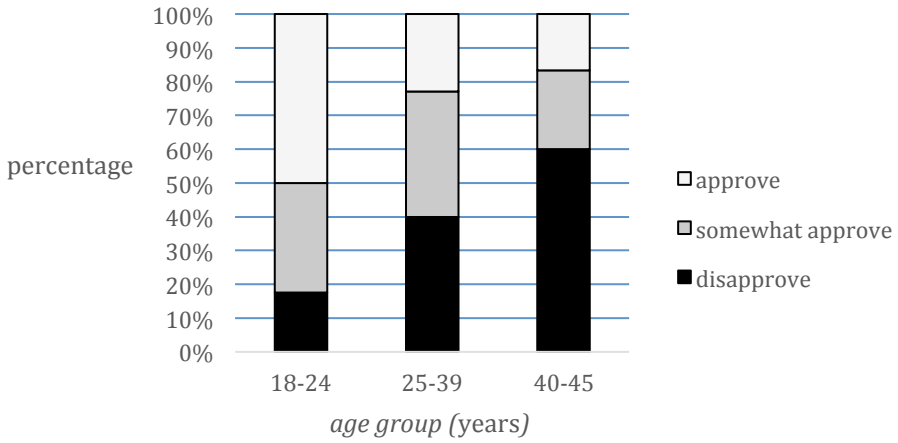
C. 23.9

D. 23.2

E. 27.9

Use the following information to answer questions 7-9

A survey was conducted to find the *opinion* (approve, somewhat approve, disapprove) of a sample of residents towards the construction of a new mountain bike track. The sample is divided into *age group* (18-24, 25-39, 40-45).
The results are displayed in the percentaged segmented bar chart below.



Question 7

The variables *opinion* (approve, somewhat approve, disapprove) and *age group* (18-24, 25-39, 40-45) are

A. both ordinal variables

B. a numerical variable and a nominal variable respectively

C. both numerical continuous variables

D. an ordinal variable and a nominal variable respectively

E. both numerical discrete variables

Question 8
There were 160 people in the 18-24 age group.
The number of this age group who somewhat approved is closest to

A. 64

B. 57

C. 80

D. 53

E. 96

Question 9

The data displayed in the segmented bar chart supports the contention that there is an association between *opinion* and *age group* because

A. around 18% of 18-24 age group disapproved while 37% of 25-49 age group approved

B. of the 18-24 age group, the percentage who approved is greater than the percentage who disapproved

C. the percentage of people in the 18-24 age group who approved is greater than the percentage of people in the 40-45 age group who approved

D. 50% of those in the 18-24 age group approved

E. there are more than two age groups in the survey

Question 10

For a set of bivariate data involving the variables x and y

$$r = -0.62, \quad \bar{x} = 5.6, \quad s_x = 1.3, \quad \bar{y} = 78.0, \quad s_y = 15.0$$

Given the information above, the least squares line with x as the explanatory variable is closest to

A. $y = 570 - 7.2x$

B. $y = 120 - 7.2x$

C. $y = 120 + 7.2x$

D. $y = 120 - 0.054x$

E. $y = 570 - 0.054x$

Use the following information to answer questions 11-12

Question 11

The *temperature*, in °C, of a cooling liquid is plotted against the *time*, in minutes after the heat source is removed.
The following least squares line and coefficient of determination were obtained from the data.

$$temperature = 95.8 - 1.82 \times time \qquad r^2 = 0.5625$$

Which one of the following is a conclusion that can be made from this information?

A. the temperature of the liquid is falling at a rate of 95.8 °C per minute

B. the correlation coefficient is 0.75

C. the initial temperature of the liquid is 1.82 °C

D. the temperature of the liquid can be determined by subtracting 1.82 minutes from the time

E. for every extra minute in time, the temperature of the liquid is expected to decrease by 1.82 °C

Question 12

22 minutes after the heat source is removed, the actual temperature of the liquid is found to be 54.6 °C.
The residual when t = 22 is closest to

A. -0.8

B. 1.2

C. -1.2

D. -14.1

E. 14.1

Question 13

A small child is given a simple jigsaw to complete.
An association between the *time*, in minutes, taken to complete the puzzle and the number of *practice sessions* she has had is found to be non-linear.
The data is linearized by applying a reciprocal transformation to the variable *time*.
The equation of the least squares line for the transformed data is

$$\frac{1}{time} = -0.013 + 0.041 \times practice\ sessions$$

Using this equation, the *time*, in minutes taken by the child to complete the puzzle after 8 practice sessions is predicted to be closest to

A. 3.2

B. 0.31

C. 0.58

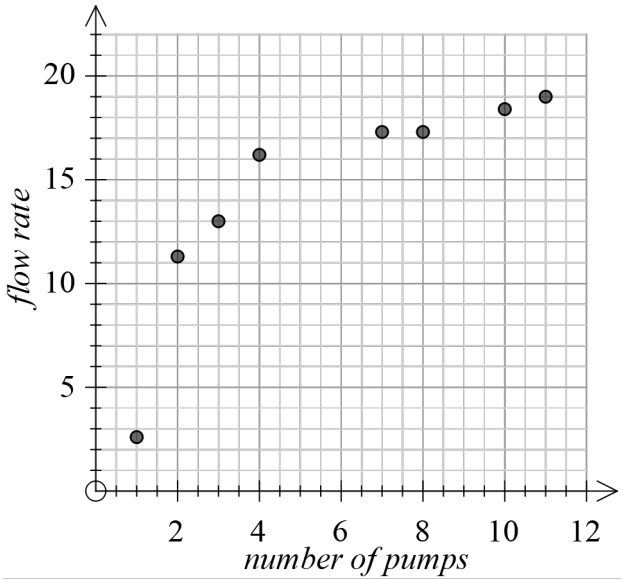
D. 2.7

E. 2.9

Question 14

The association between the *flow rate* in litres per minute through a fountain and the *number of pumps* is shown in the table and scatterplot below.

number of pumps	*flow rate* (L/min)
1	2.6
2	11.3
3	13
4	16.2
7	17.3
8	17.3
10	18.4
11	19



To linearise the data, a $\log_{10}$ transformation is applied to the variable *number of pumps*.
A least squares line is fitted to the transformed data.
With *flow rate* as the response variable, the equation of the least squares line is closest to

A. $flow\ rate = 7.5 + 1.2 \times \log_{10}(number\ of\ pumps)$

B. $number\ of\ pumps = 5.3 + 14.1 \times \log_{10}(flow\ rate)$

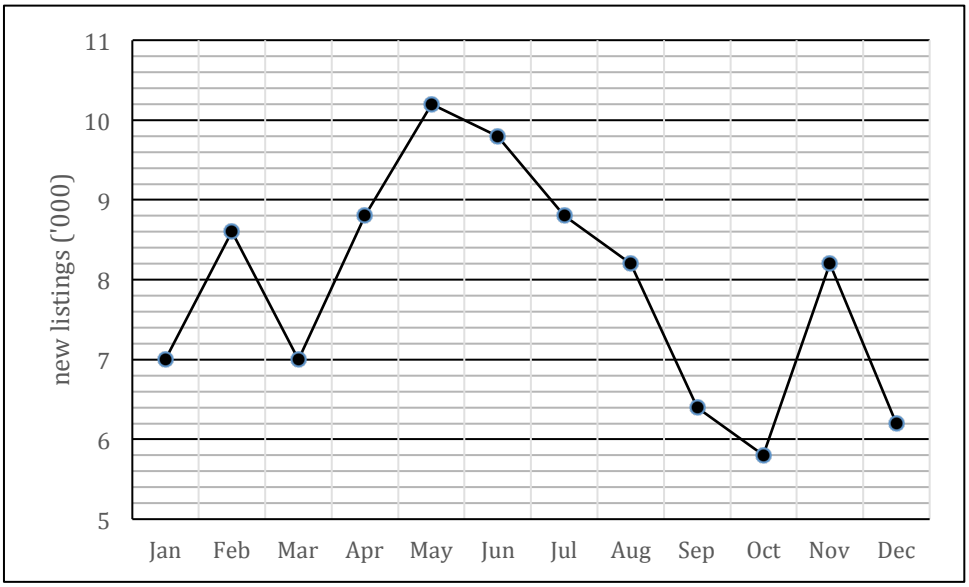
C. $number\ of\ pumps = 7.5 + 1.2 \times \log_{10}(flow\ rate)$

D. $\log_{10}(flow\ rate) = 5.3 + 14.1 \times (number\ of\ pumps)$

E. $flow\ rate = 5.3 + 14.1 \times \log_{10}(number\ of\ pumps)$

Question 15

The time series plot below shows the monthly number, in thousands, of new listings of houses on the property market over a year.



The time series is to be smoothed using five-median smoothing.
The smoothed number of listings, in thousands, for the month of October is closest to

A. 5.8

B. 6.0

C. 6.2

D. 6.4

E. 6.9

Question 16

The Warburn Golf Club pro shop is open every day except Sunday.
The table below shows the daily seasonal indices for the number of golf balls sold by the shop.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Seasonal index	0	1.04	1.43	0.76	1.53	0.52	1.72

The long-term average weekly number of golf balls sold is 513.
The expected number of golf balls sold on a Thursday is closest to

A. 65

B. 97

C. 112

D. 126

E. 130

Recursion and financial modelling

Use the following information to answer questions 17 and 18

Stewart invests \$176000 in an annuity. The value of his investment, in dollars, after *n* months, V_n, can be modelled by the recurrence relation

$$V_0 = 176000, \qquad V_{n+1} = 1.0045V_n - 950$$

Question 17

The regular amount withdrawn by Stewart from the annuity is

A. \$ 642.00

B. \$ 792.00

C. \$ 950.00

D. \$ 1004.50

E. \$ 1742.00

Question 18

Stewart wants to find the interest earned during the fourth month. He considers the following calculations.

(i) $V_4 - V_3$ (ii) $0.0045 \times V_3$ (iii) $950 - V_3 + V_4$

(iv) $0.0045 \times V_4$ (v) $950 - V_4 + V_3$

The interest earned during the fourth month can be calculated by

A. i only

B. ii and iii only

C. iii and iv only

D. ii and v only

E. None of the above

Question 19

Enya has an investment earning interest at 4.65% per annum, compounding quarterly.
The effective annual interest rate of her investment is closest to

A. 4.70 %

B. 4.73 %

C. 4.74 %

D. 4.75%

E. 4.76 %

Question 20

Which one of the following recurrence relations could be used to model the value of a perpetuity investment, P_n, after n years.

A. $P_0 = 48000, \quad P_{n+1} = 1.047 \times P_n - 3112$

B. $P_0 = 52000, \quad P_{n+1} = 1.076 \times P_n - 4010$

C. $P_0 = 56000, \quad P_{n+1} = 1.046 \times P_n - 3432$

D. $P_0 = 57000, \quad P_{n+1} = 1.076 \times P_n - 4332$

E. $P_0 = 61000, \quad P_{n+1} = 1.071 \times P_n - 4475$

Question 21

Four lines of an amortisation table for a reducing balance loan with quarterly repayments are shown below.

Repayment number	Repayment	Interest	Principal reduction	Balance of loan
5	5600	$1379.75	$4220.25	$104421.48
6	5600	$1326.15	$4273.85	$100147.63
7	5600	$1271.87	$4328.13	$ 95819.50
8	5600	$1178.58	$4421.42	$ 91398.08

The interest rate for this loan changed immediately before repayment number 8.
This change in interest rate is best described as

A. a decrease of 0.16% per annum

B. an increase of 0.04% per annum

C. a decrease of 0.004% per annum

D. an increase of 0.12% per annum

E. a decrease of 0.48% per annum

Question 22

Sally buys a machine for printing brochures.
The machine is purchased for $17000, and she prints 25000 brochures a year.
The value of the machine is depreciated using the unit cost method.
The machine reaches its scrap value of $2000 after five years.
A rule for the value, in dollars, of the machine after n brochures are printed, V_n, is

A. $V_n = 2000 - 250n$

B. $V_n = 17000 - 0.15n$

C. $V_n = 25000 - 0.15n$

D. $V_n = 17000n - 3000$

E. $V_n = 17000 - 0.12n$

Use the following information to answer questions 23 and 24

Aimee takes out a reducing balance loan of $250000.
Interest on the loan will be calculated and paid monthly at the rate of 4.25% per annum.

Question 23

If Aimee is to fully repay the loan in equal monthly instalments over 20 years, her monthly repayment will be closest to

A. $1042

B. $1417

C. $1548

D. $2970

E. $3591

Question 24

By making a lump sum payment after ten years, Aimee can reduce the principal on her loan to $130000.
At this time, her monthly repayment changes to $1650.
The interest rate remains the same at 4.25% per annum, compounding monthly.
The number of months, in total, it will take Aimee to fully repay the loan is closest to

A. 176

B. 213

C. 219

D. 227

E. 233

END OF SECTION A

SECTION B - Module

Instructions for Section B

Select **two** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.
Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
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Contents **Page**

Module 1: Matrices

Before answering these questions you **must** shade the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Which one of the following is a binary matrix?

A.

$$\begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

B.

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

C.

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

D.

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

E.

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

Question 2

P is a matrix such that $P \times \begin{bmatrix} A \\ N \\ G \\ L \\ E \end{bmatrix} = \begin{bmatrix} G \\ L \\ E \\ A \\ N \end{bmatrix}$

The matrix P is

A.
$$\begin{bmatrix} 4 \\ 5 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \end{bmatrix}$$

D.
$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

E.
$$\begin{bmatrix} 4 & 5 & 1 & 2 & 3 \end{bmatrix}$$

Question 3

Let $A = \begin{bmatrix} 3 & 4 & 5 & 6 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

The element in row i and column j of A is a_{ij}
The elements of A are determined by the rule

A. $a_{ij} = i + 2j$

B. $a_{ij} = 2j - 1$

C. $a_{ij} = i + j + 3$

D. $a_{ij} = 2i + j + 1$

E. $a_{ij} = 3i + j - 1$

Question 4
A system of simultaneous equations is to be solved using the matrix equation shown below.

$$\begin{bmatrix} 3 & a \\ 9 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ 6 \end{bmatrix}$$

For the system of equations to be inconsistent, the values of a and b could be

A. a = -3, b = 2

B. a = -1, b = -4

C. a = -1, b = 2

D. a = 1, b = -3

E. a = 9, b = -3

Module 1: Matrices

Question 5

The solution to the system of simultaneous equations below

$$\begin{aligned} 2x + 2z &= 6 \\ 4y + 2z &= 14 \\ 4x + 2y + 4z &= 10 \end{aligned}$$

is given by

A.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

B.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 2 \\ 4 & 2 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

C.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 0 & -1 \\ -2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

D.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} & 1 \\ -1 & 2 & \frac{3}{2} \\ -2 & \frac{1}{2} & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

E.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & \frac{-1}{2} & 1 \\ -1 & 0 & \frac{1}{2} \\ 2 & \frac{1}{2} & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$$

Module 1: Matrices

Question 6

Every day, a group of children choose either an apple (A) or a banana (B) for a morning snack.
On the first day, the number choosing an apple is the same as the number choosing a banana.
From day to day, the children change their choice of fruit according to a transition matrix, T.
It is expected that, in the long term, more children will choose an apple than will choose a banana.
Assuming the total number of children remains constant, a transition matrix, T, that would predict this outcome is

A.

$$T = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.7 & 0.5 \\ 0.3 & 0.5 \end{bmatrix} \end{array}$$

B.

$$T = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \end{array}$$

C.

$$T = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

D.

$$T = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \end{array}$$

E.

$$T = \begin{array}{c} \\ \text{A} \\ \text{B} \end{array} \begin{array}{c} \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \end{array}$$

Module 1: Matrices

Use the following information to answer questions 7 and 8

Sarah bakes cakes for 150 regular customers. Every week, customers each buy one of banana bread (B), carrot cake (C), or date slice (D).
Customers alter their choice from week to week as shown in the transition matrix, T, below.

this week

$$T = \begin{array}{c} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{ccc} \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.6 & 0.4 & 0.3 \\ 0.1 & 0.4 & 0.2 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \quad \text{next week}$$

The expected number of each type of cake chosen after n weeks can be determined by the recurrence relation

$$S_0 = \begin{bmatrix} 35 \\ 52 \\ 63 \end{bmatrix} \begin{array}{l} \text{B} \\ \text{C} \\ \text{D} \end{array}, \quad S_{n+1} = TS_n$$

Where S_0 is the state matrix for the first week.

Question 7

The percentage of these 150 customers who are expected to change their choice in the second week is closest to

A. 29%

B. 38%

C. 62%

D. 71%

E. 94%

Module 1: Matrices

Question 8

In the long term, the number of Sarah's 150 customers who are expected to change their choice each week from carrot cake to date slice is closest to

A. 26

B. 28

C. 20

D. 18

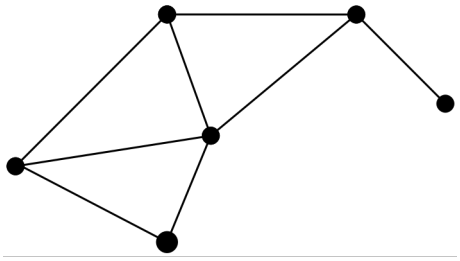
E. 39

End of Module 1

Module 2: Networks and decision mathematics

Before answering these questions you **must** shade the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



In the graph shown above, the sum of the degrees of the **odd** vertices is

A. 3

B. 6

C. 9

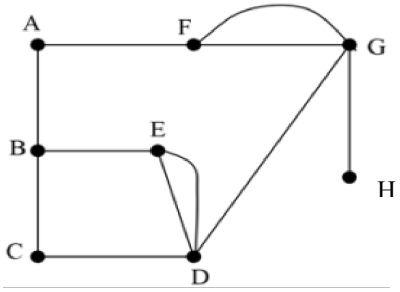
D. 10

E. 16

Module 2: Networks and decision mathematics

Question 2

Consider the graph below.



An Eulerian trail could be made possible in this graph by adding an edge between vertices

A. C and E

B. A and G

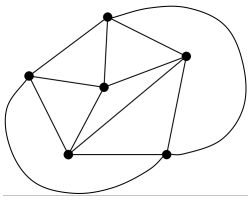
C. D and F

D. B and D

E. B and F

Question 3

Aimee draws the graph shown below to represent a solid figure.



Aimee wishes to verify Euler's formula for this graph.
What values of e, v and f should she use for this verification?

A. $e = 6,\ v = 8,\ f = 7$

B. $e = 10,\ v = 8,\ f = 8$

C. $e = 12,\ v = 6,\ f = 7$

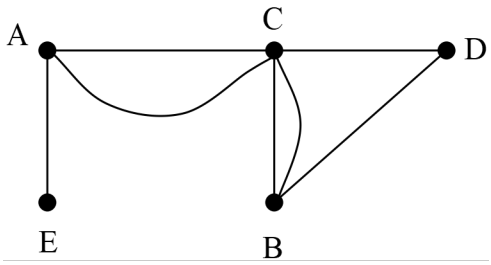
D. $e = 10,\ v = 6,\ f = 8$

E. $e = 12,\ v = 6,\ f = 8$

Module 2: Networks and decision mathematics

Question 4

Consider the graph below.



The adjacency matrix for this graph is

A.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

B.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

C.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

D.

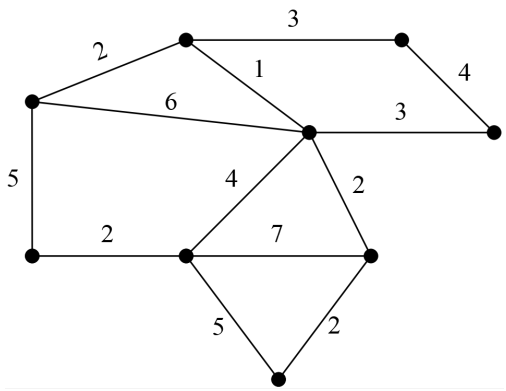
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

E.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Module 2: Networks and decision mathematics

Question 5



The minimal spanning tree for the graph shown above has a weight of

A. 16

B. 17

C. 18

D. 19

E. 20

Module 2: Networks and decision mathematics

Question 6

Emma, Frank, Guyan and Hilda are doing dishes in a kitchen. The table below shows the time, in minutes, that each person would take to complete each of four tasks.

	Rinse	**Wash**	**Dry**	**Put away**
Emma	6	5	7	8
Frank	8	5	6	7
Guyan	5	4	5	4
Hilda	7	9	11	8

The tasks must be completed one after the other.
If each person is allocated one task only, the minimum time, in minutes, in which the four tasks can be completed is

A. 21

B. 22

C. 23

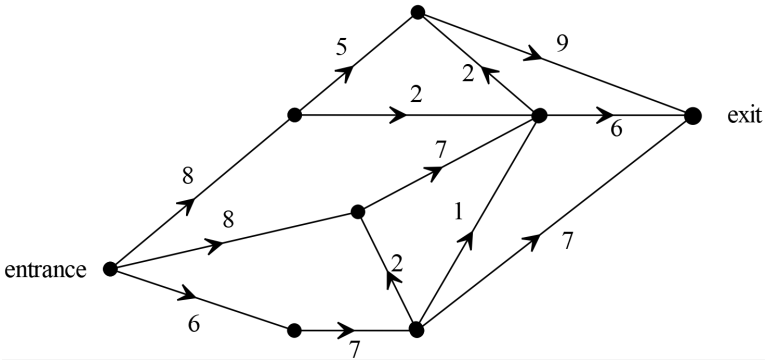
D. 24

E. 25

Module 2: Networks and decision mathematics

Question 7

A modern art exhibition involves visitors moving through a number of galleries.
The flow of people through the galleries from entrance to exit is shown in the network below.
The number on the edges show the maximum flow through each gallery in visitors per minute.



The maximum flow, in visitors per minute, from entrance to exit is

A. 19

B. 20

C. 21

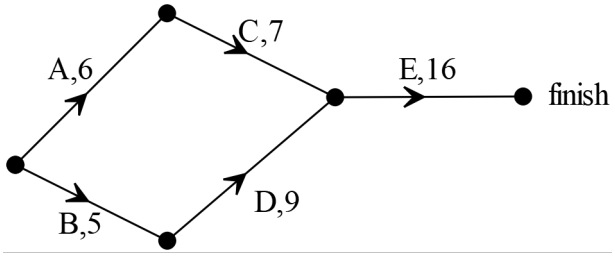
D. 22

E. 23

Module 2: Networks and decision mathematics

Question 8

The graph below shows five activities, A, B, C, D and E that must be completed to finish building a garden shed.
The time, in days, that each activity is expected to take is shown on the corresponding edge.



It is found that each of the five activities can have its completion time reduced by a maximum of one day at a cost of $250 per day.
The least cost to achieve the greatest reduction in time taken to complete the garden shed is

A. $500

B. $750

C. $1000

D. $1250

E. $1500

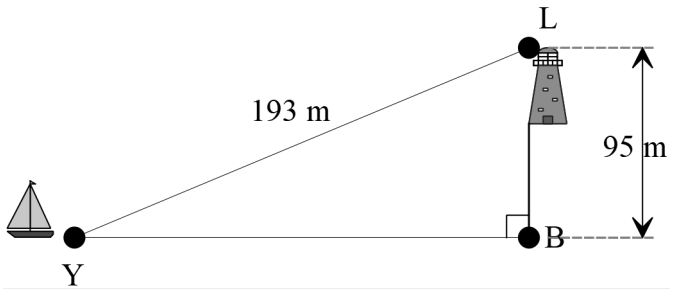
End of Module 2

Module 3: Geometry and measurement

Before answering these questions you **must** shade the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The direct distance between a yacht at point Y and the top of a lighthouse at point L is 193 metres. The base of the vertical cliff on which the lighthouse stands is at point B. The vertical distance from the base of the cliff to the top of the lighthouse at point L is 95 metres.



The distance, in metres, from Y to B is

A. 98

B. 168

C. 192

D. 215

E. 288

Question 2

Which one of the following locations is closest to the South Pole?

A. 12°S 72°W

B. 72°N 05°E

C. 42°S 125°E

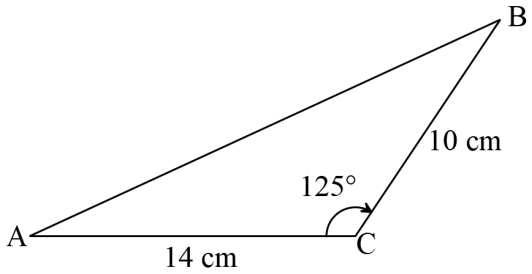
D. 15°S 42°W

E. 36°S 05°W

Module 3: Geometry and measurement

Question 3

Triangle ABC is shown below.



The perimeter, in centimetres of triangle ABC is closest to

A. 21

B. 22

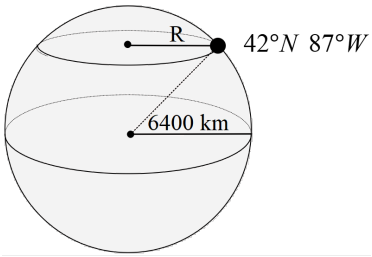
C. 37

D. 45

E. 46

Question 4

Lake Michigan in the USA is located at location 42° N 87° W as shown on the diagram below.



Assume the radius of the Earth is 6400 km.
A calculation that will give the radius, R, in kilometres of the small circle (parallel) passing through Lake Michigan is

A. $6400\times \sin 42°$

B. $6400\times \sin 93°$

C. $6400\times \cos 93°$

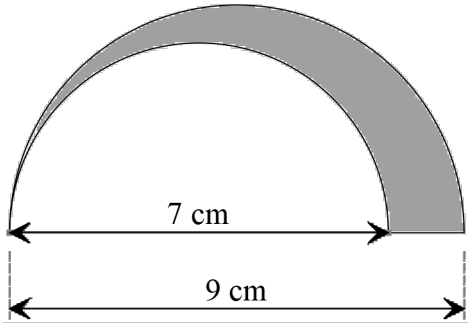
D. $6400\times \cos 87°$

E. $6400\times \cos 42°$

Module 3: Geometry and measurement

Question 5

The logo for a surfboard making company is in the shape of a semicircle with a smaller semicircle cut out as shown in the diagram below.



The diameter of the larger semicircle is 9 centimetres and the diameter of the smaller cut out semicircle is 7 cm.
The shaded area, in cm^2 is equal to

A. $\frac{5\pi}{2}$

B. 3π

C. $\frac{7\pi}{2}$

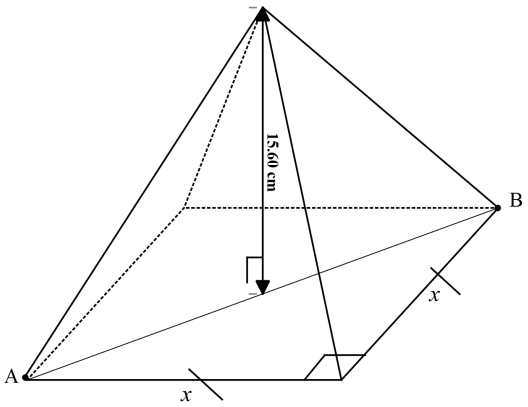
D. 4π

E. 5π

Module 3: Geometry and measurement

Question 6

A right pyramid, as shown below, has a square base with side length x cm. The vertical height of the pyramid is 15.60 cm.
The volume of the pyramid is 84.50 cm^3.



The length, in cm, of the diagonal of the base, AB, is closest to

A. 5.70

B. 4.03

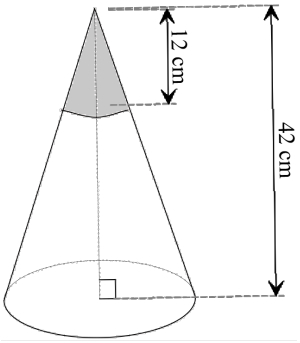
C. 6.25

D. 6.76

E. 5.25

Module 3: Geometry and measurement

Question 7
A clown's hat is made from an open-ended cone. The surface of the top part of the cone is painted, as shown in the diagram below.



The vertical height of the cone is 42 cm. The vertical height of the painted section is 12 cm.
The ratio of the area of the painted surface to the area of the non-painted surface is

A. 2 : 7

B. 2 : 23

C. 4 : 45

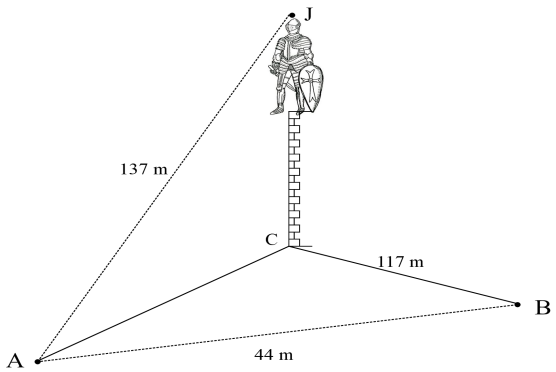
D. 1 : 4

E. 3 : 26

Module 3: Geometry and measurement

Question 8

A jacquemart stands atop a vertical tower. Points A and B are in the same horizontal plane as the base of the tower, C.
Point C is directly north of point A.
The direct distance from point A to the top of the jacquemart, J, is 137 m.
The direct distance from A to B is 44 m.
The direct distance from B to C is 117 m.



The bearing of B from A is 082°
The bearing of B from C is 172°
The distance from the base of the tower, C, to the top of the jacquemart, J is closest to

A. 71 metres

B. 56 metres

C. 108 metres

D. 113 metres

E. 63 metres

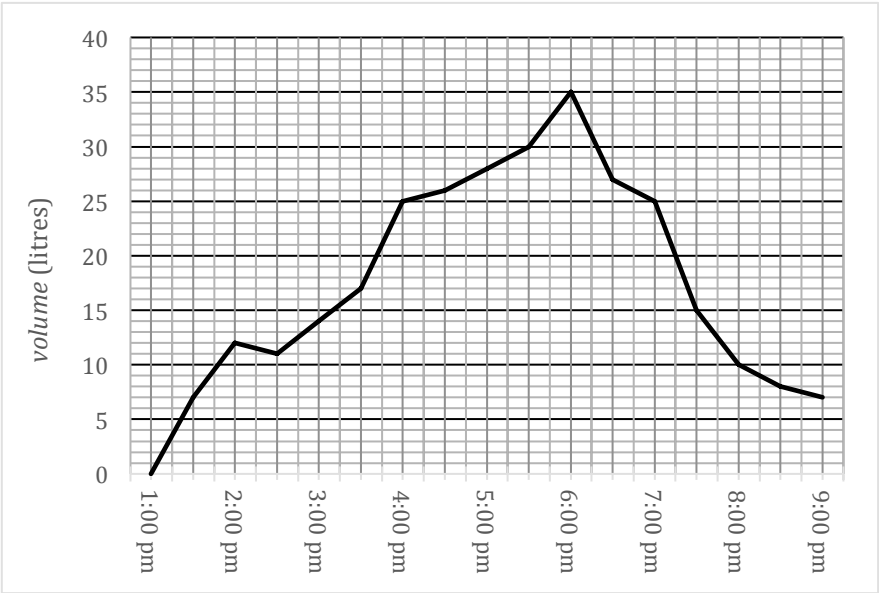
End of Module 3

Module 4: Graphs and relations

Before answering these questions you **must** shade the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Use the following information to answer questions 1 and 2

The graph below shows the volume, in litres, of water in a roof gutter from 1:00 pm to 9:00 pm on a rainy day.



Question 1

The difference in the volume of water, in litres, in the gutter between 5:00 pm and 7:30 pm is

A. 21

B. 15

C. 13

D. 10

E. 7

Module 4: Graphs and relations

Question 2
The gutter is considered blocked whenever the volume of water in it is 20 litres or more.
The time, in minutes, that the gutter is blocked is closest to

A. 215

B. 180

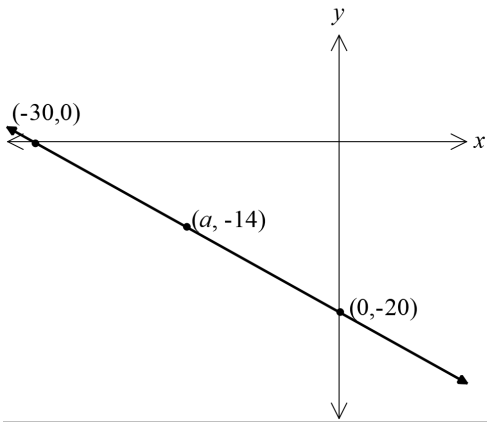
C. 340

D. 225

E. 195

Question 3

A straight line passes through the points (-30, 0) and (0, -20) as shown on the graph below.
The point (a, -14) lies on the line.



The value of a is

A. -15

B. -8

C. -9

D. -12

E. -10

Module 4: Graphs and relations

Question 4

Miles employs four workers to replace a roof gutter for a client. He pays each worker $45 per hour.
The fixed costs for this job are $560 and it will take five hours to complete the job.
To make a profit of $750 on this job, Miles' hourly charge to the client should be

A. $397

B. $442

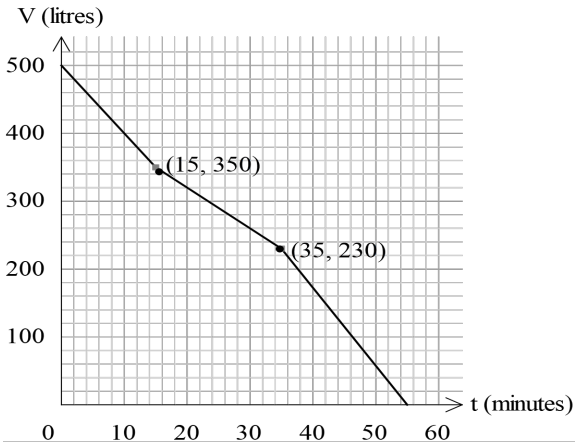
C. $330

D. $552

E. $484

Question 5

A water tank is being emptied.
The graph below shows the volume of water, V litres, in the tank at time t minutes.



The equation of this line between $t = 15$ and $t = 35$ minutes is

A. $V = 633 - 12t$

B. $V = 350 - 8t$

C. $V = 500 - 6t$

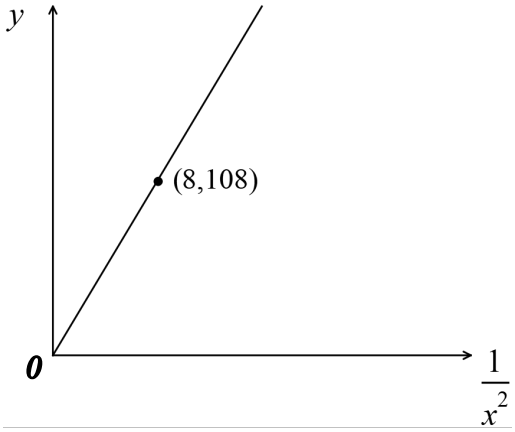
D. $V = 633 - 8t$

E. $V = 440 - 6t$

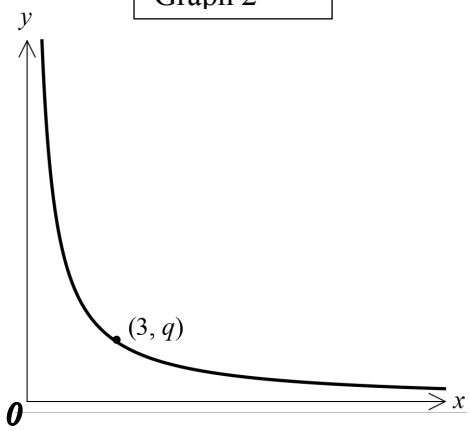
Module 4: Graphs and relations

Question 6
The graphs below represent the same relationship, $y = kx^n$.

Graph 1



Graph 2



Graph 2 passes through the point $(3, q)$.
The value of q is

A. $\frac{3}{8}$

B. $\frac{27}{2}$

C. $\frac{27}{8}$

D. $\frac{3}{2}$

E. $\frac{81}{2}$

Module 4: Graphs and relations

Question 7

Sarah is hiring cups and saucers for a tea party.
Cups cost \$2.70 each to hire and saucers cost \$1.90 each to hire.
Let x be the number of cups hired.
Let y be the number of saucers hired.
Sarah can spend no more than \$7 for each of her 40 guests on hiring cups and saucers.
An inequality that represents this constraint is

A. $2.7x + 1.9y \leq 280$

B. $1.9x + 2.7y \leq 7$

C. $1.9x + 2.7y \leq 40$

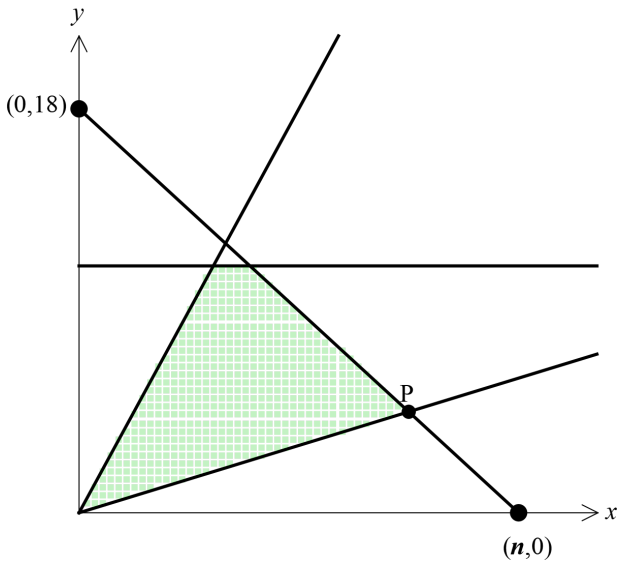
D. $2.7x + 1.9y \leq 40$

E. $1.9x + 2.7y \leq 280$

Module 4: Graphs and relations

Question 8

In the diagram below, the shaded region (with boundaries included) represents the feasible region for a linear programming problem.



For the objective function $Z = 24x + 20y$ to be a maximum at point P only, the value of n could be

A. 12

B. 15

C. 11

D. 16

E. 14

End of Module 4

End of 2021 Further Mathematics Trial Examination 1

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FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index$= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core: Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1: Matrices

determinant of a 2×2 matrix:	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix:	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$
recurrence relation:	$S_0 =$ initial state, $\quad S_{n+1} = TS_n + B$

Module 2: Networks and decision mathematics

Euler's formula:	$v + f = e + 2$

Module 3: Geometry and measurement

area of a triangle:	$A = \frac{1}{2}bc\sin(\theta^0)$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle:	$2\pi r$
length of an arc:	$r \times \frac{\pi}{180} \times \theta^0$
area of a circle:	πr^2
area of sector	$\pi r^2 \times \frac{\theta^0}{360}$
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a prism:	area of base $\times$ height
volume of a pyramid:	$\frac{1}{3}$ $\times$ area of base $\times$ height

Module 4: Graphs and relations

gradient (slope) of a straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line:	$y = mx + c$

END OF FORMULA SHEET

VCE FURTHER MATHEMATICS 2021
Trial Written Examination 1
ANSWER SHEET

NAME: _____

STUDENT NUMBER _____

SIGNATURE _____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
  If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	B	C	D	E

VCE FURTHER MATHEMATICS 2021
Trial Written Examination 1
ANSWER SHEET

NAME: _____

STUDENT NUMBER _____

SIGNATURE _____

Instructions

- Write your name in the space provided above.
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- All answers must be completed like **THIS** example.

A	B	C	D	E

Section A

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E
23	A	B	C	D	E
24	A	B	C	D	E

Please turn over . . .

VCE FURTHER MATHEMATICS 2021
Trial Written Examination 1
ANSWER SHEET

Section B
(Shade the box of the one module selected **and** write the name of the module you have selected. There are a total of four from which to choose)

	Matrices	**Module 1**	**1**	A	B	C	D	E
		_____	**2**	A	B	C	D	E
			3	A	B	C	D	E
		_____	**4**	A	B	C	D	E
			5	A	B	C	D	E
		_____	**6**	A	B	C	D	E
			7	A	B	C	D	E
			8	A	B	C	D	E
	Networks and decision mathematics	**Module 2**	**1**	A	B	C	D	E
		_____	**2**	A	B	C	D	E
			3	A	B	C	D	E
		_____	**4**	A	B	C	D	E
			5	A	B	C	D	E
		_____	**6**	A	B	C	D	E
			7	A	B	C	D	E
			8	A	B	C	D	E
	Geometry and measurement	**Module 3**	**1**	A	B	C	D	E
		_____	**2**	A	B	C	D	E
			3	A	B	C	D	E
		_____	**4**	A	B	C	D	E
			5	A	B	C	D	E
		_____	**6**	A	B	C	D	E
			7	A	B	C	D	E
			8	A	B	C	D	E
	Graphs and relations	**Module 4**	**1**	A	B	C	D	E
		_____	**2**	A	B	C	D	E
			3	A	B	C	D	E
		_____	**4**	A	B	C	D	E
			5	A	B	C	D	E
		_____	**6**	A	B	C	D	E
			7	A	B	C	D	E
			8	A	B	C	D	E

Please DO NOT fold, bend or staple this form