# Neap

Trial Examination 2021

# VCE Further Mathematics Units 3&4

Written Examination 1

## Suggested Solutions

## SECTION A – CORE

### Data analysis

### Question 1   D

**D** is correct. There are 16 pieces of data, so to find the mean, all the figures must be added together and then divided by 16. The three 0s are counted as pieces of data, but will not add to the total that is divided. **A** and **C** are incorrect. They do not count the 0s as pieces of data, counting only 13 instead of 16. **B** and **E** are incorrect. They divide by the amount of different numbers in the set (5), not the number of pieces of data.

### Question 2   E

**E** is correct. The mode is the most frequent number. All the frequency tells us is how many figures are in each interval, but not what each figure individually is. **A**, **B**, **C** and **D** are incorrect. If the three figures in the first interval were all 0 and every other figure in the set had a frequency of 1, then the mode would be 0. The same argument can be made for any number between 0 and 39, and so the mode can only be determined to be one of those numbers, but none of them more specifically.

### Question 3   E

**E** is correct. This is the only incorrect statement, as the five-number summary tells us nothing about the mode. The mode could be any number between 0 and 52. **A** is incorrect. This statement correctly calculates the IQR as 21 – 7 = 14. **B** is incorrect. This statement correctly finds the median, the centre figure, which is 20. **C** is incorrect. This statement is correct; the IQR is 21 – 7 = 14, the upper fence is at $21 + 1.5 \times 14 = 42$, and 52 is outside this and therefore an outlier. **D** is incorrect. This statement correctly identifies that there is a greater spread of figures above 20 than below.

### Question 4   E

The histogram uses a $\log_{10}$ scale. The 3 means $\log_{10}1000 = 3$. The total number of people who earn less than \$1000 per week is $25 + 50 + 150 = 225$.

### Question 5   C

$$\frac{24 + 20 + 19}{3} = 21$$

### Question 6   D

**D** is correct. A time series plot would show how the population had changed over time and reveal any pattern. **A** is incorrect. A frequency table would tell us how many times each figure appears, but this is not a useful way to display this information. **B** is incorrect. A histogram is used for continuous data and this data is discrete. **C** is incorrect. A residual plot is used for assessing the correctness of a linear assumption for bivariate data. **E** is incorrect. A scatterplot is used for bivariate data.

**Question 7   C**

**C** is correct. The junior members number $99 + 21 = 120$ members, the adult members number $64 + 66 = 130$ members (which is the largest group) and the senior members number $26 + 90 = 116$. **A** is incorrect. The club has more female members than male members. **B** is incorrect. The number of members who voted for a picnic is $21 + 66 + 90 = 177$, which is less than the number that voted for a tournament $(99 + 64 + 26 = 189)$. **D** is incorrect. There are 366 members, not 364. **E** is incorrect. There are 68 male adult members, not 63.

**Question 8   C**

**C** is correct. A positive skew means the data is not symmetrical and, in this case, there are more values at the left-hand end of the data set, which indicates a positive skew. **A** is incorrect. The range is 55 for the action film and 55 for the drama film. **B** is incorrect. The median age is 44 but the mean age to one decimal place is $\dfrac{812}{19} = 43.3$. **D** is incorrect. 16 over-30s attended the drama film while 22 attended the action film. **E** is incorrect. Since there are three equally popular figures for each set, there is no mode for either set of data.

**Question 9   A**

**A** is correct. To find if 140 is an outlier, the IQR is $70 - 20 = 50$. The upper fence is therefore $70 + 1.5 \times 50 = 145$. 140 is inside the fence, so there are no outliers. As there are no outliers, the mean and standard deviation should be used. **B**, **C** and **D** are incorrect. The IQR and median are used to remove outliers from the data set. **E** is incorrect. The mode will not represent the central tendency of this data set.

**Question 10   E**
**Method 1:**

The gradient is calculated first.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 218}{50 - 10}$$
$$= \frac{-128}{40}$$
$$= -3.2$$

Substitute one of the points into the equation.

| | |
|---|---|
| $y = a - 3.2x$ | $y = a - 3.2x$ |
| $90 = a - 3.2 \times 50$ | $218 = a - 3.2 \times 10$ |
| $90 = a - 160$ | $218 = a - 32$ |
| $a = 250$ | $a = 250$ |

So, the equation is $y = 250 - 3.2x$.

**Method 2:**

Solve using CAS.

| A x | B y | C | D |
|---|---|---|---|
| | | | =LinRegB |
| 10. | 218. | Title | Linear R... |
| 50. | 90. | RegEqn | a+b*x |
| | | a | 250. |
| | | b | -3.2 |

**Question 11   C**

**C** is correct. The average sales would occur if the seasonal index were 1. A seasonal index of 1.6 represents a 60% increase in the average sales. Since there are only two periods, the seasonal indices must add to 2. The seasonal index for the warm half of the year is therefore 2 – 1.6 = 0.4. **A** is incorrect. There is an increase of 60% in the average sales, not 60% of the total sales. **B** is incorrect. It assumes there are 4 seasons and the seasonal indices must add to 4. **D** is incorrect. 36 comes from calculating a value of $0.6^2$, implying a calculation of $r^2$, which is not appropriate. **E** is incorrect. The sales are reduced to 40% of the average, not reduced by 40%.

**Question 12   E**

**E** is correct. In a symmetrical data set (such as a normal distribution), 95% of the data points will be within two standard deviations of the mean. 95% of the packets will weigh between 200 and 220. This leaves 5%, which is split 2.5% above 220 and 2.5% below 200. 2.5% of 1000 = 25, so 25 packets of chips would be expected to weight below 200 g. **A** is incorrect. In this normal distribution, there would be expected to be more than zero packets of chips weighing less than 200 g. **B** is incorrect. 2.5 is the percentage of packets of chips that would be expected to weight less than 200 g, not the number of packets of chips. **C** and **D** are incorrect. These options may be reached by incorrectly calculating the answer.

**Question 13   C**

Substituting 80 for $x$ into the equation gives a value of $y = 314$.

$$\text{residual} = \text{actual value} - \text{predicted value}$$
$$= 320 - 314$$
$$= 6$$

**Question 14   E**

**E** is correct. The general formula is $y = a + bx$ (or $y = ax + b$). From the information given, $a = 14.6$, $b = -2.1$ and $x$ has been transformed to $\frac{1}{x}$. Substituting into the formula gives $y = 14.6 - \frac{2.1}{x}$. **A** is incorrect. The $r$ value does not give the gradient and the transformation has not been applied. **B** is incorrect. The $\frac{1}{x}$ transformation has not been applied. **C** is incorrect. The gradient and $y$-intercept have been switched and the transformation has not been applied. **D** is incorrect. The gradient has not been included.

**Question 15    D**

**Method 1:**

The formula for $z$ is $z = \dfrac{x - \bar{x}}{s_x}$.

Substitute into the equation and solve for $x$.

$$0.8 = \frac{x - 250}{10}$$
$$0.8 \times 10 = x - 250$$
$$8 + 250 = x$$
$$x = 258$$

258 is closest to 260 g.

**Method 2:**

Solve using CAS.

$$\text{solve}\left(0.8 = \frac{x - 250}{10}, x\right) \qquad x = 258.$$

258 is closest to 260 g.

**Question 16    C**

The slope of the scatterplot is negative; therefore, $r$ is negative.

Since $r^2 = 0.81$, $r = -0.9$.

The gradient can be calculated using:

$$b = r \times \frac{S_y}{S_x}$$
$$= -0.9 \times \frac{0.78}{115.6}$$
$$= -0.006$$

This means the hours of sleep are decreased by 0.006 hours for every minute of social media use.

$$0.006 \text{ hours} = 0.006 \times 60$$
$$= 0.36 \text{ minutes}$$
$$0.36 \times 60 = 21.6 \text{ seconds}$$

## Recursion and financial modelling

**Question 17   D**

**D** is correct. The first term is $t_1 = 2$; therefore, the only possible correct answers are **D** and **E**. Each next term is the previous term multiplied by 4, which eliminates **E**. **A** is incorrect. This option does not specify the first term. **B** is incorrect. This rule only works for the first pair of terms. **C** is incorrect. The first term in this option is incorrect.

**Question 18   E**

Calculate the first, second and third terms, beginning with $t_0 = 5$.

$t_{n+1} = -2t_n + 20$
$\quad t_0 = 5$
$\quad t_1 = -2 \times 5 + 20 = 10$
$\quad t_2 = -2 \times 10 + 20 = 0$
$\quad t_3 = -2 \times 0 + 20 = 20$

Therefore, $t_1$ and $t_3$ are, respectively, 10 and 20.

**Question 19   D**

Let $L$ be the length of the top rung, $n$.

The formula is $L = 80 \times 0.9^{(n-1)}$.

For $n = 6$:

$L = 80 \times 0.9^{(5)}$
$\quad = 47.2392$
$\quad \approx 47.2$

Alternatively, the length of each rung can be calculated. The rounding should only be applied on the final term in the calculation.

Rung $1 = 80$ cm
Rung $2 = 80 \times 0.9 = 72$ cm
Rung $3 = 72 \times 0.9 = 64.8$ cm
Rung $4 = 64.8 \times 0.9 = 58.32$ cm
Rung $5 = 58.32 \times 0.9 = 52.488$ cm
Rung $6 = 52.488 \times 0.9 = 47.2392$ cm

**Question 20   D**

**D** is correct. The formula $A = PR^n$ calculates the new balance. To find the interest earned, subtract the original principal of $20\,000$. $R$ is calculated using $R = 1 + \dfrac{r}{100}$, where $r$ is, in this case, the monthly interest rate of $\dfrac{3.5}{12}$. **A** and **B** are incorrect. These options do not subtract the original principal. **C** is incorrect. This option gives an incorrect value for $R$. **E** is incorrect. This option does not alter the period of 2 years to 24 months, as the interest is compounded monthly, not annually.

**Question 21    A**

**A** is correct. The total depreciation over 10 years is $2\,450\,000 - 50\,000 = \$2\,400\,000$  The depreciation each year is therefore $\dfrac{2\,400\,000}{10} = \$240\,000$  per year. This makes the value after $n$ years $2\,450\,000 - 240\,000n$. **B** is incorrect. This option adds the depreciation in place of subtracting it. **C** is incorrect. This option subtracts the selling price from the initial cost. **D** is incorrect. This option subtracts the selling price from the initial cost and adds the depreciation in place of subtracting it. **E** is incorrect. This option does not subtract the correct end-of-use sale price.

**Question 22    D**

**D** is correct. Find the values of $P$, $r$ and $t$.

$$R = 1 + \frac{r}{100}$$

$$1.06 = 1 + \frac{r}{100}$$

$$0.06 = \frac{r}{100}$$

$$r = 6\%$$

**A** and **B** are incorrect. These options give values for $R$, not $r$. **C** is incorrect. The term is 5 years, which is the same as 60 months, but the question specifically asks for the term in years. **E** is incorrect. This option gives the incorrect value for $r$.

**Question 23    D**

**Method 1:**

Use the financial package on an approved technology.

Option 1:

```
N=730
I%=4.05
PV=-100000
PMT=0
FV=108436.6024
P/Y=365
C/Y=365
PMT:END BEGIN
```

Option 2:

```
N=24
I%=4.09
PV=-100000
PMT=0
FV=108508.7811
P/Y=12
C/Y=12
PMT:END BEGIN
```

Option 2 returns a higher amount by $108\,508.78 - 108\,436.60 = 72.18$. Rounding to the nearest dollar gives $72.

**Method 2:**

Manually calculate.

Option 1:

$A = PR^n$

$n$ is the number of compounding periods, which is $365 \times 2 = 730$.

$R = 1 + \dfrac{r}{100}$

$\phantom{R} = 1 + \dfrac{\frac{4.05}{365}}{100}$

$A = 100\,000\left(1 + \dfrac{\frac{4.05}{365}}{100}\right)^{730}$

$\phantom{A} = 108\,436.60$

Option 2:

$A = PR^n$

$n$ is the number of compounding periods $12 \times 2 = 24$.

$R = 1 + \dfrac{r}{100}$

$\phantom{R} = 1 + \dfrac{\frac{4.09}{12}}{100}$

$A = 100\,000\left(1 + \dfrac{\frac{4.09}{12}}{100}\right)^{24}$

$\phantom{A} = 108\,508.78$

Option 2 returns a higher amount by $108\,508.78 - 108\,436.60 = 72.18$. Rounding to the nearest dollar gives $72.

**Question 24    C**

**C** is correct. The value after inflation is:

$A = PR^n$

$\phantom{A} = 4500\left(1 + \dfrac{1.8}{100}\right)^{10}$

$\phantom{A} = 5378.86$

The profit is therefore $6000 - 5378.86 = \$621.14$.

$\text{percentage profit} = \dfrac{621.14}{4500} \times 100$

$\phantom{\text{percentage profit}} = 13.8\%$

**A** and **B** are incorrect. The question asks for a percentage profit. **D** is incorrect. This option does not allow for inflation. **E** is incorrect. The investment will turn a profit over the given period.

## SECTION B – MODULES

### Module 1 – Matrices

### Question 1   C
C is correct. There are 3 rows and 2 columns in matrix $A$, so the order is $3 \times 2$. **A** is incorrect. This option gives 2 rows and 3 columns. **B** is incorrect. This option gives the amount of numbers in matrix $A$. **D** is incorrect. This option gives 3 rows and 3 columns. **E** is incorrect. This option gives the sum of values in matrix $A$.

### Question 2   A
$$\begin{bmatrix} -1+2 & 4+1 \\ -3+0 & 7+0.5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -3 & 7.5 \end{bmatrix}$$

### Question 3   D
D is correct. The only matrix product that is defined is $(AD)^2$. **A**, **C** and **E** are incorrect. A matrix product is possible when the number of columns in the first matrix is equal to the number of rows in the second matrix. **B** is incorrect. While the matrix product $AB$ is defined, it does not produce a matrix that is able to be squared.

### Question 4   D
$2 \times 0.5 - 0 \times 1 = 1$

### Question 5   D
D is correct. A $1 \times 3$ matrix being multiplied by a $3 \times 2$ matrix will result in a $1 \times 2$ matrix, which will show the sum of the columns of the matrix. **A**, **B** and **C** are incorrect. These matrix calculations are not possible with matrix $A$. **E** is incorrect. This calculation will not give the sum of the columns for matrix $A$.

### Question 6   C
C is correct. If every element of matrix $A$ is multiplied by 1.5, it will be equal to $\begin{bmatrix} 6 & 9 \\ 3 & 10.5 \\ 4.5 & 6 \end{bmatrix}$.

**A** is incorrect. This scalar will produce the matrix $\begin{bmatrix} 4 & 6 \\ 2 & 7 \\ 3 & 4 \end{bmatrix}$.

**B** is incorrect. This scalar will produce the matrix $\begin{bmatrix} 5 & 7.5 \\ 2.5 & 8.75 \\ 3.75 & 5 \end{bmatrix}$.

**D** is incorrect. This scalar will produce the matrix $\begin{bmatrix} 7 & 10.5 \\ 3.5 & 12.25 \\ 5.25 & 7 \end{bmatrix}$.

**E** is incorrect. This scalar will produce the matrix $\begin{bmatrix} 8 & 12 \\ 4 & 14 \\ 6 & 8 \end{bmatrix}$.

**Question 7   B**
**Method 1:**

Using the formula from the data sheet for the inverse of a matrix:

$$\frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{(-1\times 7)-(-3\times 4)}\begin{bmatrix} 7 & -4 \\ 3 & -1 \end{bmatrix}$$

$$= \frac{1}{(-7)-(-12)}\begin{bmatrix} 7 & -4 \\ 3 & -1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 7 & -4 \\ 3 & -1 \end{bmatrix}$$

**Method 2:**

Using an approved technology and the formula from the data sheet for the inverse of a matrix:

$$\begin{bmatrix} -1 & 4 \\ -3 & 7 \end{bmatrix}^{-1} \qquad \begin{bmatrix} \dfrac{7}{5} & \dfrac{-4}{5} \\ \dfrac{3}{5} & \dfrac{-1}{5} \end{bmatrix}$$

Removing the common factor of $\dfrac{1}{5}$ gives $\dfrac{1}{5}\begin{bmatrix} 7 & -4 \\ 3 & -1 \end{bmatrix}$.

**Question 8   C**

This question can be solved by process of elimination. By putting each matrix into the equation, the only

matrix that returns the result of $\begin{bmatrix} 1100 \\ 1000 \\ 900 \end{bmatrix}$ is $\begin{bmatrix} 837 \\ 691 \\ 1472 \end{bmatrix}$.

**Module 2 – Networks and decision mathematics**

**Question 1   D**

**D** is correct. There are 4 edges leaving *C* that go to another vertex and one loop that has 2 ends,
so the degree of *C* is 6. **A**, **B**, **C** and **E** are incorrect. These options do not give the correct degree of *C*.
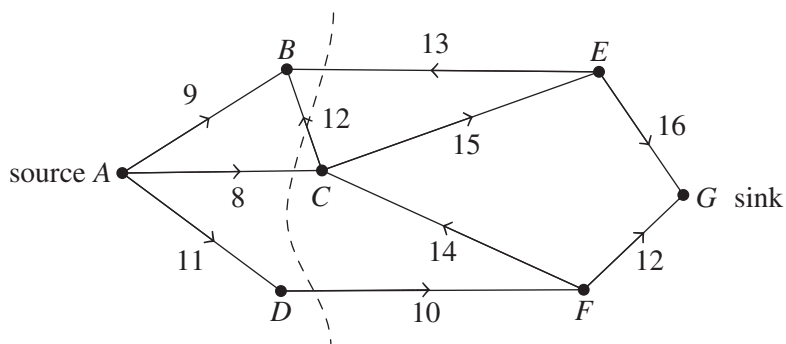
**Question 2   A**

**A** is correct. *A–B–C* is equal to 39, which is the shortest path. **B** is incorrect. *A–E–C* is equal to 40, which
is not the shortest path. **C** is incorrect. There is no path from *A* to *C* that is equal to 48. **D** is incorrect.
*A–E–B–C* is equal to 53, which is not the shortest path. **E** is incorrect. *A–D–C* is equal to 56, which
is not the shortest path.

**Question 3   C**

**C** is correct. There are 4 faces enclosed by the edges of the graph and 1 face around the graph for a total
of 5 faces. **A** is incorrect. This option may be reached by not considering the loop or the face around the
edge. **B** is incorrect. This option may be reached by not counting the face around the graph. **D** and **E** are
incorrect. These options may be reached by counting too many faces for the graph.

**Question 4    A**

The cut on the diagram below shows the minimum cut, which is the maximum flow.



The maximum flow is 18.

**Question 5    D**

**D** is correct. The critical path of this project is *B–D–C–G–I*, which has a total of 15. **A** and **B** are incorrect. *B–F–G–I* and *B–E–H–I* are not the critical path as they ignore the dummy event, task *D*. **C** is incorrect. *A–C–G–I* bypasses task *B,* as well as the dummy event. **E** is incorrect. There is no viable path that has a total of 16.

**Question 6    B**

**B** is correct. Due to the dummy event, task *C* requires task *B* to be complete before it can start, which means the earliest possible start time is 3 days. **A, C, D** and **E** are incorrect. These options do not give the correct earliest possible start time for task *C*.

**Question 7    B**

**B** is correct. Task *F* has to be complete by day 10 to not impact the critical path. Task *F* takes 4 days, so the latest it can start is day 6. Task *F* can start on day 3 at the earliest. The difference between earliest and latest start is 3 days. **A, C, D** and **E** are incorrect. These options do not give the correct float time for task *F*.

**Question 8    A**

**A** is correct. A reduction of 3 days for task *C* will reduce the overall completion time. **B, C** and **E** are incorrect. Tasks *E, F* and *H* are not on the critical path. **D** is incorrect. The reduction in time given for task *G* is less than the reduction given for task *C*.

**Module 3 – Geometry and measurement**

**Question 1    B**

$$\sqrt{8.5^2 + 6.2^2} = 10.520932$$
$$\approx 10.5$$

**Question 2    E**

$$\tan^{-1}\left(\frac{8.5}{6.2}\right) = 53.89254887082$$
$$\approx 53.9°$$

**Question 3   B**

**B** is correct. The area would increase by the square of the scale factor of the side lengths ($1.75^2 = 3.0625$).
**A**, **C**, **D** and **E** are incorrect. These options do not give the correct scale factor for the area of the triangle.

**Question 4   C**

$$\text{solve}\left(132 = \frac{x}{360} \cdot (8.4)^2 \cdot \pi, x\right)$$

$$x = 214.371964165$$

$\theta = 215.371964165$

$\approx 214.37$

**Question 5   D**

$\dfrac{48}{360} \times \pi \times 14 \times 2 = 11.7286125734$

$\approx 12$ cm

**Question 6   C**

$14\cos(24) = 12.789636407$

$\approx 12.79$ cm

**Question 7   B**

$$\sqrt{\frac{21}{2} \times \left(\frac{21}{2} - 7\right) \times \left(\frac{21}{2} - 6\right) \times \left(\frac{21}{2} - 8\right)} = 20.3331625676$$

$\approx 20$ cm$^2$

**Question 8   C**

$\dfrac{1}{3} \times 6 \times 8 \times 12 = 192$ cm$^3$

**Module 4 – Graphs and relations**

**Question 1   E**

**E** is correct. There are two periods where the temperature is less than 25°C: from 12 midnight until 12 noon (12 hours) and from 6 pm until midnight (6 hours). This is a total of 18 hours. **A**, **B**, **C** and **D** are incorrect. These options do not give the correct amount of time that the temperature was less than 25°C.

**Question 2   A**

The $y$-intercept is 15, so the equation is in the form $y = ax + 15$.

Calculate the gradient.

$\dfrac{\text{rise}}{\text{run}} = \dfrac{-15}{5}$

$= -3$

The equation is therefore $y = -3x + 15$.

This is rearranged to give $y + 3x - 15 = 0$.

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**Question 3   E**

**E** is correct. This option includes all constraints, including that $x \leq 600$. **A** and **B** are incorrect. These options may be reached by only attempting to define one constraint. **C** is incorrect. Both $x$ and $y$ must be greater than 0, and this option does not include the possibility of both being zero. **D** is incorrect. This option does not include a lower boundary for $x$.

**Question 4   E**

**E** is correct. This option gives the correct values and domains for all sections of the graph. **A, B** and **C** are incorrect. There are four parts to the graph, and these options do not have four equations. **D** is incorrect. $x \leq 1$ is not an appropriate domain, as the line has an open circle at the end.

**Question 5   D**

**D** is correct.
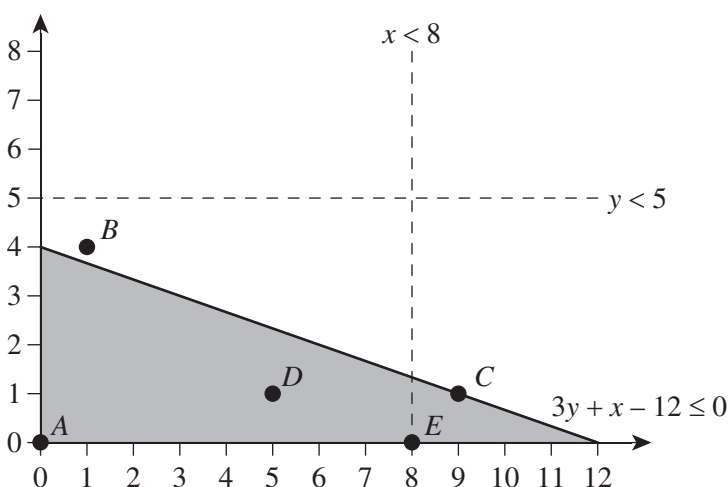


A quick sketch of the graphs shows only the point (5, 1) is within the boundaries. **A** and **E** are incorrect. These points lie on the boundary, not within it. **B** is incorrect. The point (1, 4) is above the boundary. **C** is incorrect. The point (9, 1) is to the right of the boundary.

Alternatively, find the corresponding $y$-value for $x = 2$ and $x = 5$ by substituting into the equation $3y + x - 12 \leq 0$:

$$3y + (2) - 12 = 0$$
$$3y = 10$$
$$y = \frac{10}{3}$$

Since $5 > \dfrac{10}{3}$ , this is outside the region.

$$3y + (5) - 12 = 0$$
$$3y = 7$$
$$y = \frac{7}{3}$$

Since $1 < \dfrac{7}{3}$ , this is inside the region.

**Question 6   C**

The general formula is $h = ax^2 + b.$

Using the point (0, 20):

$$h = ax^2 + b$$
$$20 = a(0)^2 + b$$
$$b = 20$$

Using the point (100, 50):

$$h = ax^2 + b$$
$$50 = a(100)^2 + 20$$
$$30 = a \times 10\,000$$
$$a = \frac{30}{10\,000}$$
$$= 0.003$$

The equation is therefore $h = 0.003x^2 + 20.$

**Question 7   B**

Substitute (10, 45) into the equation $y = kx^2$ and solve for $k$:

$$y = kx^2$$
$$45 = k(10)^2$$
$$45 = 100k$$
$$k = 0.45$$

**Question 8   D**

Create a pair of simultaneous equation from the given information.

Let $a =$ the cost of an adult ticket.

Let $c =$ the cost of a child ticket.

$$3a + 2c = 600$$
$$5a = 700$$

Since $5a = 700$:

$$a = \frac{700}{5}$$
$$= \$140$$

Substitute into $3a + 2c = 600$:

$$3a + 2c = 600$$
$$3(140) + 2c = 600$$
$$420 + 2c = 600$$
$$2c = 180$$
$$c = 90$$

The combined cost of one adult ticket and one child ticket is $140 + 90 = \$230.$