VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2021**

# FURTHER MATHEMATICS

## Written examination 1

**Thursday 27 May 2021**

**Reading time: 10.00 am to 10.15 am (15 minutes)**
**Writing time: 10.15 am to 11.45 am (1 hour 30 minutes)**

## MULTIPLE-CHOICE QUESTION BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
|---|---|---|---|---|---|
| A – Core | 24 | 24 | | | 24 |
| B – Modules | 32 | 16 | 4 | 2 | 16 |
| | | | | | Total 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question book of 35 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

**Instructions**
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**At the end of the examination**
- You may keep this question book and the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**THIS PAGE IS BLANK**

**THIS PAGE IS BLANK**

## SECTION A – Core

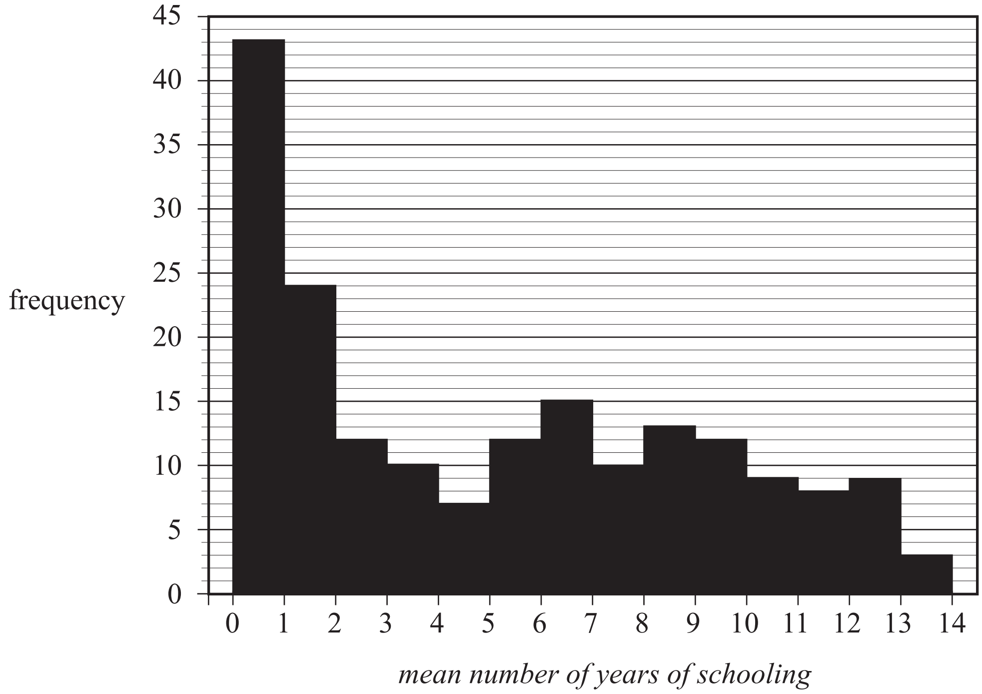
### Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

## Data analysis

*Use the following information to answer Questions 1 and 2.*

The histogram below shows the distribution of the *mean number of years of schooling* of women aged 65 years and older in 187 countries in 2015.




Data: Institute for Health Metrics and Evaluation (IHME), 'Global Educational Attainment, 1970–2015', Seattle (USA), 2015

### Question 1

The percentage of these 187 countries where the *mean number of years of schooling* of women aged 65 years and older is less than one year is closest to

**A.** 13%

**B.** 23%

**C.** 33%

**D.** 36%

**E.** 43%

**Question 2**

The median of the *mean number of years of schooling* of women aged 65 years and older in these 187 countries is closest to

**A.** 2.8

**B.** 3.2

**C.** 4.4

**D.** 5.7

**E.** 6.2

**Question 3**

The variables *number of family members* and *age group* (15–24 years, 25–34 years, …) are

**A.** both numerical variables.

**B.** both categorical variables.

**C.** a numerical variable and an ordinal variable respectively.

**D.** a numerical variable and a nominal variable respectively.

**E.** a nominal variable and a continuous variable respectively.

**Question 4**

A class of 25 children in Year 5 (15 girls and 10 boys) sat for a mathematics test.

The mean score on the test for the 15 girls in the class was 28.5

The mean score on the test for the 10 boys in the class was 25.5

The mean score on the test for the whole class was

**A.** 25.5

**B.** 26.7

**C.** 27.0

**D.** 27.3

**E.** 28.5

**Question 5**

The amount of salt in a salt container, in grams, is approximately normally distributed with a mean of 756 g and a standard deviation of 2 g.

In a random sample of 1000 of these salt containers, the number of containers expected to contain less than 750 g of salt is closest to

**A.** 0

**B.** 2

**C.** 3

**D.** 15

**E.** 30

**Question 6**

Ruth sells eggs.

From past experience, Ruth knows that:

- the mean weight of the eggs that she sells is 56 g
- 2.5% of the eggs that she sells have a weight that is greater than 65 g.

Assuming that the weights of the eggs that Ruth sells are approximately normally distributed, the standard deviation of the weights of the eggs is

**A.** 2.5 g
**B.** 3.0 g
**C.** 3.5 g
**D.** 4.0 g
**E.** 4.5 g

**Question 7**

A college awards a certificate of high achievement to a student if that student's score for a subject is in the top 10%.

Sasi studied Mathematics A, Mathematics B, Physics and Chemistry in her first year at the college.

The mean and the standard deviation of students' scores for these four subjects and Sasi's score for each of these subjects are shown in the table below.

| Subject | Mean | Standard deviation | Sasi's score |
|---|---|---|---|
| Mathematics A | 69 | 12 | 80 |
| Mathematics B | 68 | 8 | 85 |
| Physics | 69 | 13 | 81 |
| Chemistry | 62 | 9 | 81 |

Assuming that the students' scores for each of these four subjects are normally distributed, the number of certificates of high achievement that Sasi received was

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

*Use the following information to answer Questions 8 and 9.*

The *blood pressure* (normal, high) and the *age group* (60–69 years, 70–79 years, 80+ years) of a group of people were recorded. The results are shown in the two-way frequency table below.

| **Blood pressure** | **Age group** | | |
|---|---|---|---|
| | **60–69 years** | **70–79 years** | **80+ years** |
| normal | 42 | 23 | 11 |
| high | 38 | 37 | 29 |

**Question 8**

In this group of people, the percentage of people who have high blood pressure and are in the 70–79 years age group is closest to

**A.** 21%
**B.** 23%
**C.** 35%
**D.** 37%
**E.** 38%

**Question 9**

In this group of people, the percentage of people in the 60–69 years age group who have normal blood pressure is closest to

**A.** 23%
**B.** 38%
**C.** 42%
**D.** 48%
**E.** 53%

*Use the following information to answer Questions 10 and 11.*

*Bone length*, in millimetres, of one of the metacarpal bones and *height*, in centimetres, were recorded for a sample of nine people. The results are shown in the table below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ***Bone length*** **(mm)** | 45 | 51 | 39 | 41 | 48 | 49 | 46 | 43 | 47 |
| ***Height*** **(cm)** | 171 | 178 | 157 | 163 | 172 | 183 | 173 | 175 | 173 |

Data: StatSci.org, <www.statsci.org/data/general/stature.txt>

**Question 10**

The mean and the standard deviation of *bone length* for this sample of nine people, in millimetres, are closest to

**A.** mean = 45.4, standard deviation = 3.65

**B.** mean = 45.4, standard deviation = 3.88

**C.** mean = 45.5, standard deviation = 3.65

**D.** mean = 45.5, standard deviation = 3.66

**E.** mean = 45.6, standard deviation = 3.88

**Question 11**

The equation of the least squares line that enables *bone length* to be predicted from *height* is closest to

**A.** *bone length* = –28.6 + 0.431 × *height*

**B.** *height* = –28.6 + 0.431 × *bone length*

**C.** *bone length* = 94.4 + 1.70 × *height*

**D.** *height* = 94.4 + 1.70 × *bone length*

**E.** *bone length* = 94.4 + 0.431 × *height*

**Question 12**

A least squares line is used to model the association between *arm span*, in centimetres, and *height*, in centimetres.

With the values of the $y$-intercept and slope left unrounded, the equation of the least squares line was found to be

$$\text{arm span} = -67.68040231\ldots + 1.35575905\ldots \times \text{height}$$

With the numerical values rounded to five significant figures, the equation of this least squares line is

**A.** *arm span* = –67.68 + 1.3558 × *height*

**B.** *arm span* = –67.680 + 1.3558 × *height*

**C.** *arm span* = –67.700 + 1.4000 × *height*

**D.** *arm span* = –67.680 + 1.3560 × *height*

**E.** *arm span* = –67.68040 + 1.35576 × *height*

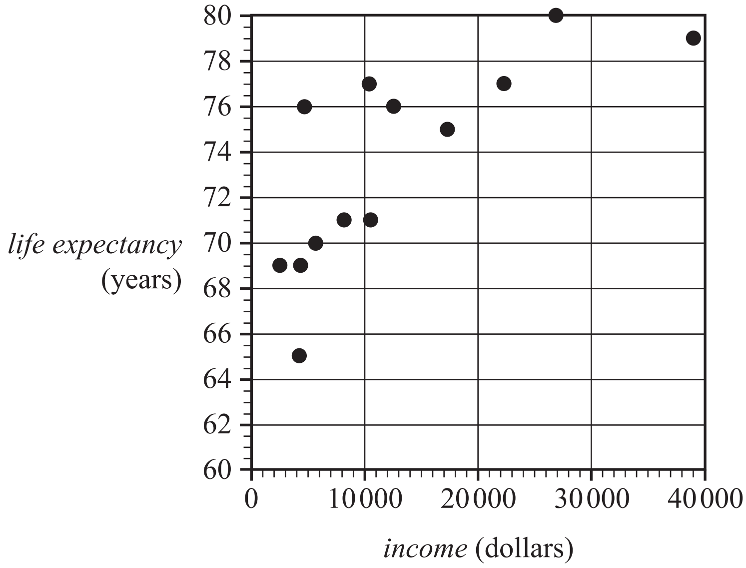
*Use the following information to answer Questions 13 and 14.*

The table below shows the mean *life expectancy*, in years, and the per capita *income*, in dollars, for 13 countries.

A scatterplot displaying this data is also shown.

| ***Life expectancy* (years)** | ***Income* (dollars)** |
|---|---|
| 69 | 2589 |
| 76 | 4717 |
| 70 | 5721 |
| 71 | 8282 |
| 71 | 10630 |
| 76 | 12610 |
| 77 | 22330 |
| 80 | 26930 |
| 75 | 17360 |
| 65 | 4261 |
| 77 | 10430 |
| 79 | 39060 |
| 69 | 4412 |

Data: Gapminder




Data: Gapminder

**Question 13**

A $\log_{10}$ transformation applied to the variable *income* can be used to linearise the scatterplot.

With $\log_{10}$(*income*) as the explanatory variable, the equation of the least squares line fitted to the linearised data is closest to

**A.** *life expectancy* = –0.5840 + 0.06219 × $\log_{10}$(*income*)

**B.** *life expectancy* = 0.06219 – 0.5840 × $\log_{10}$(*income*)

**C.** *life expectancy* = –0.5840 + 37.35 × $\log_{10}$(*income*)

**D.** *life expectancy* = 10.32 + 37.35 × $\log_{10}$(*income*)

**E.** *life expectancy* = 32.35 + 10.32 × $\log_{10}$(*income*)

**Question 14**

The scatterplot on page 9 can also be linearised using a squared transformation applied to the variable *life expectancy*.

The equation of the least squares line for this linearised data is

$$\textit{life expectancy}^2 = 4801.6 + 0.04715 \times \textit{income}$$

When this equation is used to predict *life expectancy*, in years, for a country with a per capita *income* of \$20 000, *life expectancy* is closest to

**A.** 57 years.
**B.** 69 years.
**C.** 74 years.
**D.** 76 years.
**E.** 77 years.

**Question 15**

The seasonal index for the number of cars sold at a local car dealership in June is 0.80

To correct the June car sales for seasonality, the actual number of cars sold should be

**A.** decreased by 20%
**B.** increased by 20%
**C.** decreased by 25%
**D.** increased by 25%
**E.** increased by 80%

**Question 16**

The table below shows the monthly rainfall, in millimetres, recorded at a weather station over a one-year period.

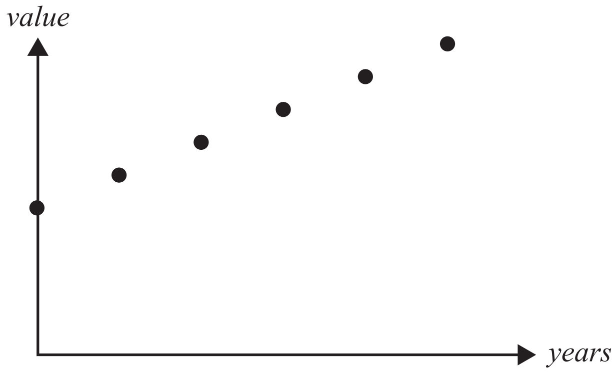
| **Month** | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Rainfall (mm)** | 15.3 | 80.6 | 56.4 | 59.3 | 93.5 | 186.4 | 83.2 | 126.4 | 112.4 | 67.0 | 97.3 | 46.2 |

The six-mean smoothed monthly rainfall with centring for May, in millimetres, is closest to

**A.** 93.2
**B.** 97.1
**C.** 100.9
**D.** 110.0
**E.** 113.1

## Recursion and financial modelling

### Question 17




The graph above could represent the value of

**A.** the balance of a savings account earning simple interest.

**B.** an asset as it depreciates using the flat rate method.

**C.** a perpetuity earning compound interest.

**D.** an annuity from which equal regular payments are withdrawn.

**E.** a reducing balance loan with interest-only repayments.

### Question 18

The value of an investment, in dollars, after $n$ years, $V_n$, can be modelled by the recurrence relation shown below.

$$V_0 = 8000, \qquad V_{n+1} = RV_n$$

This investment earns compound interest at the rate of 5% per annum, compounding quarterly.

What is the value of $R$ in this recurrence relation?

**A.** 0.05

**B.** 1.005

**C.** 1.0125

**D.** 1.05

**E.** 1.125

**Question 19**

The value of a computer, purchased for \$5000, is depreciated by 10% per annum using the reducing balance method.

Recursive calculations can determine the value of the computer after $n$ years, $V_n$.

Which one of the following recursive calculations is not correct?

**A.** $V_0 = 5000$

**B.** $V_1 = 0.9 \times 5000$

**C.** $V_2 = 0.9 \times 4500$

**D.** $V_3 = 0.9 \times 4000$

**E.** $V_4 = 0.9 \times 3645$

**Question 20**

Leonard has invested an amount of money in a perpetuity.

The perpetuity earns interest at the rate of 4.8% per annum.

Interest is calculated and paid monthly.

If Leonard receives \$1600 per month from the perpetuity, then the amount he has invested is

**A.** \$19 200

**B.** \$100 000

**C.** \$160 000

**D.** \$400 000

**E.** \$500 000

**Question 21**

Dirga invests \$250 000 in an account that pays interest at the rate of 3% per annum, compounding monthly.

He makes additional payments of \$250 each month into his account.

The value of Dirga's account, in dollars, after $n$ months, $V_n$, can be modelled by the recurrence relation shown below.

$$V_0 = 250\,000, \qquad V_{n+1} = 1.0025\,V_n + 250$$

The balance of Dirga's account first exceeds \$500 000 at the end of year

**A.** 17

**B.** 18

**C.** 19

**D.** 20

**E.** 21

**Question 22**

Cherie has purchased a boat for \$90 000.

She depreciates the value of her boat using the reducing balance method.

For the first two years of reducing balance depreciation, the annual depreciation rate was 15%.

Cherie then changed the annual depreciation rate to *d* per cent.

After three more years of reducing balance depreciation, the value of the boat is \$43 561.67

The changed depreciation rate, *d* per cent, is closest to

**A.** 7.2%

**B.** 10.3%

**C.** 12.5%

**D.** 13.0%

**E.** 14.5%

**Question 23**

Anne has taken out a personal loan of \$5000.

Interest for this loan compounds quarterly.

Anne makes no repayments.

After one year, she owes \$5325.14

The effective annual rate of interest for the first year of Anne's loan is closest to

**A.** 6.34%

**B.** 6.35%

**C.** 6.50%

**D.** 6.54%

**E.** 6.56%

**Question 24**

Vittorio has borrowed \$40 000 to go on a holiday.

Interest on this loan is charged at the rate of 8.3% per annum, compounding monthly.

Vittorio intends to repay the loan over four years with 48 monthly repayments.

After 47 equal repayments of \$982.16, he finds that a small adjustment to the final repayment is required to fully repay the loan to the nearest cent.

Compared to the 47 earlier repayments, the final repayment will be

**A.** \$6.78 lower.

**B.** \$0.03 lower.

**C.** \$0.07 higher.

**D.** \$0.13 higher.

**E.** \$6.78 higher.

**END OF SECTION A**
**TURN OVER**

**THIS PAGE IS BLANK**

## SECTION B – Modules

### Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Contents** **Page**

Module 1 – Matrices ........ 16

Module 2 – Networks and decision mathematics ........ 21

Module 3 – Geometry and measurement ........ 27

Module 4 – Graphs and relations ........ 31

## Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

The matrix expression $\begin{bmatrix} 2 & 6 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 24 \\ 16 & 2 \end{bmatrix}$ is equal to

**A.** 0

**B.** [0]

**C.** [2]

**D.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**E.** $\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$

**Question 2**

Consider the following four matrices.

$$\begin{bmatrix} 0 & 4 & 4 \\ 4 & 0 & 4 \\ 4 & 4 & 0 \end{bmatrix} \quad \begin{bmatrix} 4 & 4 & 4 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 4 \end{bmatrix}$$

How many of these matrices are diagonal matrices?

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

**Question 3**

The matrix equation $\begin{bmatrix} 4 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$ has a unique solution for $x$ and $y$.

Which one of the following matrix equations has the same unique solution for $x$ and $y$?

**A.** $\begin{bmatrix} 2 & -4 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$

**B.** $\begin{bmatrix} 2 & -4 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$

**C.** $\begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} -4 \\ -15 \end{bmatrix}$

**D.** $\begin{bmatrix} 4 & 2 \\ 8 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 16 \end{bmatrix}$

**E.** $\begin{bmatrix} -4 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$

**Question 4**

Matrix $M_1$ is the $5 \times 1$ column matrix $\begin{bmatrix} 12 \\ 3 \\ 25 \\ 15 \\ 9 \end{bmatrix}$.

A second $5 \times 1$ column matrix, $M_2$, contains the same elements as $M_1$, but the elements are ordered with the largest value at the top and the smallest value at the bottom.

Matrix $M_2 = P \times M_1$, where $P$ is a permutation matrix.

Matrix $P$ is

**A.** $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

**B.** $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

**C.** $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

**D.** $\begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

**E.** $\begin{bmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix}$

**Question 5**

A factory employs the same number of workers each day.

The workers are allocated to work in either the green ($G$) building or the white ($W$) building.

The workers may be allocated to work in a different building from day to day, as shown in the transition matrix below.

$$\begin{array}{c} \textit{this day} \\ \begin{array}{cc} G & W \end{array} \\ \begin{bmatrix} 0.52 & 0.24 \\ 0.48 & 0.76 \end{bmatrix} \begin{array}{l} G \\ W \end{array} \textit{next day} \end{array}$$

The green building must have 36 workers each day.

Each day, the number of workers in the white building will be

**A.** 24

**B.** 36

**C.** 72

**D.** 76

**E.** 78

**Question 6**

Four families visit an amusement park.

At this amusement park, there are three rides: a roller-coaster ($R$), a Ferris wheel ($F$) and go-karts ($G$).

Each ride has a different ticket price for adults ($A$) and for children ($C$).

The number of tickets that each family pays for is shown in Table 1 below.

**Table 1**

| | **Roller-coaster (*R*)** | | **Ferris wheel (*F*)** | | **Go-karts (*G*)** | |
|---|---|---|---|---|---|---|
| | **Adult (*A*)** | **Child (*C*)** | **Adult (*A*)** | **Child (*C*)** | **Adult (*A*)** | **Child (*C*)** |
| **Family 1** | 1 | 2 | 1 | 1 | 0 | 3 |
| **Family 2** | 2 | 3 | 1 | 2 | 0 | 2 |
| **Family 3** | 4 | 4 | 1 | 2 | 1 | 1 |
| **Family 4** | 2 | 2 | 1 | 2 | 0 | 0 |

The data in Table 1 can be converted into a 4 × 6 matrix, $V$.

The ticket price per ride is shown in Table 2 below.

**Table 2**

| | **Roller-coaster (*R*)** | **Ferris wheel (*F*)** | **Go-karts (*G*)** |
|---|---|---|---|
| **Adult (*A*)** | \$10 | \$4 | \$12 |
| **Child (*C*)** | \$6 | \$2 | \$8 |

The data in Table 2 can be converted into a matrix, $W$, where $W = \begin{array}{c} \\ R_A \\ R_C \\ F_A \\ F_C \\ G_A \\ G_C \end{array}\begin{array}{c} \begin{array}{cc} A & C \end{array} \\ \begin{bmatrix} 10 & 0 \\ 0 & 6 \\ 4 & 0 \\ 0 & 2 \\ 12 & 0 \\ 0 & 8 \end{bmatrix} \end{array}$, $R_A$ is the ticket price for one adult to ride the roller-coaster and $R_C$ is the ticket price for one child to ride the roller-coaster.

The matrix product $V \times W$ produces a new matrix, $L$.

The elements in matrix $L$ show the total cost of

**A.** all tickets for each family.

**B.** all adult tickets and all child tickets at the amusement park.

**C.** all adult tickets and all child tickets for each family.

**D.** all tickets for each of the three rides.

**E.** all adult tickets and all child tickets for each of the three rides.

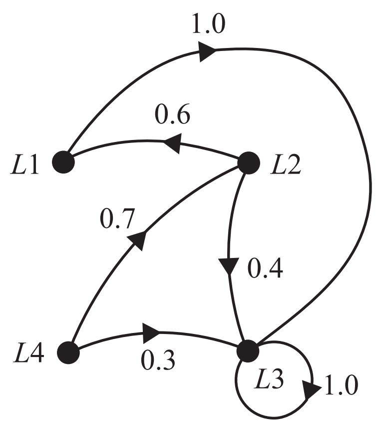
**Question 7**

Each day, all employees of a large company will work at one of four locations: *L*1, *L*2, *L*3 or *L*4.

The table below shows the number of employees at each location every Monday.

| **Location** | *L*1 | *L*2 | *L*3 | *L*4 |
|---|---|---|---|---|
| **Number of employees** | 80 | 75 | 60 | 50 |

The transition diagram below shows the change in the number of employees at each location from day to day.




Which one of the following statements is true?

**A.** On Tuesday, there will be no employees at location *L*2 only.

**B.** On Tuesday, there will be no employees at locations *L*2 and *L*4 only.

**C.** On Wednesday, there will be no employees at location *L*4 only.

**D.** On Wednesday, there will be no employees at locations *L*2 and *L*4 only.

**E.** On Thursday, there will be no employees at locations *L*2 and *L*4 only.

**Question 8**

Four teams, $J$, $K$, $L$ and $M$, compete in a tournament.

Each team plays one game against every other team.

In each game, there is a winner and a loser.

To decide the winner of the tournament, the sum of the one-step dominance matrix, $D$, and the two-step dominance matrix, $D^2$, for this tournament is determined. This sum, $D + D^2$, is shown below.

$$D + D^2 = \begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The '2' in the second row shows that the total number of one-step and two-step dominances that team $K$ has over team $M$ is two.

In the first two games:

- team $J$ defeated team $K$
- team $L$ defeated team $M$.

Which one of the following is the correct one-step dominance for this tournament?

**A.**
$$D = \begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

**B.**
$$D = \begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

**C.**
$$D = \begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

**D.**
$$D = \begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

**E.**
$$D = \begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

## Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

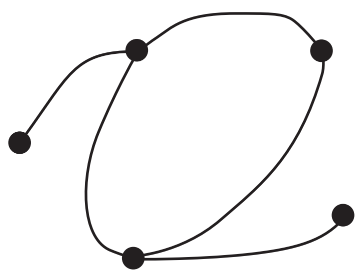
For a connected graph with four vertices and four edges, the sum of the degrees of the vertices is

**A.** 2
**B.** 4
**C.** 6
**D.** 8
**E.** 10

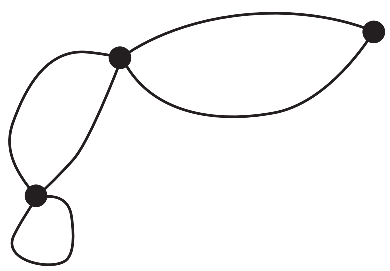
**Question 2**

Which one of the following graphs has an Eulerian circuit?

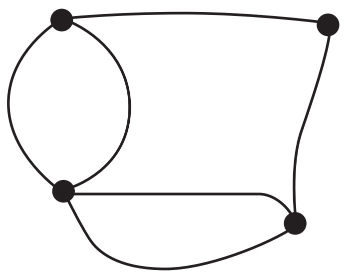
**A.**



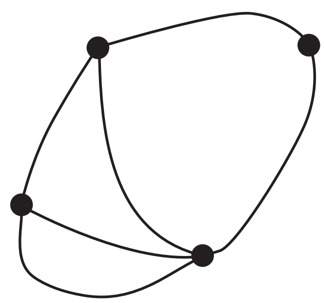
**B.**



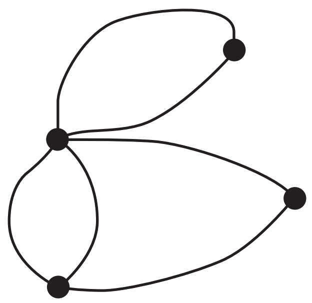
**C.**



**D.**



**E.**

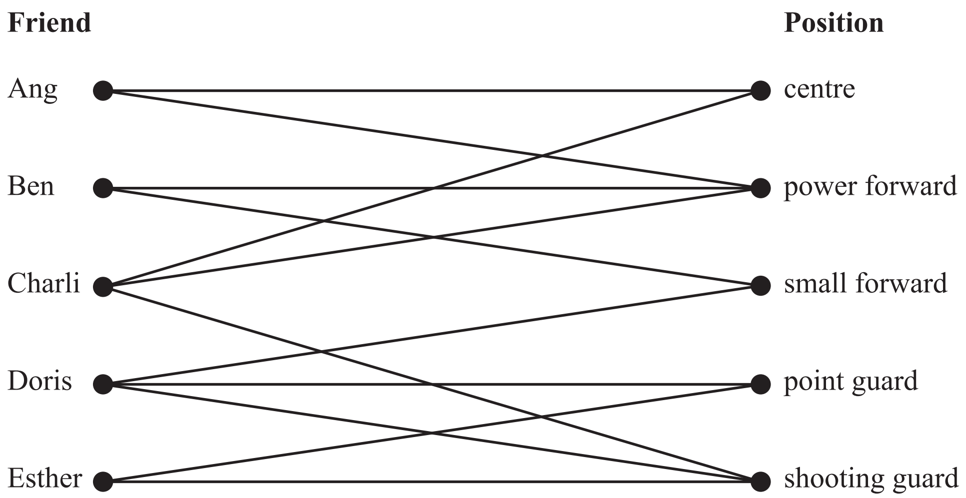


**Question 3**

The sport of basketball has five player positions: centre, power forward, small forward, point guard and shooting guard.

Five friends, Ang, Ben, Charli, Doris and Esther, play together in a team.

The bipartite graph below shows which positions each of the five friends can play.




Based on the bipartite graph, which one of the following allocations is **not** possible?

**A.**

| | |
|---|---|
| Ang | power forward |
| Ben | small forward |
| Charli | centre |
| Doris | point guard |
| Esther | shooting guard |

**B.**

| | |
|---|---|
| Ang | power forward |
| Ben | small forward |
| Charli | centre |
| Doris | shooting guard |
| Esther | point guard |

**C.**

| | |
|---|---|
| Ang | centre |
| Ben | small forward |
| Charli | power forward |
| Doris | point guard |
| Esther | shooting guard |

**D.**

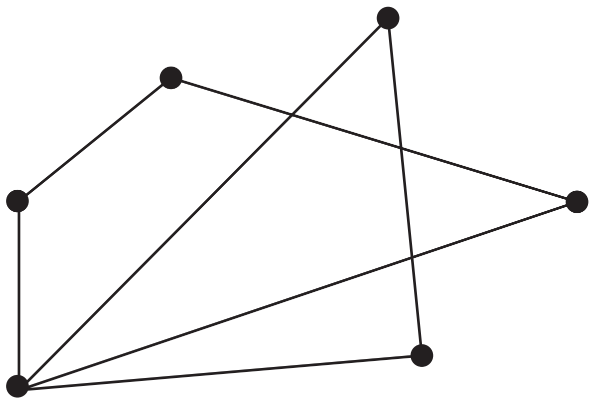
| | |
|---|---|
| Ang | centre |
| Ben | power forward |
| Charli | shooting guard |
| Doris | small forward |
| Esther | point guard |

**E.**

| | |
|---|---|
| Ang | centre |
| Ben | power forward |
| Charli | small forward |
| Doris | point guard |
| Esther | shooting guard |

**Question 4**

Consider the graph below.



How many faces does this graph have?

**A.** 3

**B.** 4

**C.** 5

**D.** 6

**E.** 7

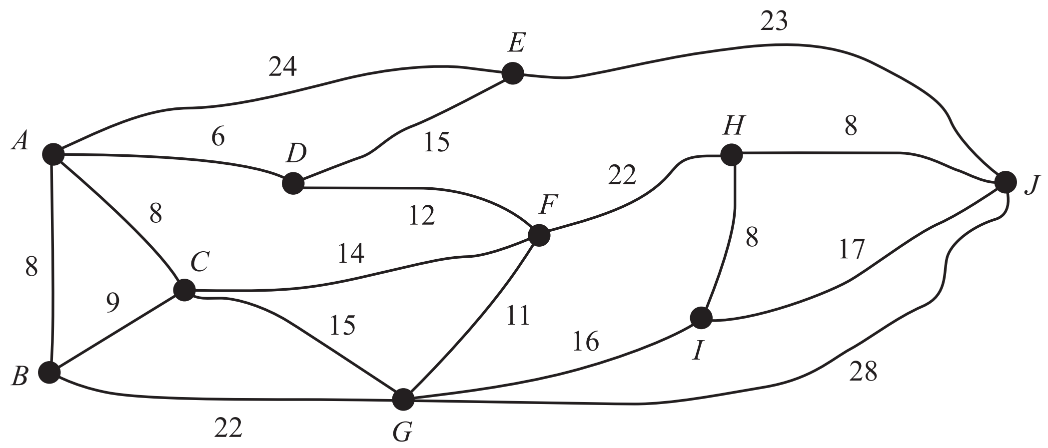
**Question 5**

Aisha works as a sales representative.

The network below shows the roads that Aisha could use to travel between her clients.

Aisha's clients are represented by the vertices labelled *A*–*J*.

The edges in the network represent the roads and the numbers on the edges show the distances, in kilometres, between her clients.




Aisha needs to travel from client *A* to client *J*.

Using Dijkstra's algorithm, she finds that the

**A.** critical path is 46 km.

**B.** critical path is 109 km.

**C.** shortest path is 44 km.

**D.** shortest path is 46 km.

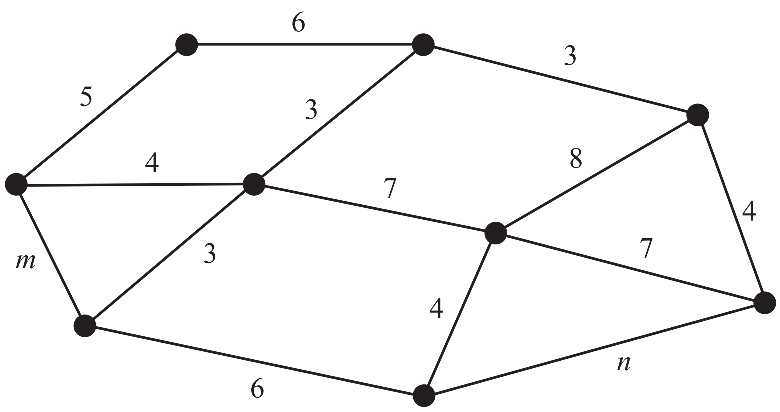
**E.** minimum spanning tree is 44 km.

**Question 6**

The vertices of the network below represent nine communication towers.

The edges of the graph represent the fibre-optic cables that connect the communication towers.

The numbers on the edges show the length, in kilometres, of fibre-optic cable between the communication towers.




The minimum length of fibre-optic cable that is required to connect the nine communication towers is 28 km except when

**A.** $m = 1$ and $n = 5$

**B.** $m = 2$ and $n = 5$

**C.** $m = 2$ and $n = 4$

**D.** $m = 3$ and $n = 3$

**E.** $m = 4$ and $n = 2$

**Question 7**

A project requires 11 activities (*A*–*K*) to be completed.

The duration, in hours, and the immediate predecessor(s) of each activity are shown in the table below.

| **Activity** | **Duration (hours)** | **Immediate predecessor(s)** |
|---|---|---|
| *A* | 9 | – |
| *B* | 7 | *A* |
| *C* | 5 | *A* |
| *D* | 3 | *C* |
| *E* | 6 | *C* |
| *F* | 4 | *B*, *D* |
| *G* | 4 | *B*, *D* |
| *H* | 2 | *E*, *G* |
| *I* | 2 | *F*, *H* |
| *J* | 5 | *I* |
| *K* | 7 | *E*, *G* |

To complete this project in the minimum time, the number of activities that **cannot** be delayed is

**A.** 4

**B.** 5

**C.** 6

**D.** 7

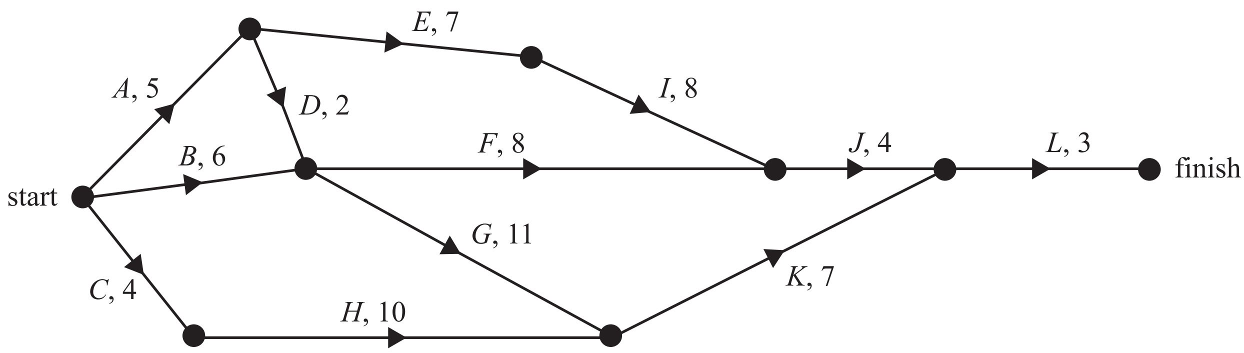
**E.** 8

**Question 8**

A company builds caravans.

The building project for the Holiday Fun caravan involves 12 activities (*A*–*L*).

The directed network below shows these activities and their completion times, in hours.




An updated caravan model, the Sunny Life caravan, is introduced.

The building project for the Sunny Life caravan is modified from the building project for the Holiday Fun caravan.

Activity *D* is no longer required and is removed. A new activity, activity *M*, is added.

Activity *M* has a duration of two hours. It has an earliest start time of 17 hours and a latest start time of 18 hours.

The minimum completion time for the Sunny Life caravan will be

**A.** the same as the Holiday Fun caravan.

**B.** one hour less than the Holiday Fun caravan.

**C.** two hours less than the Holiday Fun caravan.

**D.** one hour more than the Holiday Fun caravan.

**E.** two hours more than the Holiday Fun caravan.

## Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

A stove has one circular burner. This burner has a radius of 11 cm.

The area of the top of this burner, in square centimetres, is closest to

**A.** 35
**B.** 69
**C.** 95
**D.** 380
**E.** 1521

**Question 2**

The length and width of a rectangular photograph were increased by a linear scale factor of $k = 4$.

The area of this photograph has increased by a factor of

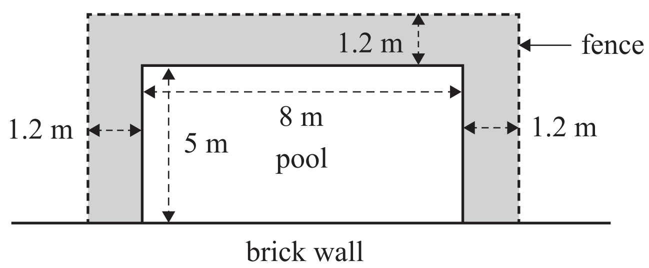
**A.** 2
**B.** 4
**C.** 8
**D.** 16
**E.** 64

**Question 3**

A rectangular swimming pool is enclosed by a brick wall on one side and a fence along the other three sides.

The pool is 8 m long and 5 m wide. The fence is 1.2 m from the pool on each side.

The area between the pool and the fence will be concreted, as shown shaded in the diagram below.

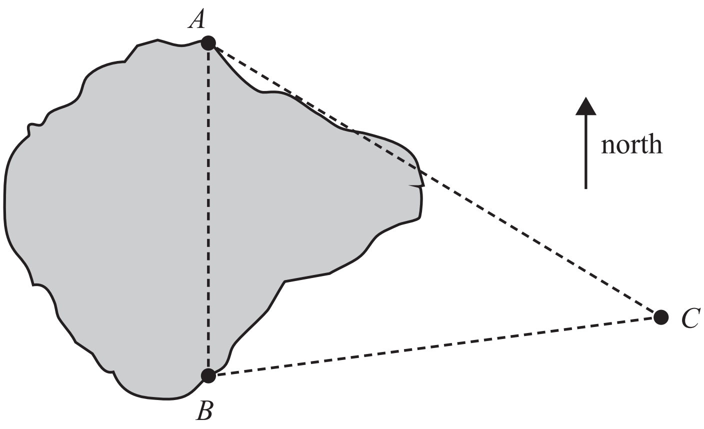



The total area that will be concreted, in square metres, is

**A.** 21.60
**B.** 24.48
**C.** 28.08
**D.** 36.96
**E.** 40.00

**Question 4**

Points $A$, $B$ and $C$ are located around a lake, as shown in the diagram below.




Point $A$ is directly north of point $B$.

Point $C$ is 46 m from point $A$ on a bearing of 133°.

Point $C$ is 35 m from point $B$ on a bearing of 074°.

The shortest distance from point $A$ to point $B$, in metres, can be found by evaluating

**A.** $\dfrac{35\sin(59^\circ)}{\sin(47^\circ)}$

**B.** $\dfrac{35\sin(74^\circ)}{\sin(47^\circ)}$

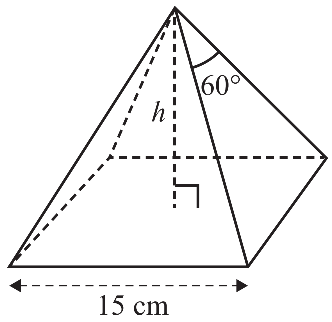
**C.** $\dfrac{35\sin(59^\circ)}{\sin(74^\circ)}$

**D.** $\dfrac{46\sin(59^\circ)}{\sin(47^\circ)}$

**E.** $\dfrac{46\sin(74^\circ)}{\sin(47^\circ)}$

**Question 5**

A square-based pyramid is shown below.




The height, $h$, in centimetres, of this pyramid is closest to

**A.** 7.5

**B.** 10.6

**C.** 13.0

**D.** 15.0

**E.** 26.0

**Question 6**

The shortest great circle distance between the Great Pyramid of Giza and the North Pole is approximately 6702 km.

Assume that the radius of Earth is 6400 km.

The latitude of the Great Pyramid of Giza is

**A.** 30° N

**B.** 35° N

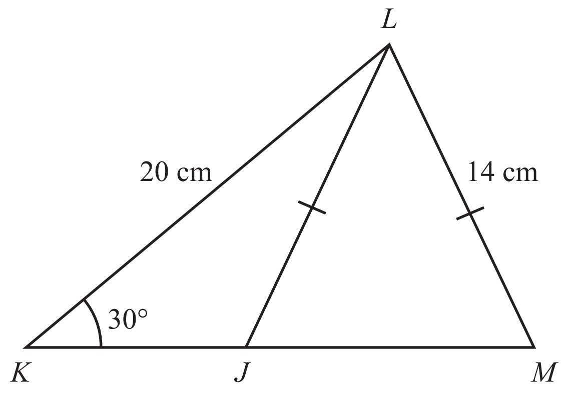
**C.** 55° N

**D.** 60° N

**E.** 120° N

**Question 7**

A triangle *KLM* is shown in the diagram below. Point *J* is located on the side *KM*.




The length of the side *KL* is 20 cm.

The length of the sides *JL* and *LM* is 14 cm.

The angle *LKM* is 30°.

The angle *JLM* is closest to

**A.** 16°

**B.** 30°

**C.** 60°

**D.** 89°

**E.** 104°

**Question 8**

Three friends will travel to Papeete (18° S, 150° W) from different cities for a wedding in June.

The friends, the city each friend is travelling from (name and coordinates) and their flight details are shown in the table below.

| | | **Flight details** | |
|---|---|---|---|
| **Friend** | **City** | **Departure (local time)** | **Journey time** |
| Harry | Tokyo (36° N, 140° E) | Monday at 5.00 pm | 12 hours |
| Josh | Auckland (37° S, 175° E) | Monday at 10.00 am | 5 hours |
| Michael | Los Angeles (34° N, 118° W) | Monday at 5.00 pm | 8 hours |

The order in which the friends will arrive in Papeete, from the first to arrive to the last to arrive, is

**A.** Michael, Josh, Harry.

**B.** Harry, Michael, Josh.

**C.** Harry, Josh, Michael.

**D.** Josh, Michael, Harry.

**E.** Josh, Harry, Michael.

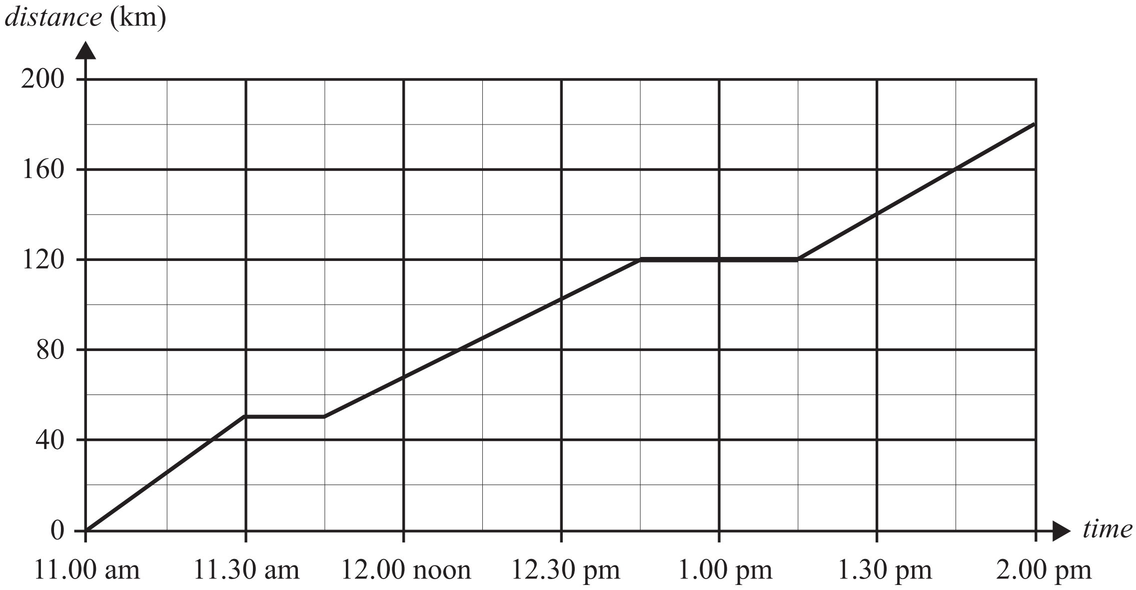
## Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

*Use the following information to answer Questions 1 and 2.*

The distance-time graph below shows a train's journey between two towns.

The train leaves the first town at 11.00 am and arrives at the second town at 2.00 pm.




**Question 1**

The average speed of the train for the entire journey, in kilometres per hour, is closest to

**A.** 45
**B.** 50
**C.** 60
**D.** 65
**E.** 80

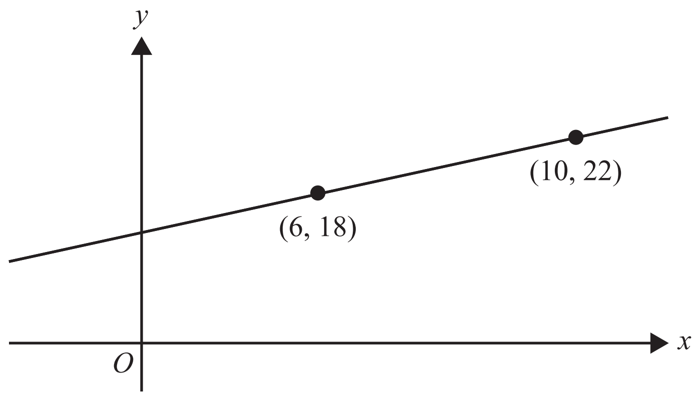
**Question 2**

For how many minutes was the train stationary during the entire journey?

**A.** 0
**B.** 15
**C.** 30
**D.** 45
**E.** 60

**Question 3**

The graph below shows a straight line that passes through the points (6, 18) and (10, 22).




The coordinates of the point where the line crosses the *y*-axis are

**A.** (0, 11)

**B.** (0, 12)

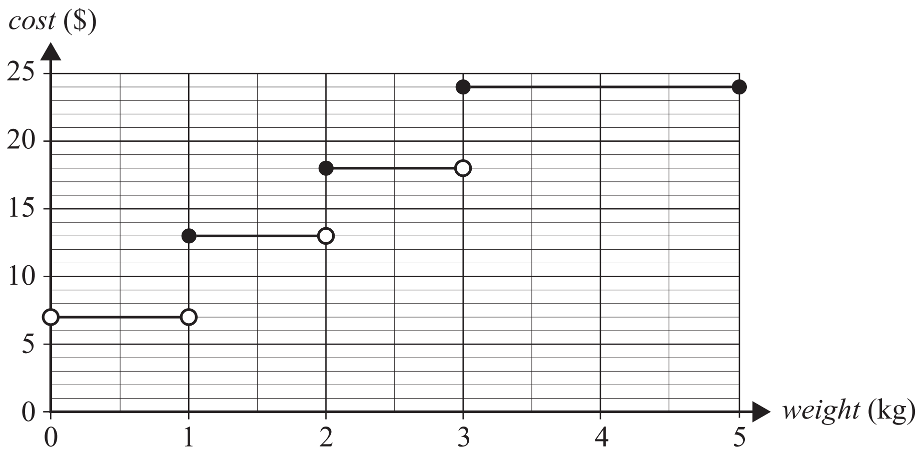
**C.** (0, 13)

**D.** (0, 14)

**E.** (0, 15)

**Question 4**

The graph below shows the *cost*, in dollars, of sending a package by a courier based on the *weight* of the package, in kilograms.




Gloria has to send three packages to three different destinations:

- one package weighing 1.5 kg
- one package weighing 2.0 kg
- one package weighing 4.5 kg

What is the total *cost* of sending all three packages?

**A.** $13.00

**B.** $24.00

**C.** $31.00

**D.** $48.00

**E.** $55.00

**Question 5**

Wong's dishwasher repair business repairs dishwashers in owners' homes.

If it takes 45 minutes to repair a dishwasher, the cost is \$155.

If it takes two hours to repair a dishwasher, the cost is \$330.

The rule for the cost of the repair service is

$$cost = a + b \times n$$

where $a$ is the fixed fee for coming to the owner's home,

$b$ is the charge, in dollars, for each 15-minute unit of time required to repair the dishwasher and

$n$ is the number of 15-minute units of time required to repair the dishwasher.

Using this rule, the cost for a 90-minute service is

**A.** \$180

**B.** \$190

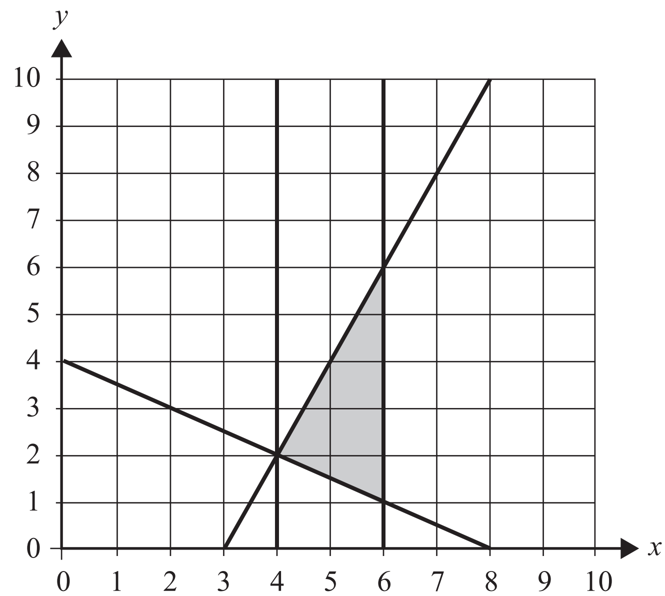
**C.** \$220

**D.** \$260

**E.** \$310

**Question 6**

The feasible region defined by four inequalities is shaded on the graph below.




One of the inequalities used to define this feasible region is

**A.** $2x + y \leq 8$

**B.** $x + 2y \geq 8$

**C.** $4 \leq y \leq 6$

**D.** $x + 2y \leq 8$

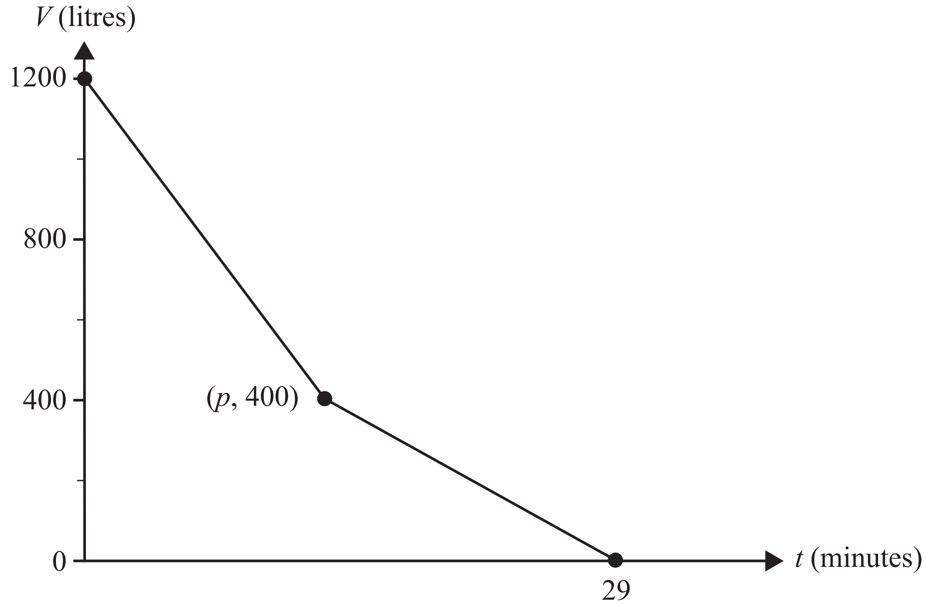
**E.** $2x - y \geq 8$

**Question 7**

A 1200-litre tank is being emptied by two pumps.

After $p$ minutes, when 800 litres has been removed, one of the pumps stops working.

The graph below shows the volume of liquid, $V$ litres, in the tank at time $t$ minutes.




The tank is empty after 29 minutes.

If both pumps had continued to work, the tank would have emptied in 24 minutes.

Over the time interval from $p$ minutes to 29 minutes, the rate at which the tank is being emptied, in litres per minute, is closest to

**A.** 25

**B.** 29

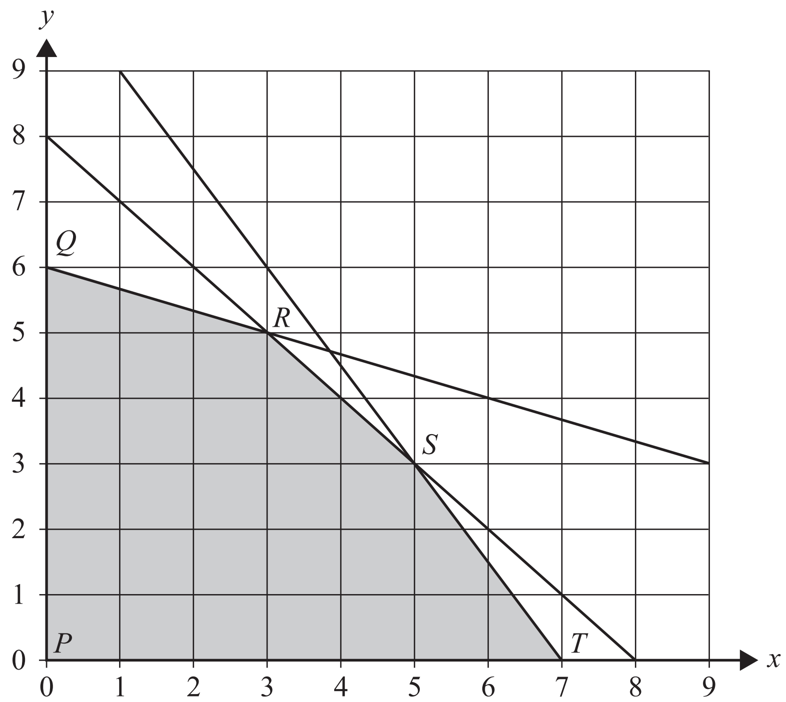
**C.** 30

**D.** 31

**E.** 50

**Question 8**

The feasible region for a linear programming problem is shaded on the graph below.




The coordinates of the points that define the boundaries of the feasible region are *P* (0, 0), *Q* (0, 6), *R* (3, 5), *S* (5, 3) and *T* (7, 0).

When maximising the objective function $Z = ax + by$, where $a > 0$ and $b > 0$, for these constraints, the solution is found at point *R* only.

The values of *a* and *b* for this objective function could be

**A.** $a = 1$ and $b = 1$

**B.** $a = 1$ and $b = 3$

**C.** $a = 2$ and $b = 3$

**D.** $a = 3$ and $b = 1$

**E.** $a = 3$ and $b = 2$

**END OF MULTIPLE-CHOICE QUESTION BOOK**

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2021**

# FURTHER MATHEMATICS

## Written examination 1

### FORMULA SHEET

**Instructions**

| |
|---|
| This formula sheet is provided for your reference.<br>A multiple-choice question book is provided with this formula sheet. |

| |
|---|
| **Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.** |

# Further Mathematics formulas

## Core – Data analysis

| | |
|---|---|
| standardised score | $z = \dfrac{x - \bar{x}}{s_x}$ |
| lower and upper fence in a boxplot | lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$ |
| least squares line of best fit | $y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$ |
| residual value | residual value = actual value – predicted value |
| seasonal index | seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$ |

## Core – Recursion and financial modelling

| | |
|---|---|
| first-order linear recurrence relation | $u_0 = a$, $u_{n+1} = bu_n + c$ |
| effective rate of interest for a compound interest loan or investment | $r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$ |

## Module 1 – Matrices

| | |
|---|---|
| determinant of a 2 × 2 matrix | $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ |
| inverse of a 2 × 2 matrix | $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$ |
| recurrence relation | $S_0$ = initial state, $S_{n+1} = T\,S_n + B$ |

## Module 2 – Networks and decision mathematics

| | |
|---|---|
| Euler's formula | $v + f = e + 2$ |

## Module 3 – Geometry and measurement

| | |
|---|---|
| area of a triangle | $A = \frac{1}{2} bc \sin(\theta^\circ)$ |
| Heron's formula | $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$ |
| sine rule | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ |
| cosine rule | $a^2 = b^2 + c^2 - 2bc \cos(A)$ |
| circumference of a circle | $2\pi r$ |
| length of an arc | $r \times \frac{\pi}{180} \times \theta^\circ$ |
| area of a circle | $\pi r^2$ |
| area of a sector | $\pi r^2 \times \frac{\theta^\circ}{360}$ |
| volume of a sphere | $\frac{4}{3}\pi r^3$ |
| surface area of a sphere | $4\pi r^2$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| volume of a prism | area of base × height |
| volume of a pyramid | $\frac{1}{3}$ × area of base × height |

## Module 4 – Graphs and relations

| | |
|---|---|
| gradient (slope) of a straight line | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| equation of a straight line | $y = mx + c$ |

**END OF FORMULA SHEET**