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YEAR 12 *Trial Exam Paper*

2022

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	9	9	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 39 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A
Answer **all** questions in the spaces provided.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (6 marks)

Farah gets a job at a supermarket.

The table below shows the sales information for each hour of trade on a particular day at the supermarket. The store was open for 14 hours.

Hour number	**No. of customers**	**Hourly total sales ($)**
1	51	520
2	82	4815
3	176	9740
4	194	15 600
5	92	6025
6	34	4010
7	120	12 985
8	105	6630
9	82	5325
10	134	6210
11	176	6510
12	86	4025
13	55	2750
14	42	2110
Total	**1429**	**87 255**

SECTION A – Question 1 – continued

a. **i.** In which hour did the store have the most customers?

1 mark

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   ii. What was the average sales per customer?

Round your answer to the nearest dollar.

1 mark

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b. The manager examines the hourly total sales and checks if any values should be considered outliers.

Determine if the sales figure of $520 is an outlier, showing calculations to justify your answer.

2 marks

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c. The age of the 38 permanent staff at the supermarket is normally distributed. The mean age is 34 years and the standard deviation is 4.6 years.

The age of the 52 casual staff is also normally distributed. The mean age is 19 years and the standard deviation is 1.2 years.

Farah's friend Toni is 25 years old. Using the 68–95–99.7% rule, Toni is in the youngest 16% of the permanent staff and oldest 0.15% of the casual staff.

Farah is 21 years old.

Using the 68–95–99.7% rule, describe her age, referring to the relevant percentages, in relation to both the permanent and casual members of staff.
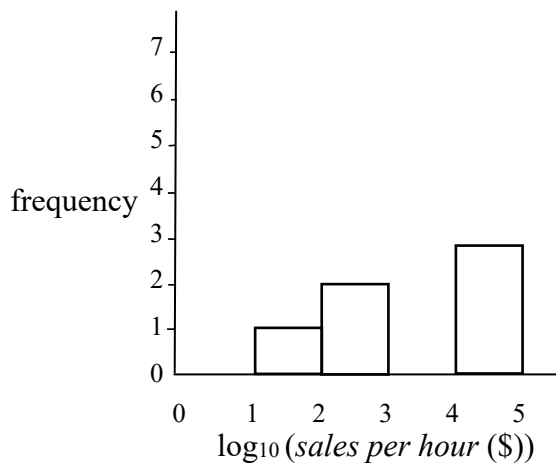
2 marks

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Question 2 (5 marks)

After work, Farah visits a homewares store next to the supermarket.

The histogram below shows the distribution of *sales per hour* ($) on a particular day at the homewares store. The histogram has been plotted on a $\log_{10}$ scale.

There were 7 hours during this day in which the *sales per hour* were between $100 and $10 000.

a. Draw a bar on the **histogram above** to represent the missing information accurately.

1 mark

(*Answer on the histogram above.*)

SECTION A – Question 2 – continued

b. The homewares store records the postcode and sex of its customers over a week. This information was used to construct the two-way frequency table below.

Distance from store	Sex		
	Male	**Female**	**Total**
Under 5 km	45	62	107
Over 5 km	13	35	48
Total	58	97	155

The information from the two-way frequency table is converted into a percentaged segmented bar chart. The data for males has been completed.
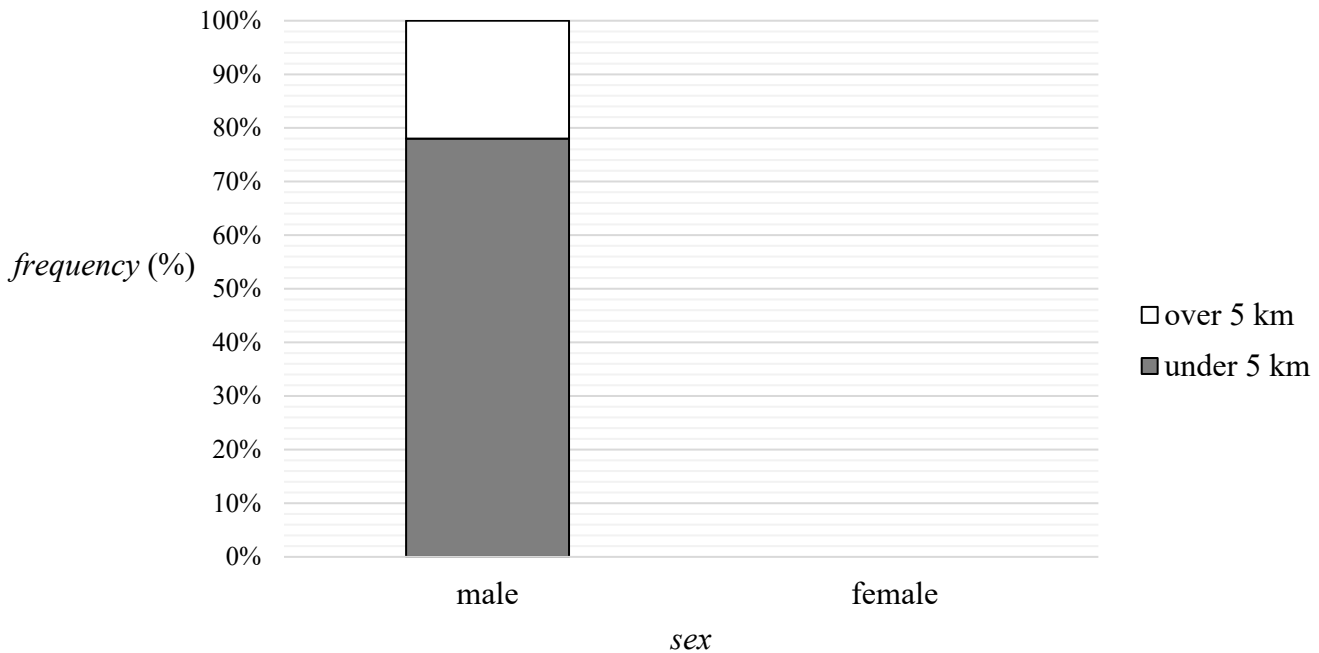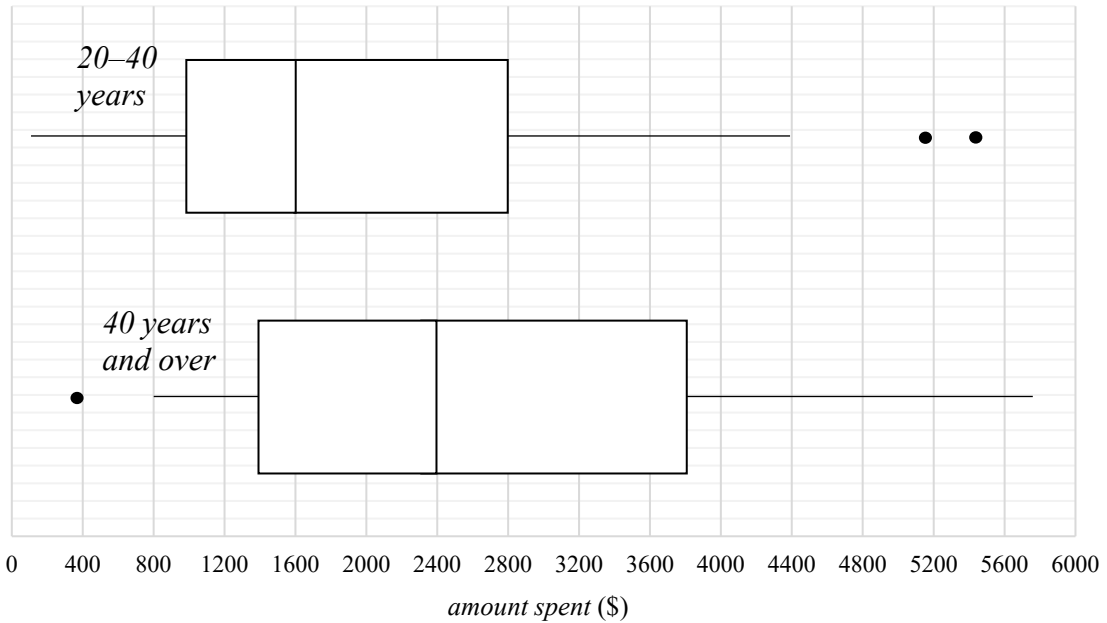

Draw the missing column for females on the **percentaged segmented bar chart above**.

2 marks

(*Answer on the percentaged segmented bar chart above.*)

c.　i.　The boxplots below show the amount spent by customers, comparing the age groups *20–40 years*, and *40 years and over*.

amount spent ($)

Describe the shape of the *amount spent* by those *40 years and over*.

1 mark

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ii.　There were 52 customers who were between *20–40 years* who spent between $2800 and $4400. There were 64 customers who were *40 years and over* who spent between $1400 and $3800.
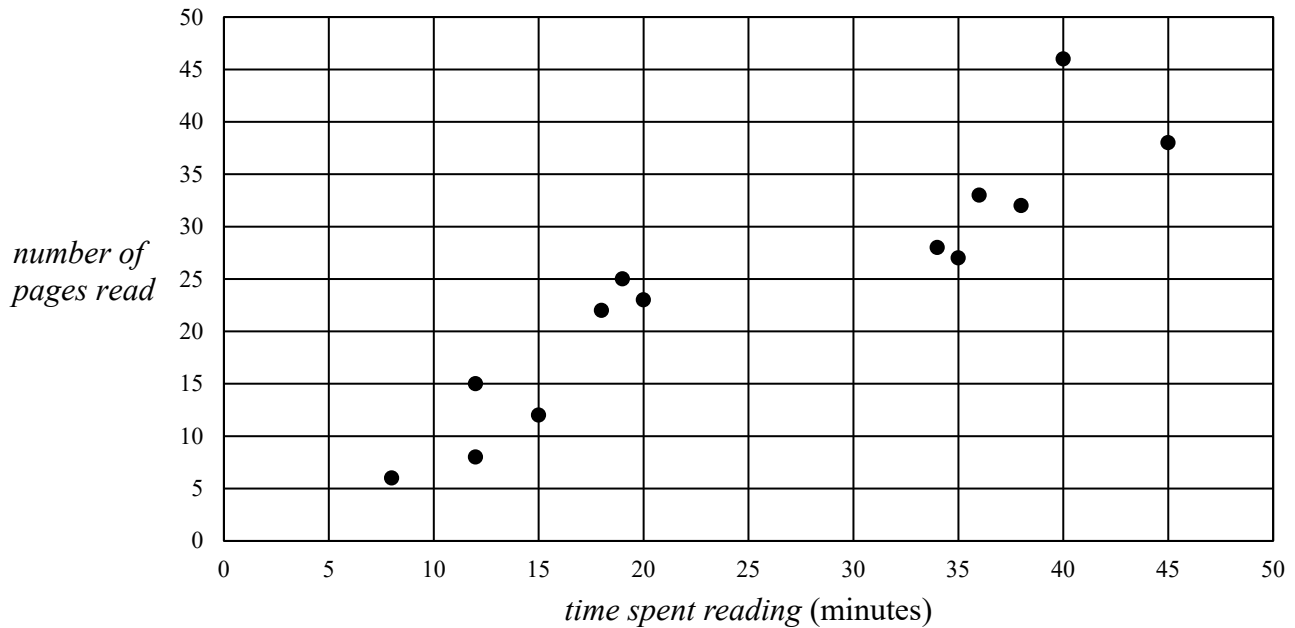
How many customers were there in total?

1 mark

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　　　　　　　　　　　SECTION A – continued

Question 3 (4 marks)

Farah reads novels during her lunch and rest breaks. The amount of *time spent reading* by Farah, in minutes, and the *number of pages read* is shown on the scatterplot below.

A least squares line is fitted to the scatterplot. The equation of this line is

number of pages read = 2.44 + 0.85 × time spent reading

a. Interpret the slope of the least squares line in terms of the *number of pages read* and *time spent reading*.

1 mark

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_____

The data from the scatterplot is shown in the table below.

Time spent reading (minutes)	15	35	45	12	8	18	12	34	40	38	19	20	36	
Number of pages read	12	27	38	8	6	22	15	28	46	32	25	23	33	23

b. A value is missing from the table and the scatterplot. The corresponding residual for this value is –0.69

Determine the missing value.

2 marks

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The correlation coefficient for the *time spent reading* and *number of pages read* is 0.9170

c. Interpret the coefficient of determination in terms of the variables *time spent reading* and *number of pages read*.

Round the percentage to the nearest whole number.

1 mark

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Question 4 (4 marks)

The number of staff rostered on for restocking at the supermarket is measured against the number of items restocked. The data is shown below.

Number of staff	4	5	12	5	4	6	9	15	8	8
Units restocked	220	960	1920	520	190	1470	1790	1990	1840	1760

In order to linearise the data, a $\log_{10}$ transformation is applied and a least squares line is fitted to the transformed equation.

a. Complete an appropriate $\log_{10}$ transformation and write down the equation of the least squares line.

Round the values to four significant figures.

2 marks

b. **i.** A reciprocal transformation is also applied to the data.

The equation of the least squares line fitted to this transformed data is

$$units\ restocked = 2942 - 10\,623 \times \frac{1}{number\ of\ staff}$$

What is the predicted number of units restocked when 16 staff are rostered on? Round your answer to the nearest whole number.

1 mark

ii. Explain why this prediction may have limited reliability.

1 mark

Question 5 (5 marks)

The total number of cars in a carpark is recorded over each day for a week. The results are shown below.

Day	Sun.	Mon.	Tues.	Wed.	Thu.	Fri.	Sat.
Number of cars	593	272	208	730	345	425	894

The data is smoothed using both a three-moving median and three-point moving mean method of smoothing.

Part of the smoothing for both methods has been graphed onto the time series plot below.
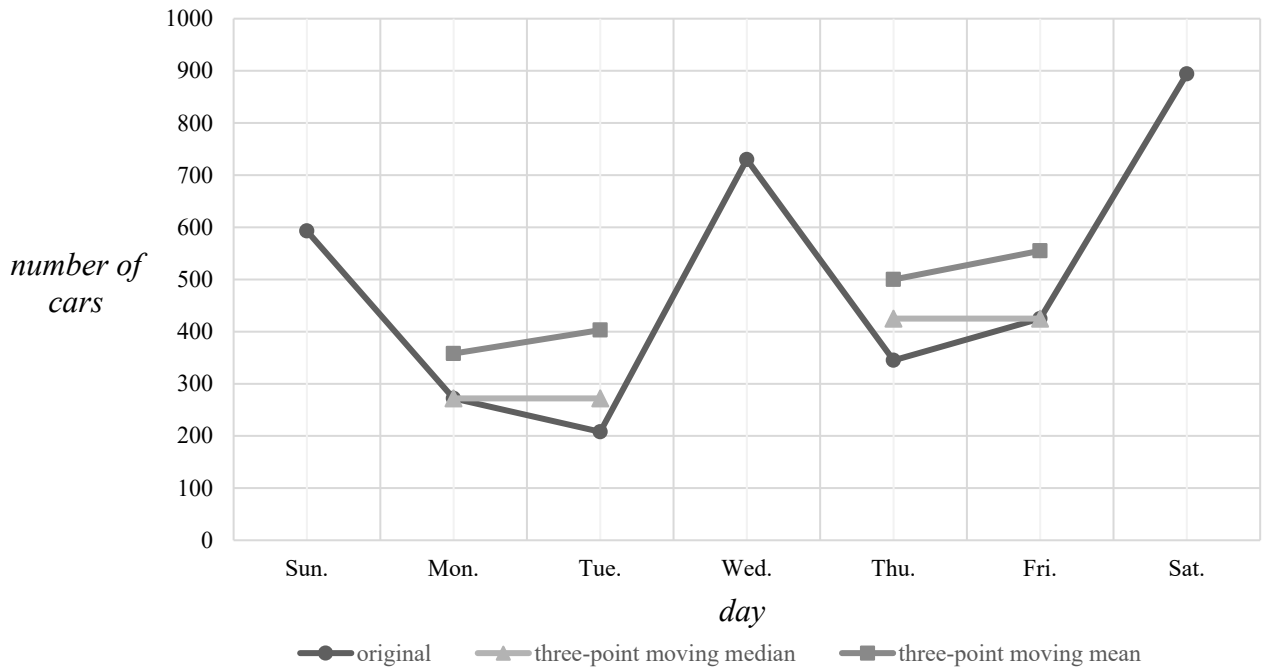
a.   i.   Complete the three-point moving median smoothing and the three-point moving mean smoothing on the **time series plot above**.

2 marks

(*Answer on the time series plot above.*)

ii.   Explain a limitation of using a moving-mean method to smooth data compared to using a moving-median method.

1 mark

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     SECTION A – Question 5 – continued

b. **i.** The average number of cars parked each month in the carpark for a year is shown in the table below.

Month	Number of cars
Jan.	22 100
Feb.	19 700
Mar.	16 400
Apr.	18 400
May.	12 500
June	12 900
July	18 800
Aug.	20 300
Sept.	21 500
Oct.	19 700
Nov.	32 400
Dec.	48 200
Average	**21 908**

Show that the calculation to determine the seasonal index for May is 0.57

1 mark

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ii. In May the following year, the average number of cars was 14 300.

What is the deseasonalised value?

Round your answer to three significant figures.

1 mark

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SECTION A – continued
TURN OVER

Recursion and financial modelling

Question 6 (3 marks)

Saskia buys a commercial washing machine for her laundromat business. The value of the washing machine after n years is shown by the equations below:

$$V_0 = 3875 \qquad V_{n+1} = V_n - 410$$

a.   **i.**   Showing recursive calculations, determine the value of the washing machine after two years.

1 mark

ii.   What is the annual flat rate of depreciation?

Write your answer as a percentage rounded to two decimal places.

1 mark

b.   An alternative model of depreciation is that the machine depreciates by $800 immediately upon purchase, then depreciates at 6% per year on a reducing balance basis.

What is the value of the machine after three years using this method?

Round your answer to the nearest dollar.

1 mark

Question 7 (4 marks)

Saskia invests $3050 in an account earning 12% per annum on a flat rate basis.

a.   **i.**   Write a calculation that shows that the interest earned each year is $366.

1 mark

_____

ii.   Write the rule that gives the balance of the investment, B_n, in dollars after n years.

1 mark

_____

iii.   How long until the balance is $7076?

1 mark

_____

b.   Saskia changes her investment account from one earning simple interest to one earning compound interest and starts making additional payments each fortnight.

She transfers the balance from the original account, $7076, into the new account, which now earns 3.56% per annum, compounding fortnightly.

What were the additional payments made if the balance is $30 482 six years later?

Round your answer to the nearest dollar.

1 mark

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Question 8 (2 marks)

Saskia's brother Tasman borrows $70 000 so that he can invest in her business. The loan is for ten years, with interest charged at 3.75% per annum, compounding monthly. The repayments are $701 per month.

a. What is the balance of the loan halfway through the loan term of ten years?

Round your answer to the nearest dollar.

1 mark

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b. What is the difference between the interest paid in the first five years and the interest paid in the last five years?

Round your answer to the nearest dollar. Accounting for an adjusted final payment is not required.

1 mark

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Question 9 (3 marks)

Tasman also has an annuity with a balance of $52 000, which earns 3.95% per annum, compounding monthly. The annuity is for 12 years.

a. i.   Tasman wants to change the terms of the annuity so that the withdrawals from it cover the $701 monthly repayments of the loan for as long as the annuity lasts.

How much must he reduce the length of the annuity by in order to obtain withdrawal payments that sufficiently cover the $701 monthly loan repayments?

2 marks

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ii.   Five years after investing the $52 000, the interest rate of the annuity increases. Two years after the rate change, the balance of the annuity is $841. The withdrawals of $701 remain constant throughout this time.

What was the interest rate, as a percentage, after the change?

Round your answer to two decimal places.

1 mark

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Module 1 – Matrices

Question 1 (4 marks)

Kyle manages a hostel for backpackers. It has a combination of single rooms, *S*, twin rooms, *T*, and dormitory rooms. The dormitory rooms are classified as large, *L*, or medium, *M,* sized.

The number of rooms available is shown in the matrix *R* below.

$$R = \begin{bmatrix} 6 \\ 8 \\ 1 \\ 2 \end{bmatrix} \begin{matrix} S \\ T \\ L \\ M \end{matrix}$$

a.   How many dormitory rooms are there?

1 mark

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b.   The large room is out of order. The matrix calculation below shows that the number of single, twin and medium rooms is 16 in total.

$$T \times R = \begin{bmatrix} 16 \end{bmatrix}$$

Write down matrix *T* below.

1 mark

$T =$

c.   Kyle considers remodelling the hostel.

He proposes increasing the number of rooms, as shown in matrix *I* below.

$$I = \begin{bmatrix} 9 \\ 14 \\ 2 \\ 4 \end{bmatrix} \begin{matrix} S \\ T \\ L \\ M \end{matrix}$$

The number of people that can be accommodated in each room is shown in matrix *P* below.

$$P = \begin{matrix} S & T & L & M \end{matrix} \\ \begin{bmatrix} 1 & 2 & 12 & 6 \end{bmatrix}$$

The council regulations require that there is a minimum of one toilet for every nine people in the hostel.

How many toilets are required in Kyle's hostel?

Show the matrix calculation, including a scalar multiplication, to justify your answer.

2 marks

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TURN OVER

Question 2 (2 marks)

As backpackers travel through different cities and locations, they book into different hostels.

The transition matrix, T, below shows the probability that a backpacker will pay for a single room (S), a twin room (T) or a dormitory style room (D) when moving from one hostel to the next.

$$\begin{array}{c} \textit{this hostel} \\ \begin{array}{ccc} S & T & D \end{array} \\ T = \begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array} \quad \textit{next hostel} \end{array}$$

Kyle's hostel is considered to be the first hostel. There are currently 38 guests at Kyle's hostel and this is represented by the matrix S_0.

$$S_0 = \begin{bmatrix} 5 \\ 12 \\ 21 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array}$$

a. **i.** Complete the following by filling in the boxes provided, showing that there are approximately 19 backpackers staying in a dormitory room at the second hostel.

1 mark

$$\boxed{} \times 5 + \boxed{} \times 12 + \boxed{} \times 21 = 19$$

ii. How many of the 38 backpackers changed their preference of room from Kyle's hostel to the second hostel?

1 mark

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SECTION B – Module 1 – continued

Question 3 (3 marks)

A number of the original 38 guests no longer need to book a room and a number of additional travellers need to book one.

The expected number of people booking the different rooms at each hostel can be determined from the matrix recurrence relation

$$S_{n+1} = T \times S_0 + B$$

where S_0 is the number of guests in the first hostel, and B is the number of guests joining or leaving.

$$
T = \begin{array}{c} \quad\quad\;\; \textit{this hostel} \\ \begin{array}{ccc} S & T & D \end{array} \\ \begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \begin{array}{c} S \\ T \\ D \end{array} \end{array} \;\textit{next hostel}
$$

$$
S_0 = \begin{bmatrix} 5 \\ 12 \\ 21 \end{bmatrix} \begin{array}{c} S \\ T \\ D \end{array} \text{ and } B = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} \begin{array}{c} S \\ T \\ D \end{array}
$$

a. Interpret the element B_{31} shown in matrix B.

                                                                      1 mark

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b. The third hostel has space for 8 people in single rooms, 12 people in twin rooms and 22 people in dormitory rooms.

Determine whether all the guests expected to book this hostel can be accommodated in the room type they wish to book.

Justify your answer with calculations.

                                                                     2 marks

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Question 4 (3 marks)

Kyle arranges for games to be provided for guests staying at his house on a Tuesday night.

A round robin game is played with the guests broken into five teams. The table below outlines the number of direct and indirect wins of each team.

Team	One-step dominance	Two-step dominance
A	2	3
B	3	5
C	1	2
D	2	3
E	2	5

a.   **i.**   From this information, it can be determined with certainty that Team E beat which other team?

1 mark

_____

ii.   Teams A and B compared their wins and losses and saw that there were only two other teams they had both beaten in common.

Using the information and the table above, determine which teams Team D beat.

2 marks

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Module 3 – Geometry and measurement

Question 1 (4 marks)

Olympia is a product designer. She is purchasing new labels for her business.

The pattern on the labels consists of numerous crosses that are identical in shape and size, as shown below. An enlarged version of the cross is also shown, with the dimensions, in millimetres, labelled.
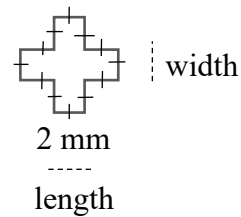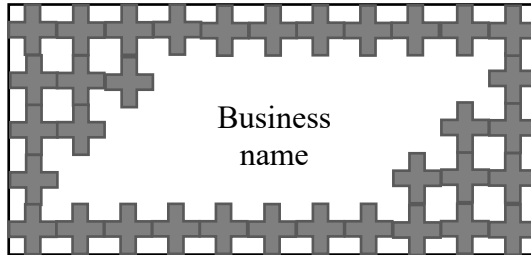
a. What is the length and width, in millimetres, of each cross?

1 mark

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_____

b. What is the length and width, in millimetres, of the label?

1 mark

_____

c. What is the area, in square millimetres, that is made up by the crosses?

1 mark

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d. The label is enlarged to become a flyer. The area of the flyer is now 25 times the area of the label.

What are the dimensions of the flyer, in centimetres?

1 mark

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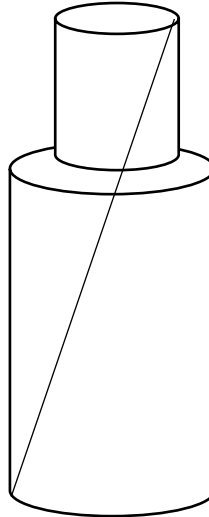
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       SECTION B – Module 3 – continued

Question 2 (3 marks)

Olympia has designed a new vase consisting of two cylinders, a smaller one on top of a larger one. The small cylinder is 8 cm high.

Olympia places a flower stem into the vase, as illustrated by the diagonal line in the diagram below.

The part of the stem that can be seen in the top cylinder is 9 cm long. The section of the stem in the bottom cylinder of the vase is three times the length of the stem in the top cylinder of the vase.

a. Determine the full height, in centimetres, of the vase.
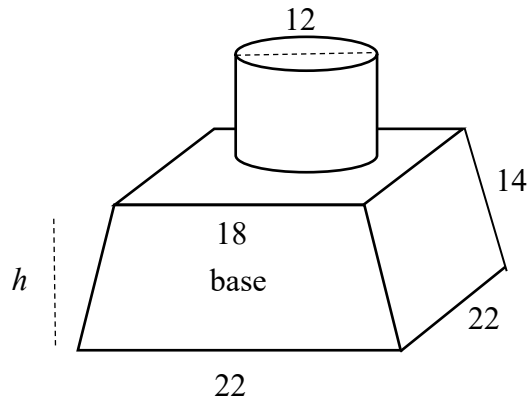
1 mark

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TURN OVER

Olympia alters the shape of the vase so that it has a symmetrical trapezium solid base and a cylindrical top.

The height of the cylinder is 10 cm, and its diameter is 12 cm.

Dimensions of the base, in centimetres, are illustrated below.

b. Show that the perpendicular height of the base, h, is approximately 13.9 centimetres.

1 mark

c. The volume of the cylinder is 1130.97

Calculate the total volume, in cubic centimetres, of the vase.

Round your answer to the nearest cubic centimetre.

1 mark

Question 3 (5 marks)

Olympia is on holiday, travelling on a boat that is located at 26° S, 134° W in the Pacific Ocean, travelling between New Zealand and Chile.

East Cape (38° S, 179° E) is one of the most eastern points in New Zealand.

Moto Nui (27° S, 109° W) is one of the most western points of Chile.

a.   Which of the locations, East Cape or Moto Nui, is closest to the Greenwich Meridian?

1 mark

_____

The boat's captain calls the port located at East Cape. It is 11 am on Sunday 10 February at the boat's location.

b.   What time is it at East Cape?

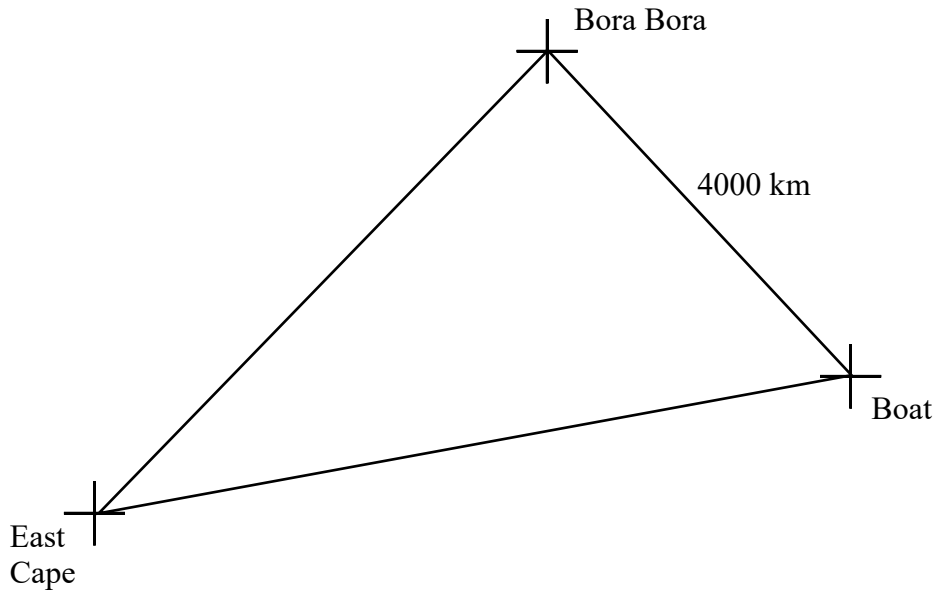Round your answer to the nearest hour.

1 mark

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_____

The boat is travelling from East Cape to Chile; however, along the way it changes course so it can visit Bora Bora.

The captain determines that Bora Bora is on a bearing of 335° from the boat's current location.

The boat is on a bearing of 70° from East Cape.

East Cape is on a bearing of 210° from Bora Bora.

The distance between Bora Bora and the boat is 4000 kilometres.

c. Determine the total distance travelled from East Cape to the boat's current location, and from there to Bora Bora.

Round your answer to the nearest kilometre.

2 marks

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Olympia's father is in Homer, Alaska (60° N, 152° W).

d. What is the shortest distance between them when Olympia is in Bora Bora (16° S, 152° W)?

Round your answer to the nearest kilometre.

1 mark

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End of Module 3 – SECTION B – continued