2022
VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

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Quality educational content

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Section A Core – Data Analysis

Question 1 C

Fruit name is nominal. Age group is ordinal (makes sense to order it; younger, older etc.)

Question 2 C

66/110 i.e. 60% of under 12 prefer grapes
72/160 i.e. 45% of 12 and over prefer grapes

Option C shows this.

Question 3 E

Under 6 attempts means 5 attempts or less.
This adds to 15 occasions.
15/21 = 0.7142… ~ 71%

Question 4 B

Range = 8 – 1 = 7
IQR = 6 – 3 = 3

Question 5 B

Number of attempts is numerical data.
There are only **two** categories of *amount of sleep*
so this information can be shown on a back-to back stem plot.

Question 6 A

Adding frequencies gives us 20 insects.
The median mass is somewhere between 10^{-1} and 10^{0}, that is between 0.1 gram and 1gram.
0.78 is the only option that satisfies this.

Question 7 C

46 is 2 standard deviations below the mean.
2.5% of the sample is expected to be 2 standard deviations below the mean.

$$2.5\% \text{ of } 640 = 16$$

Question 8 B

48 and 54 are 1 standard deviation below the mean and 2 standard deviations above the mean respectively.
Expected % of boxes to be within these limits is 68 + 13.5 = 81.5%

Section A Core – Data Analysis

Question 9 A

The gradient of the least squares line will give the expected change in *height* for every one unit change in *weight*.

$$gradient\ of\ least\ squares\ line = r\frac{S_y}{S_x}$$

$= 0.7062 \times \frac{7.426}{8.241} = 0.63635 \ldots$

Question 10 D

$$r = \pm\sqrt{0.87329} \sim \pm 0.934$$

The gradient of the least squares line is **negative**, so *r* will be **negative**.

r = - 0.934

Question 11 E

Predicted success rate

$= 84.2 - 0.327 \times 150 = 35.15$

Residual = actual – predicted

$= 42.30 - 35.15$

$= 7.15$

Question 12 B

Choose two points that the line passes through, or **very** near to.

For example, (35,70) and (25, 59)

Gradient = $\frac{70-59}{35-25} = 1.1$

Substitute (35,70) into $y = mx + c$

$70 = 1.1 \times 35 + c$

$c = 31.5$

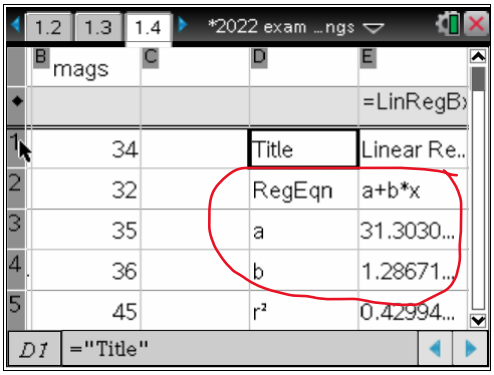
Question 13 D

Need the median of 35, 36, 45, 38, 40

When arranged in ascending order, 38 is the middle value.

Question 14 D

Use technology to find the equation of the least squares regression line.



Section A Core – Data Analysis

Question 15 C

Long term yearly average =

$$\frac{27.2 + 19.9 + 12.6 + 21.5}{4} = 20.3$$

SI $_{\text{autumn}} = \frac{12.6}{20.3} = 0.62$

Question 16 A

Deseasonalised value = $\frac{actual}{SI}$

$$actual = deseasonalised\ value \times SI = 22.2 \times 0.98$$
$$= 21.756 \sim 21.8$$

Section A Core – Recursion and financial modelling

Question 17 D

Number of copies per year
$= \frac{960}{0.03} = 32000$

Question 18 B

Flat rate =

$$\frac{960}{24000} \times 100 = 4\%$$

Question 19 B

The sequence is:
5, 15, 35, 75, 155, … where $T_0 = 5$
$T_4 - T_3 = 155 - 75 = 80$

Question 20 E

Jim's number of followers is 97% of the last month's number (a loss of 3%) with 200 added on each time.

Question 21 A

Amount of principal reduction
= payment – interest.

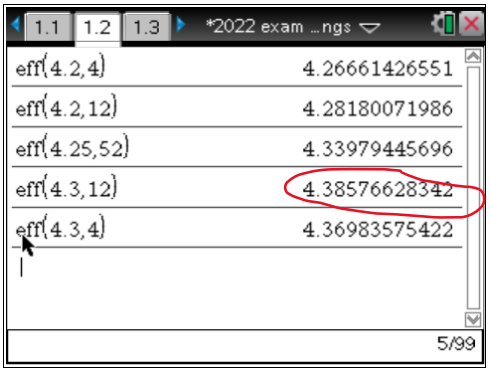
Payment = principal reduction + interest
= 1115.45 + 2184.55 = 3300

Question 22 C

Using figures from the table, interest rate per month $= \frac{2187.50}{750000} \times 100 \sim 0.219667$
Annual interest rate $= 0.219667 \times 12 = 3.5\%$

Section A Core – Recursion and financial modelling (continued)

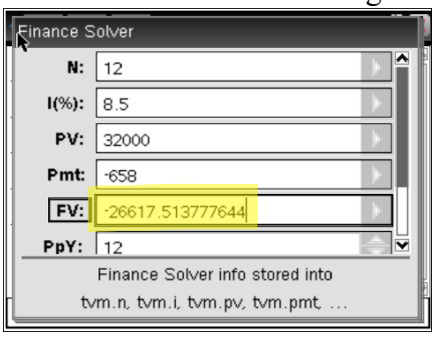
Question 23 D



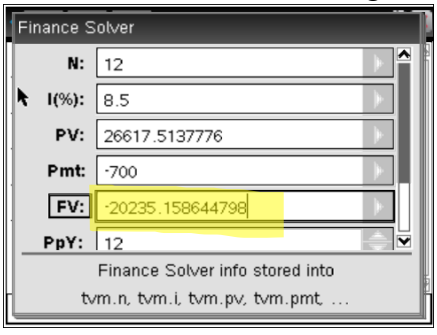
Use technology to convert the nominal rates to effective rates.
4.3% p.a. compounding monthly gives the highest effective rate of about 4.39%

Question 24 C

Calculate how much is owing after the first year



This amount now becomes the present value.
Calculate how much is owing after the second year.



Principal reduction = 32000 – 20235.16 = \$11764.84

Total payments = 12×658 + 12×700 = \$16296
Total interest paid = 16296 – 11764.84 = \$4531.16

End of solutions Section A Core

Module 1 – Matrices

Question 1 C

The matrix is symmetric about its leading diagonal, so it is an example of a symmetric matrix.

Question 2 A

$$A = \begin{bmatrix} 9 & 7 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$$

Question 3 D

Inspecting the position of the '1' in each row of the permutation matrix:

5th letter becomes 1st
3rd letter becomes 2nd
2nd letter becomes 3rd
1st letter becomes 4th
4th letter becomes 5th

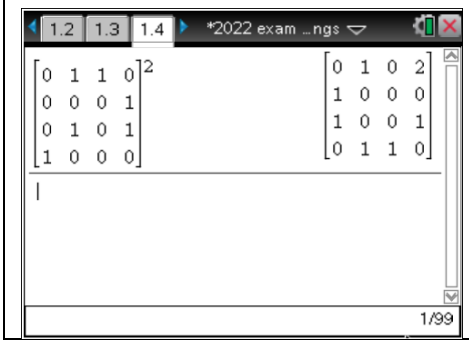
AGENT becomes TEGAN

Question 4 D

Can do by inspection, or by squaring the one-step dominance matrix.
One-step dominant matrix is

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Squaring it results as follows.



Question 5 E

Columns must add to 1.0, so eliminate options C and D.
Now, 68% of Warburn viewers don't change, and 83% of Yarragrove viewers don't change.
This is indicated in option E.

Question 6 B

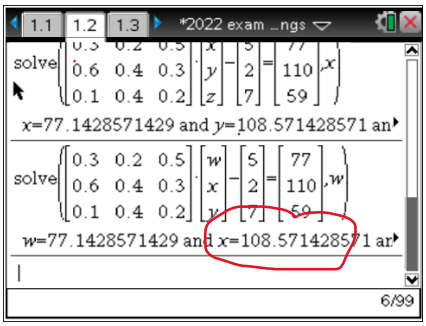
Option B does not follow the rule.
n_{21} must be 5, not 1 as is shown.
$3 \times 2 - 1 = 5$

Module 1 – Matrices (continued)

Question 7 B

Solving $T \times F_2 - B = \begin{bmatrix} 77 \\ 110 \\ 59 \end{bmatrix}$

Gives X = 108.571…~ 109



Question 8 C

The numbers in the rooms remain constant, so if 27 participants change from room A to room B for session 2, 27 must change from room B to room A.

36% of x = 27

$$x = \frac{27}{0.36} = 75$$

25% of y = 27

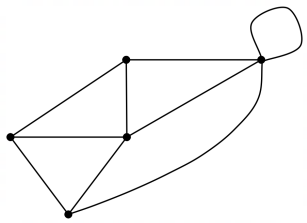
$$y = \frac{27}{0.25} = 108$$

Total number of participants = 183

End of Module 1 Solutions

Module 2 – Networks and decision mathematics

Question 1 E



Only the middle vertex has an even degree.
(remember that a loop counts as 2)

Question 2 D

e = v + f -2
= 9 + 4 – 2 = 11

Question 3 B

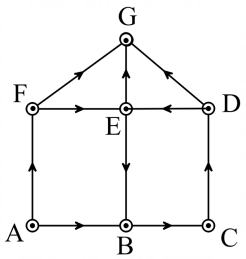
Option B involves an edge (connecting C and G) being used twice.

AGCDEFCGABCFH

Question 4 D

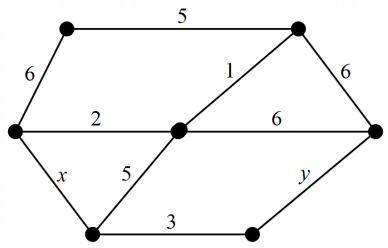
C and D both have A and B as immediate predecessors, so a dummy must run from either A or B to start of both C and D.
E has both C and D as immediate predecessors, so a dummy must run from end of either C or D to the start of E.
This is satisfied in option D.

Question 5 A



Following the arrows, we see that B
D, E and G are all reachable from C.
A and F are unreachable.

Question 6 B

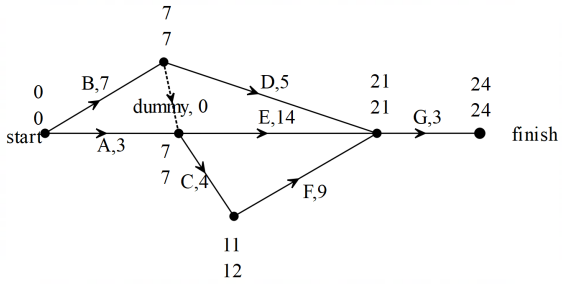


The spanning tree includes x and y. The smallest edge coming from the end of y weighs 3. Smallest edge coming from end of x weighs 2, so spanning tree length = $1 + 2 + 3 + 5 + x + y = 19$.
$x + y = 8$, so eliminate options A, D and E.
Now if $y = 7$, this edge would **not** be in the minimal spanning tree (as its greater than 6), so eliminate option C.
A quick check will show that if $x = 3$ and $y = 5$, the minimal spanning tree will have a length of 19.

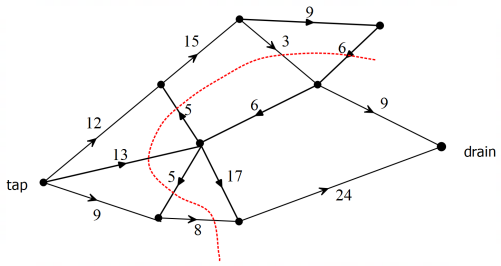
Module 2 – Networks and decision mathematics (continued)

Question 7 C

Forward and backward scanning show that the earliest completion time is 24 hours and that the critical path is BEG. Activity E has two immediate predecessors, A and B.
Float time for activity D = 21 – 7 – 5 = 9 hours.
Latest start time for activity F is 12 hours, **not** 11.



Question 8 B



Minimum cut gives maximum flow. The minimum cut, as shown, has capacity of
6 + 3 + 13 + 8 = 30