Neap

Trial Examination 2022

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	6	6	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 30 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Data analysis

Question 1 (11 marks)

The We Want Money (WWM) investment company is looking to build a new tourist attraction to generate profits. They research two options.

- Option 1 is an ocean wave pool that will generate waves strong enough to surf in, located in a section of Port Phillip Bay.
- Option 2 is a 500 m sky trail that is suspended over the head of Port Phillip Bay, originating from Queenscliff. The trail will have glass floors and walls to allow visitors to view the ocean below.

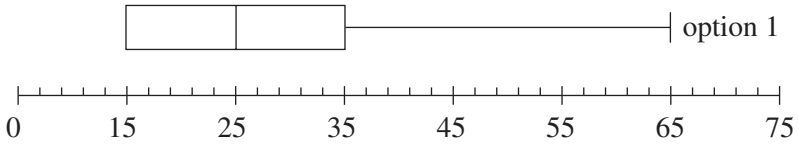
Over several months, 10 000 people are surveyed at different venues and times to see how likely people would be to use each attraction.

a. Give **one** reason as to why the survey may have been administered across different venues and times. 1 mark

The responses for option 1 or 2 are broken down by age group, as shown in the table below.

Age group	Option 1	Option 2
10 < 20	1300	200
20 < 30	1500	800
30 < 40	1200	1300
40 < 50	500	1500
50 < 60	300	900
60 < 70	200	300

b. A box plot representing the option 1 responses was constructed without including any possible outliers, as shown in the following diagram.



Draw a parallel boxplot that represents the option 2 responses on the diagram above. Do not include any outliers. Show your working. 2 marks

c. Use your parallel box plot from **part b.** to comment on the shape of the data for options 1 and 2. 2 marks

d. Are there any outliers in the data for option 2? Justify your answer with calculations. 1 mark

Option 1 is cheaper than option 2, but is only popular with the 20 < 30 and 30 < 40-year-olds.

A total of 1400 people from these two age groups said they were extremely interested in option 1, so a more detailed follow-up survey was conducted with them.

One of the questions in the survey was:

'How many hours a month would you surf if the price were $50 an hour?'

The table below shows the responses to this question.

Hours per month	Frequency
0	100
1	100
2	300
3	400
4	250
5	150
6	80
7	20

e. Calculate the following. Give your answers correct to two decimal places if necessary.

i. the mean number of hours each person would surf per month 1 mark

_____

ii. the standard deviation of the number of hours each person would surf per month 1 mark

_____

iii. the mode of the number of hours each person would surf per month 1 mark

_____

f. Assuming the data is normally distributed, calculate the 95% confidence limits for the income expected from a single 20 < 40-year-old per month.

Give your answer correct to the nearest whole number. 2 marks

_____

_____

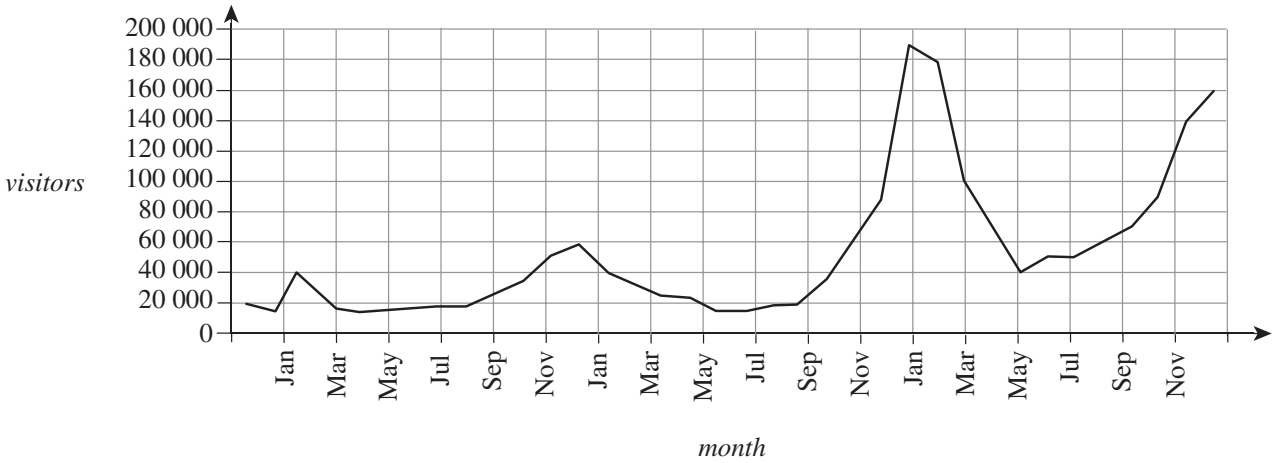
_____

_____

Question 2 (4 marks)

After extensive research into the popularity and construction costs of each option, the WWM investment company chooses to build the sky trail. Construction is expected to cost $3.2 million.

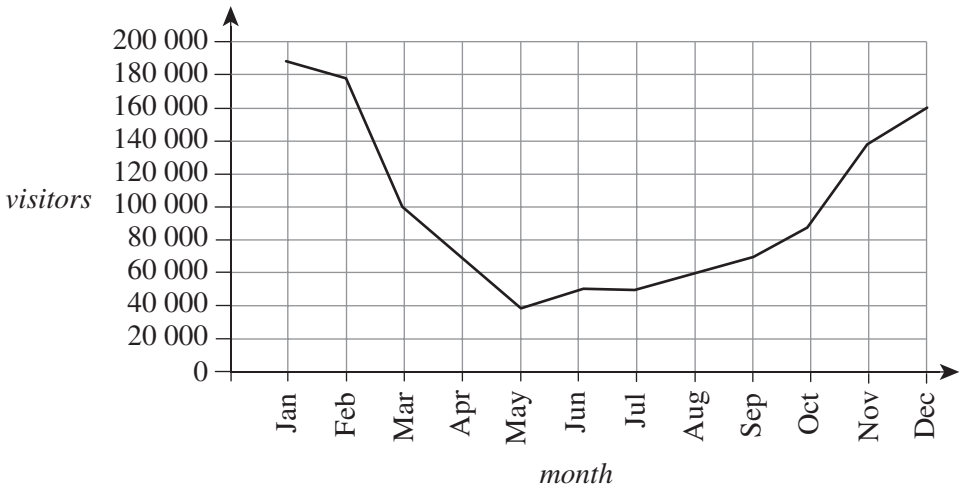
The projected numbers of visitors to the sky trail each month over the first three years of operation are shown below.



a. What type of graph is shown? 1 mark

b. Describe the shape of the graph. 1 mark

The data for the third year is shown in the graph below.



c. To smooth the curve, a three-point median graph is drawn on the same axes.

Which is the only point on the graph to shift? 1 mark

The actual number of visitors per month for the first nine months of operation is shown in the table below.

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep
Visitors	190 000	180 000	100 000	70 000	40 000	50 000	50 000	60 000	70 000

d. Find the three-point moving average, centred on February.
Round your answer to three significant figures. 1 mark

Question 3 (9 marks)

The daily temperature plays an important part in the number of visitors at the sky trail on a particular day. The number of visitors and maximum temperatures over a number of days at a similar project overseas were recorded, as shown in the table below.

Maximum temperature	12	25	32	15	29	33	10	18	23	27
Visitors	800	2000	4000	1500	3000	4100	900	1300	2200	3200

a. Find the equation of the least squares line.

Round your answer to the nearest whole number. 1 mark

b. A temperature of 45° is forecast for a particular day.

Is it reasonable to expect over 6000 visitors on this day? Explain your answer. 2 marks

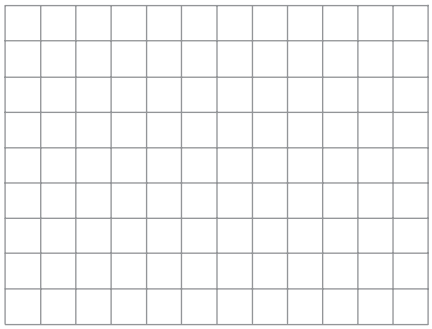
On a different set of days, the data produces the least squares line equation

$$\text{visitors} = 150 \times \text{max temperature} - 30, \text{ with } r = 0.85.$$

c. Complete the table of residuals below for a selected set of five points from this data. 2 marks

Maximum temperature	10	15	20	25	30
Number of visitors	1670	2020	3070	4020	4070
Predicted number of visitors	1470	2220	2970		
Residual	200	–200	100		

d. Draw a residual plot on the grid below. 1 mark



e. Comment on what the residual plot drawn in **part d.** shows. 1 mark

f. Perform a $\log_x$ transformation on the data from the five points to find the new correlation coefficient. 1 mark

g. Using your answers from **parts a–f.**, determine whether a linear or $\log_x$ transformation best models the data. Justify your answer. 1 mark

Recursion and financial modelling

Question 4 (4 marks)
Of the $3.2 million estimated cost to build the sky trail, $1 million is borrowed as a loan and the rest is provided by investors. The investors expect the value of their holding to increase by an average of 4% per annum over the first 10 years of the sky trail's operation.

a. Write an expression for the expected value, V, of their investment after n years for ≤ 10. 1 mark

b. After how many years does the investment first exceed $2 500 000? 1 mark

The loan compounds monthly with an interest rate of 4.4% per annum. The loan is to be repaid over a ten-year period with monthly repayments.

c. Determine the amount of the monthly repayments. 1 mark

After making loan repayments and covering the 4% increase in value expected by the investors, the company is predicting to break even in the first year and make an increasing profit annually for the next five years.

The profit after the first year can be described by the regression equation

$$t_{n+1} = 1.075t_n,\ t_2 = \$250\ 000.$$

d. Complete the following table of profits. 1 mark

Year	1	2	3	4	5
Profit ($)	0	250 000			

Question 5 (5 marks)

The ticketing system used for the sky trail cost $50 000 and is expected to last for five years or 10 million tickets sold, whichever comes first. The scrap value when the system needs to be replaced is $4000.

The system can be depreciated using two different methods.

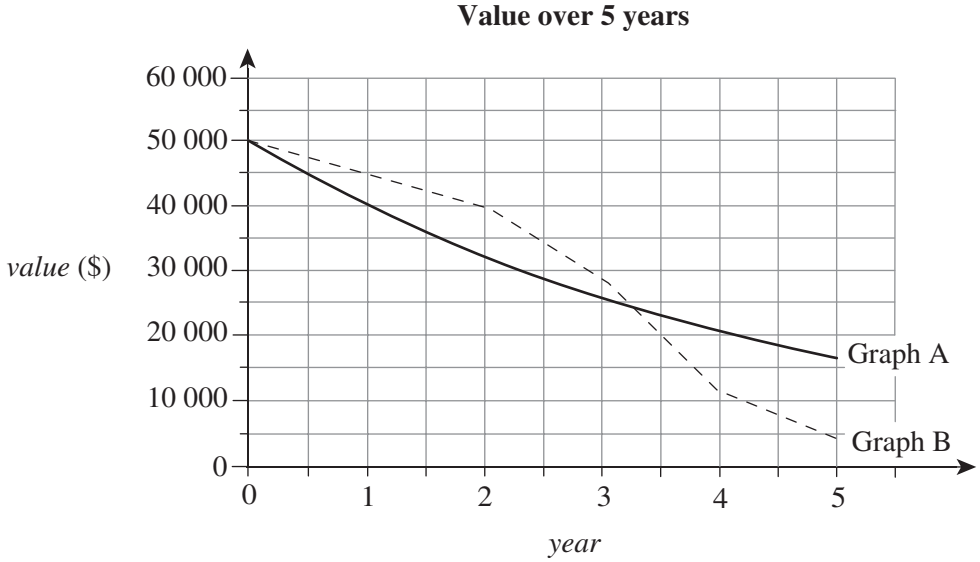
a. Calculate the depreciation cost per ticket if the ticketing system lasts until 10 million ticket are sold. 2 marks

b. What is the scrap value of the ticketing system after five years if it is depreciated at 20% per annum using the reducing balance method? 1 mark

The projections for the numbers of tickets processed over the first five years of operation are shown in the following table.

Year	Number of tickets
1	1 000 000
2	2 200 000
3	4 600 000
4	8 500 000
5	10 000 000

The value of the ticketing system over the first five years of operation is shown for both methods of depreciation on the graph below.



c. Which method of depreciation is represented by graph A **or** B? Explain your choice with at least **one** piece of mathematical evidence. 2 marks

Question 6 (3 marks)

After six years, the WWM investment company sells the sky trail for $7 million and pays out the initial investors, leaving $4.8 million of profit. The owners of the company invest $2 million in a perpetuity account that earns interest at a fixed rate of 2.8% per annum. The interest is calculated and paid monthly.

a. What is the value of the monthly payment the owners will receive? 1 mark

b. The remaining profit of $2.8 million is used as a deposit on a $6 million loan for another business venture. The interest on the 10-year loan is 8.3% per annum, compounding monthly. The company can either borrow the full $6 million or use the $2.8 million deposit to reduce their loan to $3.2 million.

How much interest would be saved by borrowing $3.2 million instead of $6 million? 2 marks

SECTION B – MODULES

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

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Contents **Page**

Module 1 – Matrices

Question 1 (2 marks)
The Year 7 class timetable on a particular day at Central High School can be represented by the following matrix, T.

$$T = \begin{array}{c} \\ 7A \\ 7B \\ 7C \end{array} \begin{array}{c} \begin{array}{ccccc} P1 & P2 & P3 & P4 & P5 \end{array} \\ \begin{bmatrix} ENG & MAT & ART & PE & SCI \\ PE & SCI & MAT & ENG & HE \\ MAT & MUS & ENG & SCI & PE \end{bmatrix} \end{array}$$

a. What is the order of matrix T? 1 mark

b. Is it possible for the same teacher to teach all the Year 7 Maths and Science classes? Explain your answer. 1 mark

Question 2 (6 marks)

Central High School's canteen sells two types (A and B) of lunches, each containing a healthy meal, snack and drink.

On the first day of school, class 7A orders five type A and three type B lunches, which costs $58.50, and class 7B orders seven type A and six type B lunches, which costs $96.75.

Let a = the number of type A lunches ordered and b = the number of type A lunches ordered.

a. Write a set of simultaneous equations to represent the orders from both classes. 1 mark

b. Express the set of simultaneous equations in matrix form. 1 mark

c. Find the unit cost of the type A and type B lunches. 2 marks

On the first day of school, the lunch orders for the entire school included 240 type A lunches and 160 type B lunches.

After the first day, a student who had a type A lunch is 80% likely to reorder the same lunch and 20% likely to swap to a type B lunch. A student who had a type B lunch is 90% likely to reorder the same lunch and 10% likely to swap to a type A lunch.

d. Write the transition matrix that represents this information. 1 mark

e. After a further 10 days, how many of each type of lunch will be ordered?
Round your answer to the nearest whole number. 1 mark

Question 3 (4 marks)

Every student at Central High School belongs to one of four houses. Each house is named after a famous ex-student: Alexander, Barrymore, Cavendish and Del Rio.

A volleyball tournament was played between the four houses. The results of the tournament are listed below.

- Alexander won the matches played against Barrymore and Del Rio.
- Barrymore won the match played against Del Rio.
- Cavendish won the matches played against Alexander and Barrymore.
- Del Rio won the match played against Cavendish.

a. Write a matrix to represent the results of the tournament. 1 mark

b. Based upon one-step dominance only, which two houses would be equally ranked the highest? Justify your answer using calculations. 1 mark

c. Based upon one- and two-step dominance, which team is ranked the highest? Justify your answer using calculations. 2 marks

END OF MODULE 1

Module 2 – Networks and decision mathematics

Question 1 (3 marks)
Central High School is renovating its Science wing. The tasks that need to be done before building begins are shown in the table below, along with the time for each task and the predecessors.

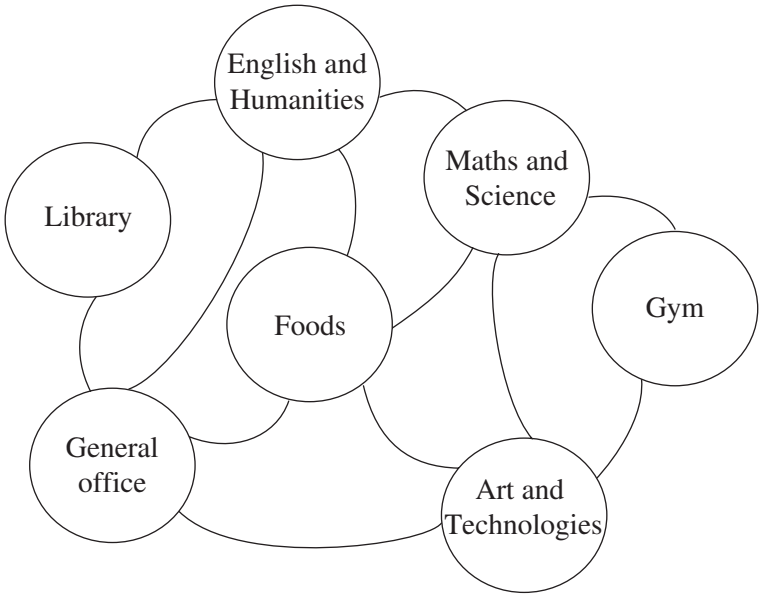
Task	Time (days)	Predecessors
h	3	–
i	2	–
j	1	*i*
k	3	*j*
l	4	*i*
m	5	*j*
n	6	*h*
o	3	*k*, *l*
p	2	*n*, *m*, *o*

a. Draw a directed graph to represent this information in the space below. 2 marks

b. What is the minimum time needed to complete this project? 1 mark

Question 2 (2 marks)

The builders visit the school to decide where they will set up their office and where deliveries will be made. They are taken on a tour of the school that begins and ends at the general office. The tour must travel every pathway and visit every building.

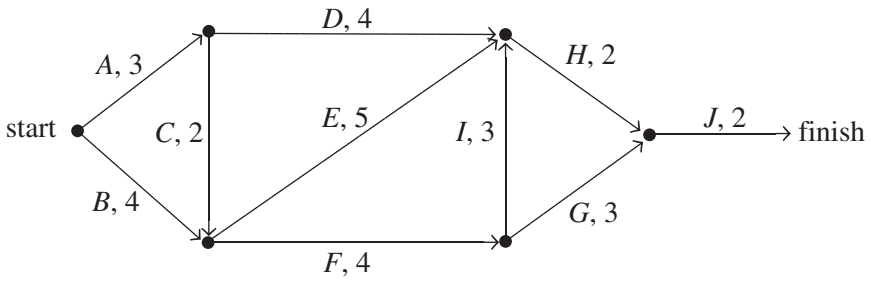


a. What is the mathematical name for this type of tour? 1 mark

b. Is it possible for the tour to begin and end at the general office while travelling every pathway? Explain your answer. 1 mark

Question 3 (2 marks)

The major building tasks are scheduled as follows. The times are given in weeks.



The minimum time to complete the project is 16 weeks.

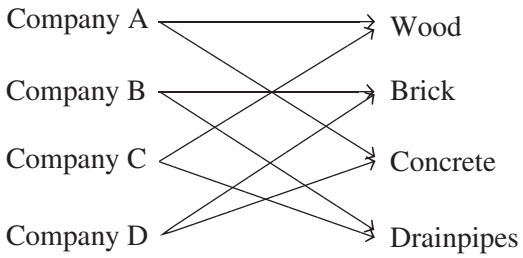
a. There is an option to employ an additional labourer to reduce the length of task *B* from 4 weeks to 2 weeks.

Explain the effect of employing the additional labourer on the completion time. 1 mark

b. How long is the float time for task *I*? 1 mark

Question 4 (5 marks)

The builder uses four suppliers and wants each company to supply one building material. A bipartite graph of the companies and what they supply is shown below.



a. The preferred supplier of the concrete is company A.

Complete the table below to assign each of the other three companies a material to supply. 2 marks

Company	Building material allocated
A	
B	
C	
D	

Four labourers are to be employed to complete the tasks. The number of hours each labourer has estimated they will take to complete the four tasks is shown below.

Labourer	Painting	Plastering	Concreting	Plumbing
Archie	27	30	34	40
Belinda	32	36	32	38
Charli	31	33	35	37
Depak	28	32	31	34

b. Use the Hungarian Algorithm to allocate each labourer to the task that will minimise the time spent on each task for this project. Show all steps. 3 marks

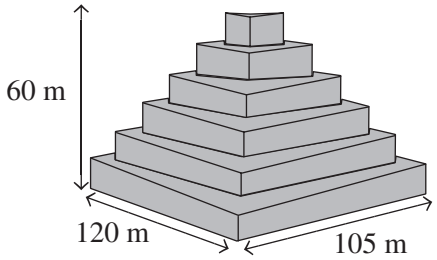
END OF MODULE 2

Module 3 – Geometry and measurement

Question 1 (3 marks)
Jacob, who lives in Darwin, wishes to visit the stepped pyramid of Djoser in Egypt.

a. The coordinates of Djoser are (31° N, 35° E). The coordinates of Darwin are (12° S, 131° E).
Using the coordinates, determine which location is closer to the equator. 1 mark

Djoser was built in 27 BC. The base has dimensions 105 m × 120 m and the height of the pyramid is 60 m, as shown in the diagram below. The pyramid can be thought of as six rectangles stacked on top of each other. Each horizontal step is 10 m wide.



b. The vertical faces of the lowest three steps were originally covered in marble.
How much marble was required?
Give your answer correct to the nearest square metre. 2 marks

Question 2 (5 marks)

Jacob enjoys hiking and wants to take a short hike through the desert. Leaving from his hotel, he walks 1.2 km on the bearing of N 25° W, then turns to a heading of N 75° E and walks a further 2 km.

a. How far is Jacob from his hotel? Show your working.

Give your answer correct to two decimal places. 2 marks

b. What is the bearing of Jacob from his hotel? Show your working.

Give your answer correct to one decimal place. 2 marks

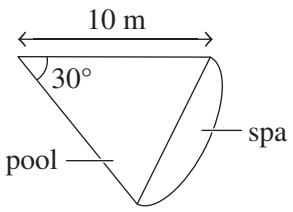
c. Jacob can see a 20 m high flagpole that is 800 m directly west of his position. Assume Jacob's eyes are 1.6 m above the ground.

Calculate the angle of elevation from Jacob's eyes to the top of the flagpole.

Give your answer correct to one decimal place. 1 mark

Question 3 (4 marks)

Jacob's hotel has an outdoor pool and spa as shown in the diagram below. The pool area is in the shape of a cone and the spa area is in the shape of a semicircle.



a. Find the area of the spa. Show your working.

Give your answer correct to one decimal place. 2 marks

b. On average, the pool section is 1.2 m deep.

Find the volume of the water required to fill the pool. 2 marks

END OF MODULE 3

Module 4 – Graphs and relations

Question 1 (4 marks)

There are two parts to the cost of running a school presentation dinner.

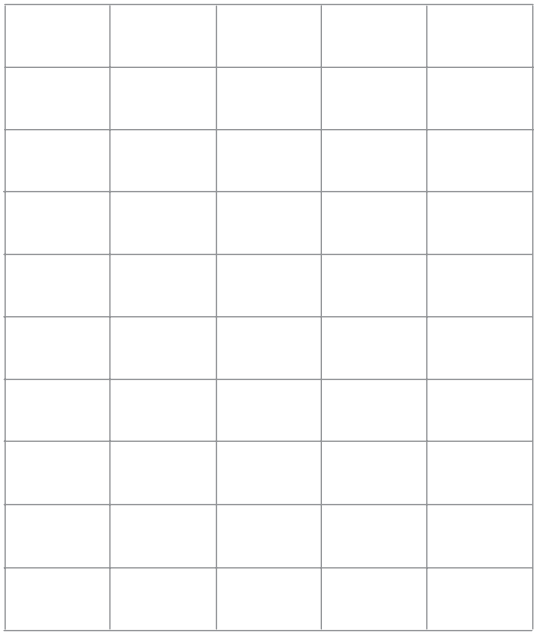
- Fixed costs: awards and certificates ($2000) and hiring of the venue ($1500)
- Variable costs: meals ($45 per person)

Let n = the number of guests attending.

a. Write an expression to show the cost, c, of running the dinner in terms of the fixed and variable costs. 1 mark

The school asks the guests attending to pay $70 per person for their meal. The income is calculated using the equation $I = 70n$.

b. Plot and label the graphs of the cost of and income from the dinner on the grid below. 2 marks



c. What does the intersection of the two graphs show? 1 mark

Question 2 (5 marks)

A company that supplies decorative square and triangular boxes for gifts gets a rush order for the school presentation dinner. They are asked to supply at least 25 boxes, including at least five of each type of box. The boxes need to be ready in one hour.

A square box takes 2 minutes to make and sells for a profit of $4. A triangular box takes 3 minutes to make and sells for a profit of $5.

Let x = the number of square boxes made, and y = the number of triangular boxes made.

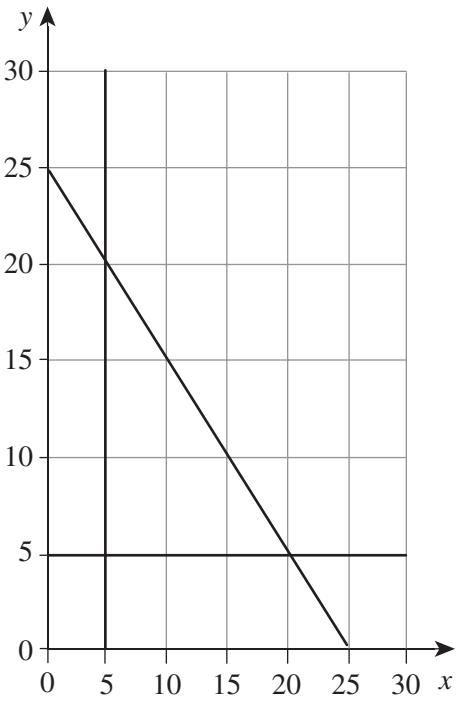
Some of the constraints for the order are:

- $x \geq 5$
- $y \geq 5$
- $x + y \geq 25$.

a. Write the final constraint caused by time. 1 mark

b. The graph below shows the first three constraints plotted.

Add the time constraint and shade the required region. 1 mark



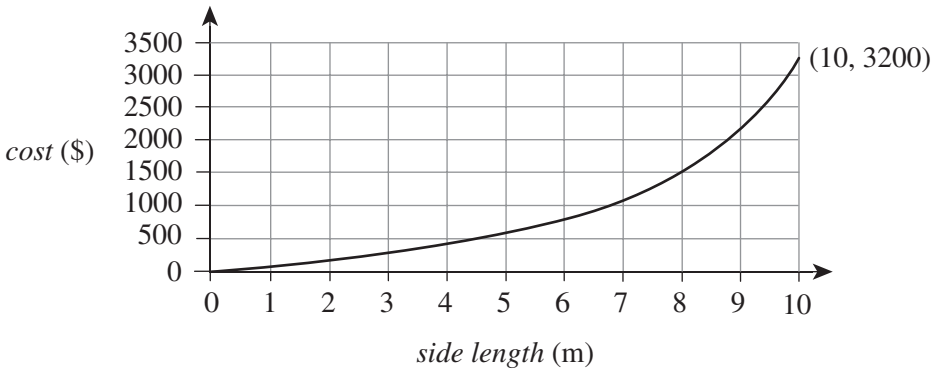
c. Write the profit function for the order. 1 mark

d. What is the number of square and triangular boxes that will earn the maximum profit? 2 marks

Question 3 (3 marks)

The cost of supplying a carpet to use as a square dance floor during the school presentation dinner to protect the existing floor surface is given by a formula in the form $y = kx^2$, where x is the length of each side.

a. The graph below shows the cost of the carpet for a given side length.



Calculate the value of k. 1 mark

b. What does the value of k represent? 1 mark

c. An alternative to supplying the carpet is a wooden surface, which is priced at a fixed cost of $250 plus $75 per square metre.

Write the cost formula in terms of x, the length of each side. 1 mark

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2022

VCE Further Mathematics Units 3&4

Written Examinations 1 & 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.

A multiple-choice question booklet and a question and answer booklet are provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

FURTHER MATHEMATICS FORMULAS

Core – Data analysis

standardised score	$z = \dfrac{x - \overline{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \overline{y} - b\overline{x}$
residual value	residual value = actual value – predicted value
seasonal index	$\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matricies

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 = \text{intital state}, \quad S_{n+1} = TS_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta°)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}, \quad$ where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta°$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta°}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times$ area of base $\times$ height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET

Neap

Trial Examination 2022

VCE Further Mathematics Units 3&4

Written Examination 1

Multiple-choice Answer Sheet

Student's Name: _____

Teacher's Name: _____

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out. Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: [A] B C D E

Use pencil only

Core: Data analysis
ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E

Core: Recursion and financial modelling
ONE ANSWER PER LINE

17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E
23	A	B	C	D	E
24	A	B	C	D	E

Continues over page

Answer TWO modules. Show modules answered by shading the appropriate box and writing the name of the module in the box provided.

Use pencil only

Module:

- ☐ **Matrices**
- ☐ **Networks and decision mathematics**
- ☐ **Geometry and measurement**
- ☐ **Graphs and relations**

Module:

ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E

Module:

- ☐ **Matrices**
- ☐ **Networks and decision mathematics**
- ☐ **Geometry and measurement**
- ☐ **Graphs and relations**

Module:

ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E