Neap

Trial Examination 2023

VCE General Mathematics Units 3&4

Written Examination 2

Suggested Solutions

DATA ANALYSIS

Question 1 (4 marks)

a. ordinal data A1

b. **i.**

	Responses				
	SD	**D**	**N**	**A**	**SA**
Parents of award winners	0	0	0	22	46
Parents of non-award winners	12	14	16	24	22

A1

ii. Adding the frequency of each response gives:

$22 + 46 + 12 + 14 + 16 + 24 + 22 = 156$ A1

iii. $\frac{22+46}{156} \times 100 = 43.59\%$

$= 44\%$ A1

*Note: Consequential on answer to **Question 1b.ii.***

Question 2 (10 marks)

a. **i.** The five-number summary is the lowest value, Q_1, median, Q_3 and the highest value.

As there are 200 responses to the survey:

Q_1 is between the 50th and 51st responses; as both responses are 4, $Q_1 = 4$.

The median is between the 100th and 101st responses; as both responses are 5, the median is 5.

Q_3 is between the 150th and 151st responses; as both responses are 6, $Q_3 = 6$.

Therefore, the five-number summary is:

1, 4, 5, 6, 7 A1

ii. negatively skewed A1

iii. $\text{IQR} = Q_3 - Q_1$

$= 6 - 4$

$= 2$ M1

$\text{lower fence} = Q_1 - 1.5 \times \text{IQR}$

$= 4 - 1.5 \times 2$

$= 1$

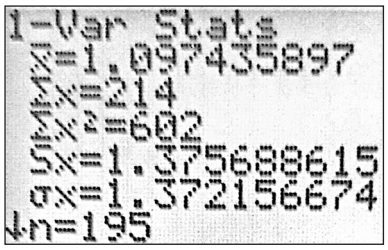
$\text{upper fence} = Q_3 + 1.5 \times \text{IQR}$

$= 6 + 1.5 \times 2$

$= 9$

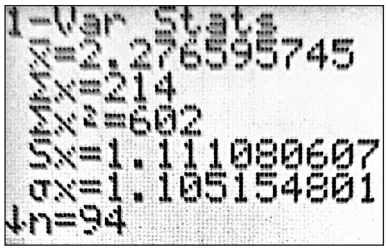
As all responses are between 1 and 9, there are no outliers. A1

b. **i.** Entering the number of current or past students into L_1 and the frequency into L_2 on a CAS calculator gives:



mean = 1.097 A1

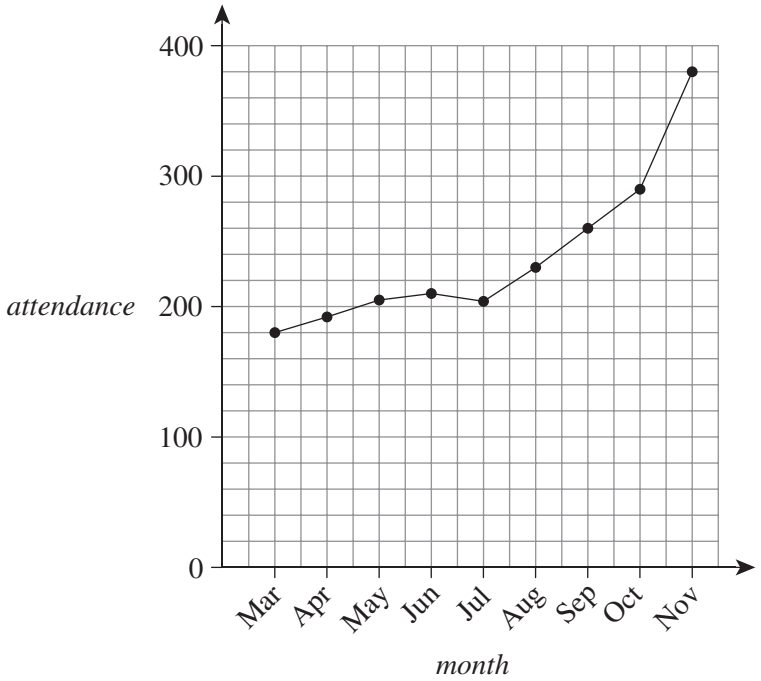
ii. Removing the values of 0 and 101 from the first row of the data from **Question 2b.i.** gives:



mean = 2.28 A1

standard deviation = 1.11 A1

c.



plots the data points A1

uses a suitable scale A1

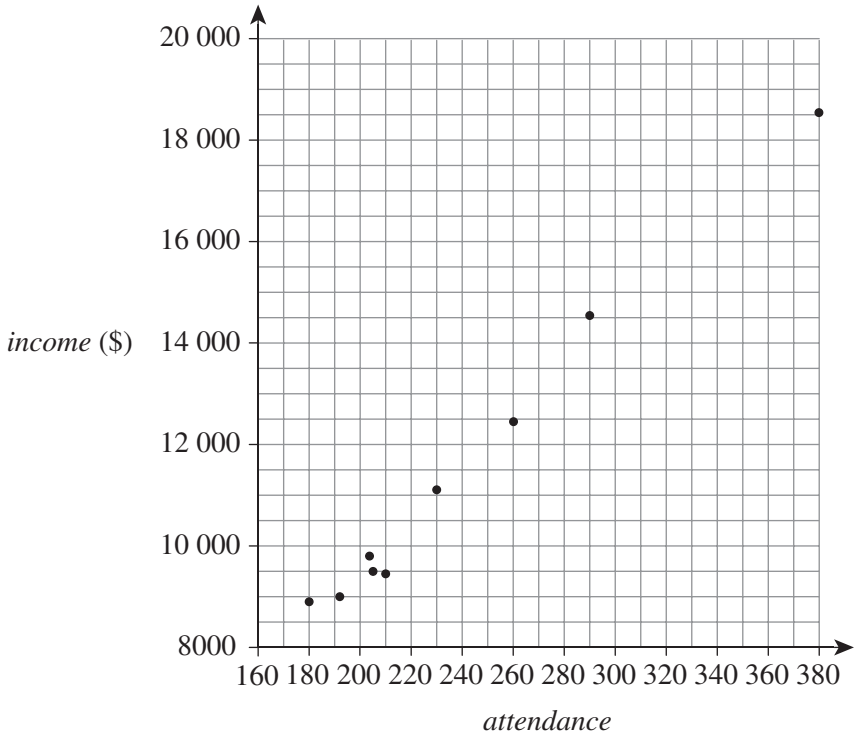
Note: Accept graphs that use different scales if they are accurate.

d. The graph has an overall increasing trend. A1

Question 3 (7 marks)

a. The attendance is the explanatory variable A1
because the income depends on attendance. A1

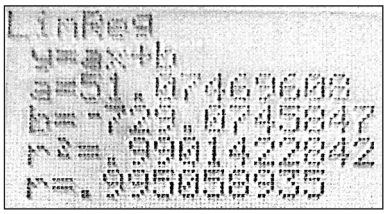
b.



identifies and plots the missing data points A1

c. There is a strong, positive association between attendance and income. A1

d. Entering the values for attendance into L_1 and the values for income into L_2 of a CAS calculator gives:



$income = -729.1 + 51.1 \times attendance$ A1

e. Using the data entered into a CAS calculator in **Question 3d.** gives:

total attendance = 2151

total income = 103 300

Finding the mean amount gives:

$$\text{mean amount} = \frac{\text{total income}}{\text{total attendees}}$$

$$= \frac{103\,300}{2151}$$ M1

$$= \$48.02$$ A1

Question 4 (3 marks)

a. Substituting *attendance* = 1500 into the least squares line to find the predicted income gives:

$$income = -1550 + 64 \times 1500$$
$$= \$94\ 450$$

Calculating the residual gives:

$$\text{residual} = \text{actual value} - \text{predicted value}$$
$$= 84\ 750 - 94\ 450 \quad \text{M1}$$
$$= -9700 \quad \text{A1}$$

b. The maximum attendance for the market was 380 people. Therefore, comparing the data from the market with the carnival, which had an attendance between 500 and 2000 people, is an extrapolation and thus may not be a good comparison. A1

Note: Responses must use the term 'extrapolation'.

RECURSION AND FINANCIAL MODELLING

Question 5 (9 marks)

a. \$5850 A1

b. $\frac{5850}{85\ 000} \times 100 = 6.88\%$ A1

c. $1 - 0.87 = 0.13$

Therefore, the equipment annually depreciates by 13%. A1

d. Finding the value of the equipment using flat rate depreciation gives:

$$V_2 = 85\ 000 - 2 \times 5850$$
$$= \$73\ 300$$
$$R_2 = 0.87^2 \times 85\ 000$$
$$= \$64\ 336.50 \quad \text{M1}$$
$$\text{difference} = 73\ 300 - 64\ 336.50$$
$$= \$8963.50 \quad \text{A1}$$

e. **i.** As the interest rate is 0.75% per month:

$$R = 1 + \frac{0.75}{100}$$
$$= 1.0075$$

The initial invoice was \$2000; therefore, $I_0 = 2000$.

Hence, the recurrence relation is:

$I_0 = 2000$, $I_{n+1} = 1.0075 I_n$ A1

ii. $I_6 = 1.0075^6 \times 2000$

$= \$2091.70$ A1

iii. The following amortisation table shows the balance of the invoice.

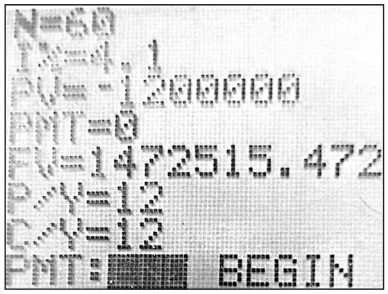
Month	Balance	Interest added	Payment	Balance at end of month
1	2000	$1.0075 \times 2000 = 2015$	700	1315
2	1315	$1.0075 \times 1315 = 1324.86$	700	624.86
3	624.86	$1.0075 \times 624.86 = 629.55$	629.55	

M1

The final payment is \$629.55. A1

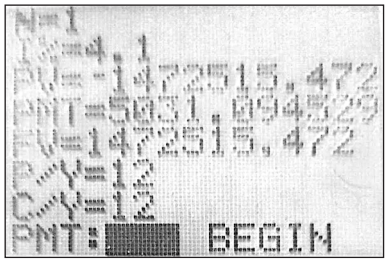
Question 6 (3 marks)

a. Using the financial solver on a CAS calculator gives:



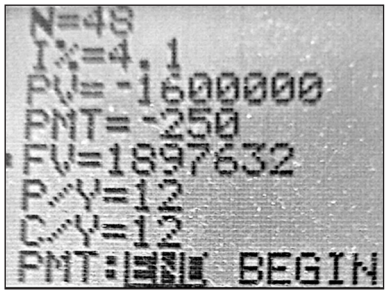
The value after five years is \$1 472 515.47. A1

b. Using the financial solver on a CAS calculator gives:



\$5031.09 can be withdrawn per month. A1

c. Using the financial solver on a CAS calculator gives:



The final balance is \$1 897 632. A1

MATRICES

Question 7 (5 marks)

a. The sales of all three items are lower in week 3 than the other weeks. Therefore, it is likely that the public holiday was in week 3, as the lower sales could be explained by the canteen only operating for four days in that week. A1

b. $26 + 22 + 14 + 29 = 91$ A1

c. Finding the mean number of burritos sold per week in the four-week period gives:

$$\text{mean sales} = \frac{34 + 30 + 23 + 36}{4}$$
$$= 30.75$$

Therefore, finding the number of burritos sold over 30 school weeks gives:

$$\text{number of burritos} = 30.75 \times 30$$
$$= 922.5$$
$$\approx 923$$

A1

Note: Accept responses of either 922.5 or 923.

d. $$\text{income for rolls} = (35 + 30 + 22 + 42) \times 4.80$$
$$= \$619.20$$
$$\text{income for sandwiches} = (26 + 22 + 14 + 29) \times 3.90$$
$$= \$354.90$$
$$\text{income for burritos} = (34 + 30 + 23 + 36) \times 4.50$$
$$= \$553.50$$

income for each item M1

$$\text{total income} = 619.20 + 354.90 + 553.50$$
$$= \$1527.60$$ A1

Question 8 (3 marks)

a. $T = \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}$ A1

b. The fourth day means that the transition matrix has been applied three times. Therefore:

$$S_4 = T^3 \times S_0$$
$$= \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}^3 \begin{bmatrix} 80 \\ 30 \end{bmatrix}$$
$$= \begin{bmatrix} 70 \\ 40 \end{bmatrix}$$ A1

c. Choosing two large values, such as $n = 15$ and $n = 16$, gives:

$$\begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}^{15} \begin{bmatrix} 80 \\ 30 \end{bmatrix} = \begin{bmatrix} 70 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix}^{16} \begin{bmatrix} 80 \\ 30 \end{bmatrix} = \begin{bmatrix} 70 \\ 40 \end{bmatrix}$$

As the two values of n result in the same matrix, the steady state matrix is $\begin{bmatrix} 70 \\ 40 \end{bmatrix}$. A1

Question 9 (4 marks)

a. I and L A1

b. Letting the dominance matrix be D and using a CAS calculator gives:

$$D + D^2$$

$$\begin{array}{c} \\ I \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} + \begin{array}{c} \\ I \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 2 \\ 1 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

M1

Calculating the total of the one- and two-step dominances for each team gives:

$$\begin{aligned} I &= 5 + 3 \\ &= 8 \\ J &= 4 + 2 \\ &= 6 \\ K &= 3 + 2 \\ &= 5 \\ L &= 2 + 1 \\ &= 3 \\ M &= 4 + 2 \\ &= 6 \end{aligned}$$

Therefore, I is first (strongest), J and M are second, K is fourth and L is fifth (weakest). M1

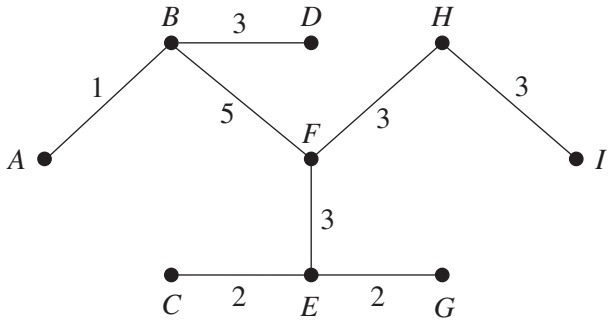
Since J defeated M, J has one-step dominance over M.

Therefore, the final ranking is I, J, M, K and L. A1

NETWORKS AND DECISION MATHEMATICS

Question 10 (3 marks)

a.



A1

The length of the minimum spanning tree is $1 + 3 + 5 + 3 + 3 + 3 + 2 + 2 = 22$ km. A1

b. The minimum cost of the project is $22 \times 22\,500 = \$495\,000$. A1

Question 11 (4 marks)

a. Performing a row reduction gives:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 18 & 9 & 3 \\ 9 & 24 & 0 & 22 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix} \end{array}$$

M1

The zeroes can be covered with less than four lines, as shown below.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 18 & 9 & 3 \\ 9 & 24 & 0 & 22 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix} \end{array}$$

However, as there are four tasks, four lines are required to find the optimum allocation.

Therefore, performing a column reduction gives:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 14 & 9 & 3 \\ 9 & 20 & 0 & 22 \\ 23 & 0 & 3 & 0 \\ 9 & 12 & 14 & 0 \end{bmatrix} \end{array}$$

M1

(continues on next page)

(continued)

The zeroes can now be covered by four lines, as shown below.

	W	*X*	*Y*	*Z*
~~*A*~~	~~0~~	~~14~~	~~9~~	~~3~~
~~*B*~~	~~9~~	~~20~~	~~0~~	~~22~~
~~*C*~~	~~23~~	~~0~~	~~3~~	~~0~~
~~*D*~~	~~9~~	~~12~~	~~14~~	~~0~~

Allocating the tasks gives:

	W	*X*	*Y*	*Z*
A	**0**	14	9	3
B	9	20	**0**	22
C	23	**0**	3	0
D	9	12	14	**0**

Contractor *A* should be allocated to job *W* (8 hours).

Contractor *B* should be allocated to job *Y* (4 hours).

Contractor *C* should be allocated to job *X* (19 hours).

Contractor *D* should be allocated to job *Z* (10 hours).

Therefore, the minimum number of hours is 8 + 4 + 19 + 10 = 41 hours. A1

b. The minimum uncovered value in the matrix would be added to the elements covered by two lines. A1

Question 12 (5 marks)

a. tasks *F*, *G* and *H* A1

b. Using a forward scan to find the length of the project gives:

27 days A1

c. *B–D–H–K–N* A1

d. Task *J* can start after 12 days but must start by day 15. Therefore, task *J* has a float time of 3 days. A1

e. As the length of task *H* is reduced by 3 days, this change will reduce the length of the project to 24 days. A1

