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Trial Examination 2023

# VCE General Mathematics Units 1&2

Written Examination 2

**Suggested Solutions**

**AREA OF STUDY 1**

- Data analysis, probability and statistics

**Question 1** (5 marks)

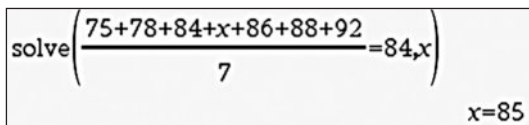
- a. Given that the mean of Aiden's scores is 84:

$$\frac{75 + 78 + 84 + x + 86 + 88 + 92}{7} = 84$$

$$x = 85$$

A1

Using a CAS calculator gives:



Given that the mode of Bell's scores is 84:

As 84 and 87 both appear twice in Bell's scores, for 84 to be the mode,  $y$  must be 84.

A1

- b. mean score for Bell =  $\frac{75 + 84 + 84 + 84 + 87 + 87 + 93}{7}$   
 $= 85.43$

A1

As Bell's mean score of 85.43 is higher than Aiden's mean score of 84, Bell will progress to the next round.

A1

- c. new mean score for Aiden =  $\frac{78 + 84 + 85 + 86 + 88}{5}$   
 $= 84.2$

$$\text{new mean score for Bell} = \frac{84 + 84 + 84 + 87 + 87}{5}$$

$$= 85.2$$

*both mean scores A1*

**Question 2** (5 marks)

a. 6.9 seconds A1

b. There are 21 data points in the data set. Excluding the median (11th data point) and dividing the remaining 20 values into bottom half and upper half gives:

Bottom half: 2.1, 5.7, 5.7, 5.9, 6.1, 6.2, 6.2, 6.5, 6.5, 6.9

Upper half: 7.2, 7.4, 7.5, 7.6, 7.6, 7.7, 7.8, 7.9, 8.2, 8.5

Finding Q1 (the median of the bottom half) gives:

$$\frac{6.1 + 6.2}{2} = 6.15 \text{ seconds}$$

Finding Q3 (the median of the upper half) gives:

$$\frac{7.6 + 7.7}{2} = 7.65 \text{ seconds} \quad \text{A1}$$

Therefore:

$$\text{IQR} = Q3 - Q1$$

$$= 7.65 - 6.15$$

$$= 1.5 \text{ seconds} \quad \text{A1}$$

c. 2.1 is the outlier. A1

Calculating the lower fence of the data set gives:

$$\begin{aligned} \text{lower fence} &= Q1 - 1.5 \times \text{IQR} \\ &= 6.15 - (1.5 \times 1.5) \\ &= 3.9 \text{ seconds} \end{aligned}$$

Since 2.1 is less than the lower fence, 2.1 is the outlier. A1

*Note: Consequential on answer to Question 2b.*

**Question 3** (5 marks)

a. The explanatory variable is the maximum temperature and the response variable is the number of ice creams sold. A1

b. ice creams sold =  $20.14 + 2.75 \times$  maximum temperature A1

*Note: Consequential on answer to Question 3a.*

c. Reading from the scatterplot, the direction of the association is positive as the data has an increasing trend. A1

The association is strong as all of the data points are relatively close to each other. A1

d. Substituting the maximum temperature of 32 into the equation found in **part b.** gives:

$$\begin{aligned} \text{ice creams sold} &= 20.14 + 2.75 \times 32 \\ &= 108.14 \end{aligned}$$

Therefore, Ling does not have enough stock for the day. A1

*Note: Consequential on answer to Question 3b.*

**AREA OF STUDY 2**

- Algebra, number and structure
- Discrete mathematics

**Question 4** (6 marks)

- a. i.  $B_0 = 1500, B_{n+1} = B_n + 500$  A1
- ii. After two years (24 months),  $B_{24} = 13\,500$ . A1

- b. Using a CAS calculator gives:

Finance Solver	
N:	60.
I(%):	8.
PV:	23000.
Pmt:	-466.35706863351
FV:	0.
PpY:	12

The monthly repayment is \$466.36.

A1

- c. Calculating the monthly interest rate gives:

$$\begin{aligned} \text{monthly interest rate} &= \frac{8}{12} \\ &= 0.67\% \end{aligned}$$

A1

$$V_0 = 23\,000, V_{n+1} = 1.0067V_n - 466.36$$

A1

- d. interest =  $466.36 \times 60 - 23\,000$   
 $= 4981.6$

A1

*Note: Consequential on answer to Question 4b.*

**Question 5** (5 marks)

- a. expected value =  $46\,000 - 1000 \times 0.7$   
 $= 45\,300$

A1

- b. expected value =  $46\,000 \times \left(1 - \frac{2.1}{100}\right)^6$   
 $= \$40\,499.90$

A1

- c. Using method 1 to calculate the value of the car after 20 months gives:

$$\begin{aligned} \text{expected value} &= 0.7 \times 1000 \times 20 \\ &= \$14\,000 \end{aligned}$$

A1

Using method 2 to calculate the value of the car after 20 months gives:

$$\begin{aligned} \text{expected value} &= 46\,000 - 46\,000 \times \left(1 - \frac{2.1}{100}\right)^{20} \\ &= \$15\,910.73 \end{aligned}$$

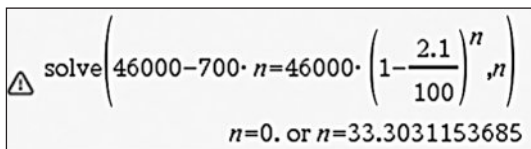
A1

Therefore, the car will have a greater value if method 2 is used.

- d. Equating both methods to find the number of months gives:

$$46\,000 - 700n = 46\,000 \times \left(1 - \frac{2.1}{100}\right)^n$$

Using a CAS calculator gives:



$\text{solve}\left(46000 - 700 \cdot n = 46000 \cdot \left(1 - \frac{2.1}{100}\right)^n, n\right)$   
 $n = 0. \text{ or } n = 33.3031153685$

Therefore, after 33 months the car will have the same value using both methods.

A1

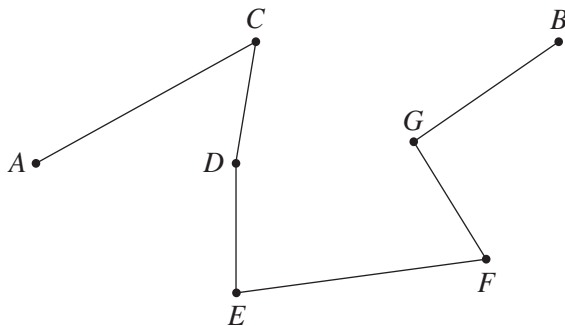
**Question 6** (4 marks)

- a. The shortest path is  $A-C-G-B$ .

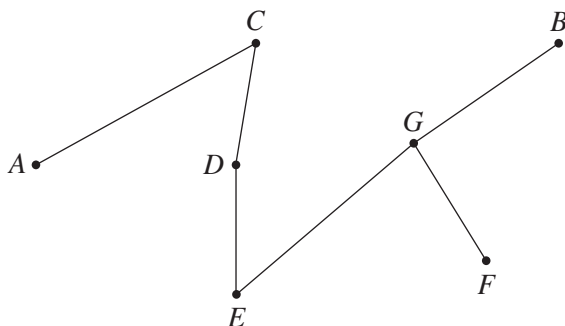
Thus, the shortest distance is  $70 + 40 + 70 = 180$  km.

A1

- b.



OR



A1

- c. To start and end at different vertices and visit every vertex exactly once, a Hamiltonian path is required.

Therefore, the shortest Hamiltonian path is  $A-C-D-E-F-G-B$ .

A1

The length of the path is  $70 + 10 + 20 + 20 + 10 + 70 = 200$  km.

A1

**AREA OF STUDY 3**

- Functions, relations and graphs

**Question 7** (4 marks)

a.  $F = 4.20 + (1.34 \times d) + 1.50 + (0.43 \times 0)$   
 $= 5.70 + 1.34d$

A1

b.  $F = 5.20 + (1.75 \times 15) + 1.50 + (0.49 \times 5)$   
 $= \$35.40$

A1

c. Calculating the fare for route 1 gives:  
 $F = 4.20 + (1.34 \times 30) + 1.5 + (0.43 \times 10)$   
 $= \$50.20$

Calculating the fare for route 2 gives:  
 $F = 4.20 + (1.34 \times 24) + 1.50 + (0.43 \times 30)$   
 $= \$50.76$

*both fares* A1

Since  $\$50.20 < \$50.76$ , route 1 will result in a lower fare.

A1

**Question 8** (7 marks)

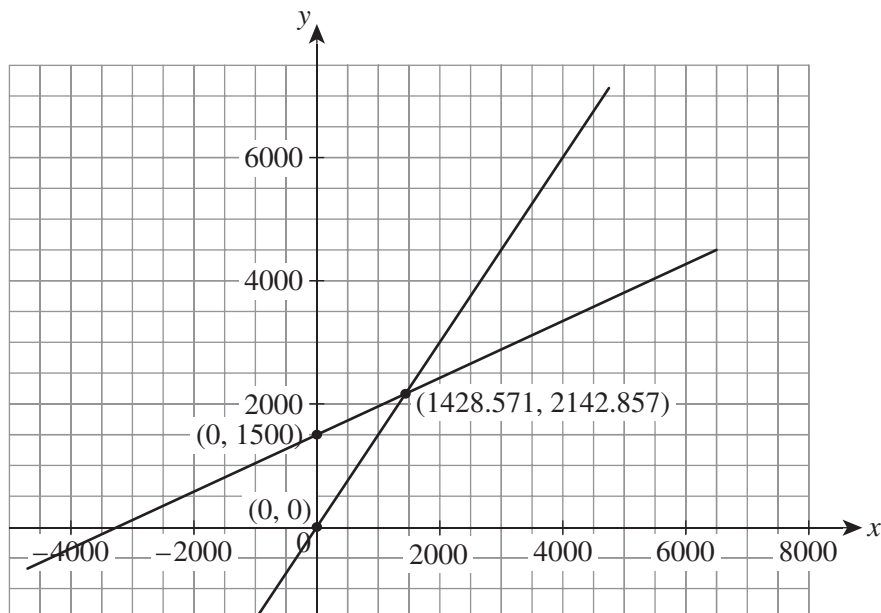
a.  $C = 1500 + 0.45x$

A1

b.  $R = 1.5x$

A1

c.



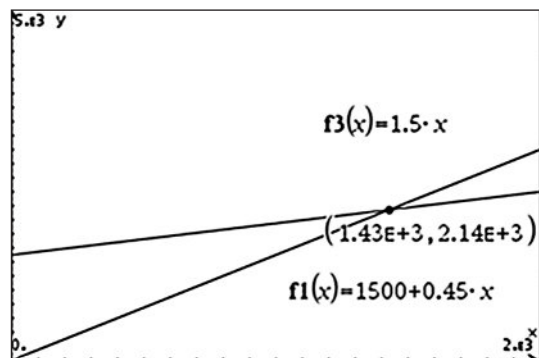
*correct graph line and y-intercept for C* A1

*correct graph line and y-intercept for R* A1

*correct point of intersection* A1

- d. The point of intersection between the graphs of  $C$  and  $R$  is the point where Rio will break even; that is, where he will start making a profit.

Hence, using a CAS calculator to find the point of intersection gives:



OR

Using a CAS calculator to solve the equation  $C = R$  gives:

```
solve(1500+0.45*x=1.5*x,x)
x=1428.57142857
```

*shows understanding of the point of intersection* A1

Thus, Rio needs to sell 1428.57 doughnuts to break even. However, as the number of doughnuts is discrete, Rio needs to sell at least 1429 doughnuts to make a profit.

A1

### Question 9 (4 marks)

- a. As the variables are inversely related:

$$P = \frac{k}{V}$$

Substituting  $(0.8, 120)$  into the equation to find the value of  $k$  gives:

$$120 = \frac{k}{0.8}$$

$$k = 96$$

$$\text{Therefore, } P = \frac{96}{V}.$$

A1

- b. Substituting  $V = 1.5 \text{ m}^3$  into the equation gives:

$$P = \frac{96}{1.5}$$

$$= 64 \text{ kPa}$$

A1

*Note: Consequential on answer to Question 9a.*

- c. The balloon will explode when  $P > 144 \text{ kPa}$ .

Therefore:

$$P \leq 144$$

$$\frac{96}{V} \leq 144$$

$$V \geq \frac{2}{3}$$

M1

Therefore, the maximum volume of the balloon is  $\frac{2}{3} \text{ m}^3$ .

A1

**AREA OF STUDY 4**

- Discrete mathematics
- Space and measurement

**Question 10** (5 marks)

- a. Oscar is still in contact with two people, Leanne and Natalia. A1

b.

$$C^2 = \begin{array}{c} \text{To} \\ \begin{array}{c} L \\ M \\ N \\ O \\ P \end{array} \end{array} \begin{array}{c} \text{From} \\ L \ M \ N \ O \ P \\ \left[ \begin{array}{ccccc} 2 & 0 & 2 & 0 & 2 \\ 1 & 3 & 1 & 2 & 0 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 2 & 0 \\ 2 & 1 & 1 & 1 & 1 \end{array} \right] \end{array}$$

A1

- c. From the two-step communication matrix, Natalia can ask two people to pass on a message to Leanne. A1

From the first-step communication matrix, Natalia can ask Marco or Oscar to pass on a message to Leanne. A1

d.

$$C + C^2 = \begin{array}{c} \text{To} \\ \begin{array}{c} L \\ M \\ N \\ O \\ P \end{array} \end{array} \begin{array}{c} \text{From} \\ L \ M \ N \ O \ P \\ \left[ \begin{array}{ccccc} 2 & 1 & 2 & 1 & 2 \\ 2 & 3 & 2 & 2 & 1 \\ 3 & 2 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 & 1 \end{array} \right] \end{array}$$

$$\begin{array}{c} L \ M \ N \ O \ P \\ [11 \ 11 \ 10 \ 8 \ 7] \end{array}$$

Therefore, Leanne and Marco have the most contacts. A1



**Question 11** (6 marks)

a.  $p = b - a$

A1

b. When Sera sells 15 shirts per day for 30 days:

$$15 \times 30 \times (b - a) = 22\,500$$

Using a CAS calculator gives:

$$\text{expand}(15 \cdot 30 \cdot (b-a)=22500)$$

$$450 \cdot b - 450 \cdot a = 22500.$$

$$450b - 450a = 22\,500$$

A1

When Sera sells 25 shirts per day at a 20% discount for 30 days:

$$25 \times 30 \times \left[ \left( 1 - \frac{20}{100} \right) \times b - a \right] = 22\,500$$

Using a CAS calculator gives:

$$\text{expand}\left(25 \cdot 30 \cdot \left( \left( 1 - \frac{20}{100} \right) \cdot b - a \right) = 22500\right)$$

$$600 \cdot b - 750 \cdot a = 22500.$$

$$600b - 750a = 22\,500$$

A1

c.  $\begin{bmatrix} 450 & -450 \\ 600 & -750 \end{bmatrix} \times \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 22\,500 \\ 22\,500 \end{bmatrix}$

correct  $2 \times 2$  matrix A1

correct  $2 \times 1$  matrix A1

Note: Consequential on answer to **Question 11b**.

d. Using a CAS calculator gives:

$$\text{solve}\left(\left\{ \begin{array}{l} 450 \cdot b - 450 \cdot a = 22500 \\ 600 \cdot b - 750 \cdot a = 22500 \end{array} \right\}, \{a, b\}\right)$$

$$a=50. \text{ and } b=100.$$

$$a = 50 \text{ and } b = 100$$

A1

Note: Consequential on answer to **Question 11b**.

**Question 12** (4 marks)

- a. Given that the volume of water is given by  $V = b \times h$ :

$$\begin{aligned}V_A : V_B &= (b_A \times h_A) : (b_B \times h_B) \\ &= (5 \times 6) : (3 \times 4) \\ &= 30 : 12 \\ &= 5 : 2\end{aligned}$$

A1

- b. The water in tank B is initially 2 m lower than the water in tank A. Therefore, when the water in tank A increases by  $x$  m, the water in tank B must increase by  $x + 2$  m so that they are the same height.

M1

As the same amount of water is poured into both tanks, the expressions can be multiplied by the ratio and equated. Therefore:

$$5x = 3(x + 2)$$

M1

Using a CAS calculator to find the value of  $x$  gives:

$\text{solve}(5 \cdot x = 3 \cdot (x + 2), x)$	$x = 3.$
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A1