VCE General Mathematics Units 1&2

OUTCOMES 1, 2 & 3

Number Patterns and Recursion Test

Name: _____

Circle teacher's name:　　　Ms Jabeen　　　Mr Rossignolo　　　Ms Le　　　Ms Yang

Total _____/ 60

Satisfactory Completion? S/N:　_____

Assessment Criteria

Students should be able to:

- Define and explain key concepts and apply a range of related mathematical routines and procedures.
- Apply mathematical processes in non-routine contexts including situations requiring problem-solving, modelling or investigative techniques or approaches and analyse and discuss these applications of mathematics.
- Use numerical, graphical and symbolic functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Instructions

A single bound reference and a CAS and scientific calculator permitted.

Answer all questions in the spaces provided.

Round final solutions to 2 decimal places unless specified otherwise.

In questions where more than one mark is available, appropriate working must be shown.

Multiple choice questions are worth one mark each.

Section A	Multiple Choice Questions	15 marks

Circle the letter corresponding to the correct response.

1. The twelfth term, t_{12}, in the sequence 56, 48, 40, 32, 24, ... is:

A. 144

B. 152

C. -32

D. -24

E. -40

2. Use the recurrence relation $t_0 = -5, t_{n+1} = t_n + 7$, then t_7 is:

A. 30

B. 33

C. 37

D. 44

E. 47

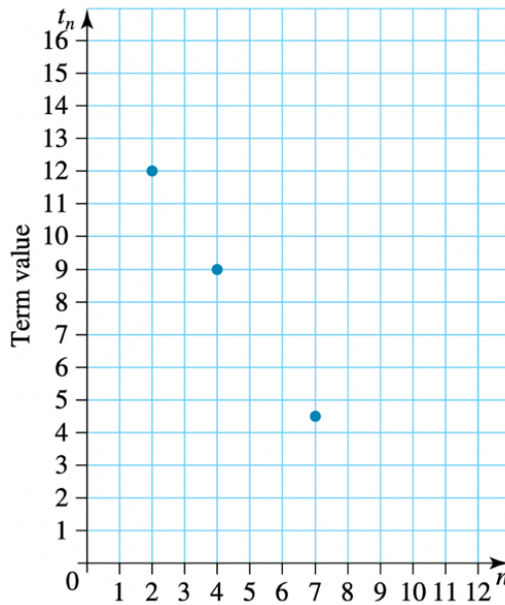**3.** The common ratio, r, of the sequence 128, 32, 8, 2,... is:

A. 4

B. $\dfrac{1}{4}$

C. 0.75

D. $\dfrac{1}{2}$

E. 2

4. The recurrence relation for the sequence 100, 10, 1, ... is:

A. $t_1 = 100, t_{n+1} = t_n + 0.1$
B. $t_1 = 100, t_{n+1} = t_n + 100$
C. $t_1 = 100, t_{n+1} = 0.1t_n$
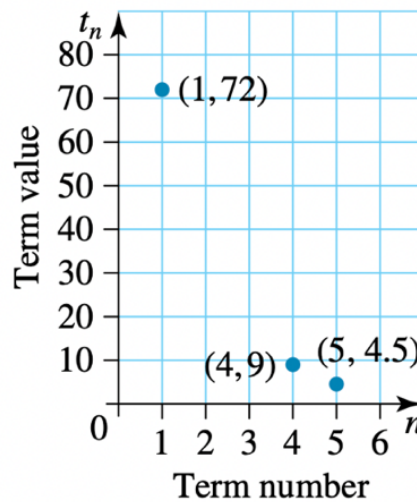D. $t_0 = 100, t_{n+1} = 0.1t_n$
E. $t_0 = 100, t_{n+1} = 0.01t_n$

5. The graph shows some points of an arithmetic sequence. What is the common difference between consecutive terms?

A. 1.5
B. -1.5
C. 3
D. -3
E. 4

6. Find the value of the 2nd term of the geometric sequence shown in the following graph:

A. 144
B. 90
C. 57.6
D. 36
E. 0.5

7. A 2.5% decrease is made by using a common ratio of:

A. 0.025

B. 0.975

C. 1.975

D. 1.25

E. 1.025

8. $25000 is invested for 5 years in an account that pays 6.36% p.a. simple interest. Calculate interest earned each year:

A. $15900

B. $1590

C. $159

D. $15.9

E. $7950

9. A gas bill including GST is $394.02. The price before GST was:

A. $354.62

B. $39.40

C. $433.42

D. $360.2

E. $358.2

10. What is the annual simple interest rate per annum for a deposit of $3000 that earns interest of $600 over a period of one and a half years?

A. 13.33%

B. 6.67%

C. 9.89%

D. 20.59%

E. 5%

11. $8000 is invested at a rate of 7.8% per annum compounded quarterly. The value of this investment after 5 years is closest to:

A. $8811

B. $11728

C. $11772

D. $11801

E. $11646

12. Henry has $1000 to invest in a bank paying compound interest monthly and the investment becomes $1450 within 3 years, what interest rate is the account paying?

A. 30%

B. 3%

C. 12%

D. 1.2%

E. 0.12%

13. The closing price of a share on Wednesday was $160. The closing price of the same share on Thursday was 3% less than its closing price on Wednesday. The closing price of the same share on Friday was 4.5% more than its closing price on Thursday. The closing price of the share on Friday is closest to

A. $157.38

B. $161.98

C. $162.18

D. $162.40

E. $172.22

14. Eva starts a new job with a salary of $73,000 per year and the promise of a 3.5% pay raise for each subsequent year in the job. The equation to determine Eva's salary in her nth year of the job is:

A. $V_n = 73{,}000 + 0.035n$

B. $V_n = 73{,}000 + 1.035n$

C. $V_n = 73{,}000 \times 0.035^n$

D. $V_n = 73{,}000 \times 0.965^n$

E. $V_n = 73{,}000 \times 1.035^n$

15. Manu invests $3000 in an account that pays interest compounding annually.

The balance of his investment after n years, B_n, can be determined using the recurrence relation.

$$B_0 = 3\,000, \qquad B_{n+1} = 1.0048 \times B_n$$

Determine the interest rate of this investment:

A.   1.0048
B.   48%
C.   4.8%
D.   0.48%
E.   0.048%

Section B　　　　　　　　　Short Answer Questions　　　　　　　　15 marks

Include working throughout.

1. Find the term in brackets for each of the following sequences.

(a) $15, 22, 29, 36, \ldots (t_8)$　　　　　　　　**(b)** $1000, 200, 40, 8 \ldots (t_5)$

2 marks　　　　　　　　　　　　　　　　　　　　2 marks

2. State the term name for the term that has a value of 51 for the recurrence relation.

$$t_0 = 15, \quad t_{n+1} = t_n + 4$$

1 mark

3. Find the starting value of the arithmetic sequence in which $t_7 = 90$ and $d = 15$.

2 marks

4. Find the 50th term of the geometric sequence $50, 47.5, 45.125, \ldots$ Give your answer correct to 2 decimal places.

2 marks

5. Amie invests $4000 in a simple interest investment. She will be paid interest at the rate of 5.5% per year. Find the value of the investment after 10 years.

2 marks

6. A photocopier costs $6800 when new. Its value depreciates at a flat rate of 12.5% per year. Find its value after 4 years.

2 marks

7. A hairdryer in a salon was purchased for $755. The value of the hairdryer depreciates by 20 cents for every hour it is used. What is the value of the hairdryer after 50 hours of use?

2 marks

Include working throughout.

Question 1 (8 marks)

A car is valued at $35 400 at the start of the year, and at $25 700 at the end of that year. During that year, the car travelled 25 000 kilometres.

(a) Find the total depreciation of the car in that year in dollars.

2 marks

(b) Find the depreciation per kilometre for this car.

2 marks

(c) Calculate the value of the car after it has travelled 51 000 km.

2 marks

(d) How many kilometres have been travelled if the car has a value of $6688?

2 marks

Question 2 (10 marks)

Sophie started working at A.J. Smith's in 2006. Her annual salary, A_n increased by $1200 each year until in 2010 it was $84800.

(a) What was her annual salary, A_0, for the year 2006?

_____

_____

<div align="right">2 marks</div>

(b) Write down a recurrence relation, which will model Sophie's salary, A_n, after n years.

_____

_____

<div align="right">2 marks</div>

(c) What salary will Sophie receive in 2016?

_____

_____

_____

<div align="right">2 marks</div>

(d) After working for the firm for 10 years, Sophie receives a 5% increase in salary for the next five years.

   i.   Write down a recurrence relation to model the value of her new salary structure, V_{n+1}

_____

_____

_____

<div align="right">2 marks</div>

   ii.   What is her salary during the fifth year of this new salary structure? Give your answer to the nearest cent.

_____

_____

_____

<div align="right">2 marks</div>

Question 3 (12 marks)

Emma plans to start her business which is a delivery company. She wants to buy a new van for $85600. After she has done some research on the van. She found there were two different models of depreciation to choose from. Once chosen she must stick with that model for the life of the van.

- Model A is straight line depreciation at a rate of 5% of the Initial value of the van per year.
- Model B uses the reducing balance method of 6.1% of the previous year's book value each year.

(a) The depreciation schedules for both models over the first five years are below with some entries left blank. Fill in the blank entries.

| Year | MODEL A | | MODEL B | |
	Depreciation ($)	End Year Book Value ($)	Depreciation ($)	End Year Book Value ($)
0	-	85600.00	-	85600.00
1	4280.00	81320.00	5221.60	80378.40
2	4280.00		4903.08	
3	4280.00	72760.00	4604.00	70871.32
4	4280.00			

4 marks

(b) Write below the recurrence relation equations that would give the book value for Model A for consecutive years.

_____

_____

_____

2 marks

Question is continued on the following page.

The nth term formulas for both models are given below

MODEL A	MODEL B
$V_n = 85600 - 4280 \times n$	$V_n = 85600 \times 0.939^n$

(c) Using these formulas or otherwise, draw two graphs to show the bookvalue of both models over the first nine years of its life.

Use an open circle: o for Model A and a star * for Model B

4 marks

It is more beneficial for Emma to claim a greater amount of depreciation.
This means she needs to choose the model that will have the lowest book value when she sells the van.

Use your graph and the previous nth term formulae to answer the following questions

(d) If Emma was going to sell the van after 6 years, which model should she choose?

2 marks

END OF TEST