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Answers

GENERAL MATHS UNITS 3 & 4 TRIAL EXAMINATION 1 SOLUTIONS

2024

Data analysis			Recursion and financial modelling		Matrices		Networks and decision mathematics		
1.	D	9.	А	17.	С	25.	С	33.	D
2.	В	10.	С	18.	В	26.	D	34.	А
3.	С	11.	А	19.	D	27.	В	35.	А
4.	В	12.	С	20.	С	28.	В	36.	D
5.	D	13.	С	21.	В	29.	А	37.	С
6.	D	14.	В	22.	А	30.	В	38.	С
7.	D	15.	А	23.	С	31.	D	39.	D
8.	С	16.	А	24.	С	32.	А	40.	А

Solutions

Data analysis

Question 1

range = 196 - 115= 81The answer is D.

Question 2

The distribution is negatively skewed.

To check whether the data value 115 might be an outlier, we need to find the value of the lower fence.

<u>Method 1</u> – finding the lower fence using a calculator Enter the 34 data values into your calculator. Calculate One – Variable statistics for this data set.

lower fence $= Q_1 - 1.5 \times IQR$ (formula sheet)

$$= 160 - 1.5 \times (Q_3 - Q_1)$$
$$= 160 - 1.5 \times (186 - 160)$$
$$= 121$$

Since 115 < 121 then 115 is an outlier. The shape is best described as negatively skewed with one or more possible outliers. The answer is B.

The answer is D.

<u>Method 2</u> – finding the lower fence by hand

The median data point is at location $\frac{n+1}{2} = \frac{34+1}{2} = 17.5$, that is, the median is between the 17th and 18th data values (counted in from either end). The median is 175. The first quartile (Q₁) is the 9th data value in the lower 17 data values and is 160. The third quartile (Q₃) is the 9th data value in the upper 17 data values and is 186. IQR = Q₃ - Q₁ = 186 - 160 = 26 lower fence = Q₁ - 1.5 × IQR (formula sheet) = 160 - 1.5 × 26

Since 115 < 121 then 115 is an outlier. The shape is best described as negatively skewed with one or more possible outliers.

The answer is B.

Question 3

The variable *latest spend* is a numerical variable not a categorical variable and therefore cannot be a nominal variable.

The variable *age*, in this context, is a categorical variable because it organizes age into three categories. It is an ordinal variable because there is an order to the categories.

The variable *postcode* is a **categorical variable** because it does not measure or count anything but rather it places various localities into particular categories.

It is also a **nominal variable** because the four digit number names a postal locality.

It cannot be an ordinal variable because the four digit number doesn't place the localities in any order, it simply names them.

The variable *repeat shopper* is a **categorical variable**. It is also a **nominal variable** because there is no order to the categories of yes or no.

The variable *shopping type* is a **categorical variable**. It is also a **nominal variable** because there is no order to the categories of delivery or click and collect.

In total there are three nominal variables.

The answer is C.

Question 4

$$z = \frac{x - \bar{x}}{s_x} \qquad (\text{ formula sheet})$$
$$-1.5 = \frac{x - 8.2}{0.9}$$

Solve this equation for *x*. x = 6.85

The answer is B.

5.5 kg is three standard deviations below the mean of 8.2 kg., i.e. $8.2 - 3 \times 0.9 = 5.5$.

50% - 0.15% =49.85% 49.85% of 268=133.598

The closest answer is 134. The answer is D.



Question 6

The modal class interval is 4.5 - 5.0 (i.e. the highest column on the histogram).

$\log_{10}(price) = 4.5$	$\log_{10}(price) = 5$
<i>price</i> $= 10^{4.5}$	price $=10^5$
=31 622.78	=100 000

We want those horse prices between \$31 622.78 and \$100 000. They include \$47 300, \$75 800, \$82 700 and \$95 000. There are four horse prices included in the modal interval. The answer is D.

Question 7

Enter the data into your CAS and calculate the linear regression line equation. Note that when you are only interested in calculating the value of r (as we are in this question), it doesn't matter which of the two variables you use as the explanatory (x) variable or the response (y) variable. It is about the only time when this is the case! When you are calculating a least squares regression line equation you must be very clear about which is the explanatory variable and which is the response variable.

The value of r will be given with the other information relating to the least squares line equation. r=0.96512...

The closest answer is 0.965. The answer is D.

Question 8

As *height* increases, *wrist circumference* tends to decrease so the value of r will be negative. Option A is true. Since *wrist circumference* is predicted by *height*, then *height* is the explanatory variable. Option B is true. The association is negative and linear, but since $r^2 = 0.3889$, then $r = -\sqrt{0.3889} = -0.62361...$ Since $-0.75 < -0.62361... \le -0.5$, then the strength of the association is moderate not strong. Option C is **not** true. Note that since $r^2 = 0.3889$, then option D is true. The answer is C.

Choose any two points on the least squares line, for example the two endpoints (90, 14.27) and (190, 13.77).

$$gradient = \frac{13.77 - 14.27}{190 - 90}$$
$$= -0.005$$

Using $y - y_1 = m(x - x_1)$ where $(x_1, y_1) = (90, 14.27)$ y - 14.27 = -0.005(x - 90)y = -0.005x + 0.45 + 14.27

$$y = 14.72 - 0.005x$$

Since *wrist circumference* can be predicted from *height*, then *wrist circumference* is the response variable.

So the least squares line equation is *wrist circumference* = $14.72 - 0.005 \times height$. Note that the point (90, 14.27) that lies on the graph showing the least squares line, is **not** the *y*-intercept of the graph. The *y*-intercept of the graph occurs at the point where *height* equals zero.

The answer is A.

Question 10

Looking at the scatterplot, there are six data values that lie within 0.1 cm (i.e. one vertical grid marking) above or below the least squares line. They are circled in the diagram below.



The answer is C.

Method 1 - using CAS

Solve 774.85 = $-2247.6 + 1913.9 \times \log_{10}(age)$ for age.

age=37.9498...

The closest answer is 37.9. The answer is A.

<u>Method 2</u> – by hand

 $774.85 = -2247.6 + 1913.9 \times \log_{10}(age)$

$$\frac{774.85 + 2247.6}{1913.9} = \log_{10}(age)$$
$$1.5792... = \log_{10}(age)$$
$$age = 10^{1.5792...}$$
$$= 37.9498...$$

The closest answer is 37.9. The answer is A.

Question 12

The summary statistics can be used to find the intercept and the slope of the least squares line equation which is of the form y = a + bx.

Note that the explanatory variable (x) is *medication* and the response variable (y) is *pain relief*. It is important to get these correct!

So our least squares line equation will be of the form *pain relief* $= a + b \times medication$. The slope *b*, tells us by how much the duration of the *pain relief* was increased (or decreased if *b* is negative) for every extra milligram of *medication* taken.

For y=a+bx, $b=r\frac{s_y}{s_x}$ (formula sheet) =0.974× $\frac{1.64}{2.83}$ =0.5644...

Since b is positive, we have an increase of 0.56 hours. The answer is C.

Question 13

The time series plot shows a decreasing trend. It does not show seasonality because there is no periodic movement, that is, there is no pattern that repeats itself each year or each two years or over any set time period. There are irregular fluctuations. The answer is C.

Look at the energy consumption for quarter number 12 and for the two quarters to either side of it.

The middle energy consumption value for these five data values is closest to 3200 kWh, i.e. this is the energy consumption for quarter number 11. The two values above this value occur for quarter numbers 12 and 14 and the two values below this occur for quarter numbers 10 and 13.

The answer is B.

Question 15

From the time series plot, we see that the actual energy consumption for quarter number 5 is 5000 kWh.

seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$ (formula sheet) $1.12 = \frac{5000}{\text{deseasonalised figure}}$ deseasonalised figure = $\frac{5000}{1.12}$ = 4464.28...

The closest value is 4464. The answer is A.

Question 16

The seasonal index for Saturday is 7 - (1.28 + 1.12 + 1.01 + 1.05 + 0.93 + 0.79) = 0.82

deseasonalised number = $439.57 - 11.392 \times 6$

seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

=371.218

(formula sheet)

$$0.82 = \frac{\text{actual figure}}{371.218}$$

actual figure =
$$0.82 \times 371.218$$

The closest answer is 304.

The answer is A.

Recursion and financial modelling

Question 17

Use trial and error.

For option A, the sequence generated is 10, 18, 26, ... Reject option A. For option B, the sequence generated is 10, 22, ... Reject option B. For option C, the sequence generated is 10, 18, 42, 114, 330, ... This is correct. If you have time, check option D. For option D, the sequence generated is 10, 18, 50, ... Reject option D. The answer is C.

Question 18

Method 1

The purchase price of the van is \$50 000 and it will be sold when it has depreciated by \$50 000 - \$35 000 = \$15 000. The van is depreciated by \$0.80 per kilometre travelled. Since $15\ 000 \div 0.80 = 18\ 750$, the van will have travelled 18 750 km by the time it is sold. The answer is B.

Method 2

The recurrence relation can be expressed as the rule $V_n = 50\ 000 - 0.8n$. When the value of the van is \$35 000, we have $35\ 000 = 50\ 000 - 0.8n$ Solving for *n* gives $n = 18\ 750$. The van will have travelled 18 750 km by the time it is sold. The answer is B.

Question 19

Method 1

Using CAS, generate the sequence represented by the recurrence relation. 15 000, 15 237.50, 15 475.59, 15 714.28, 15 953.57, 16 193.45, 16 433.94 Note that $D_0 = 15\ 000$ and $D_6 = 16\ 433.94$, so after six months the value of this investment is \$16\ 433.94 The answer is D.

<u>Method 2</u> Interest per annum is $0.25 \times 12 = 3\%$. Using finance solver:

```
N: 6
I(%): 3
PV: - 15 000
Pmt: - 200
FV: ?
Ppy: 12
Cpy: 12
```

FV:16 433.9359... The closest answer is \$16 433.94. The answer is D.

Method 1 – using CAS

The effective rate of interest can be found using the menus 8 Finance, 5 Interest Conversion, 2 Effective Interest rate.

Note that the interest rate is 1.6% per quarter or $4 \times 1.6\% = 6.4\%$ per annum. Alternatively, $1.016 = 1 + \frac{r}{100 \times 4}$ where *r* is the interest rate per annum

$$r = 6.4$$

eff(nominal interest rate per annum, number of times the interest compounds per year) i.e. eff(6.4, 4) which gives 6.55524... The closest answer is 6.56%. The answer is C.

Method 2 - Using the formula

Use the effective rate of interest formula from the formula sheet. Note that the interest rate per quarter is 1.6% or $4 \times 1.6\% = 6.4\%$ per annum.

$$r_{eff\ ective} = \left[\left(1 + \frac{6.4}{100 \times 4} \right)^4 - 1 \right] \times 100\%$$

= 6.55524...%

The closest answer is 6.56%. The answer is C.

Question 21 Method 1

For a perpetuity investment, the interest earned per period is equal to the amount paid to the investor per period, therefore the balance of the perpetuity remains constant. For option A, $P_1 = 0.09 \times 65\ 000 - 5850 = 0$. Reject option A. For option B, $P_1 = 1.09 \times 65\ 000 - 5850 = 65\ 000$. The answer is B.

If you have time, check the other options just to be sure. For option C, $P_1 = 1.001 \times 65\ 000 - 5850 = 59\ 215$. Reject option C. For option D, $P_1 = 1.01 \times 65\ 000 - 6500 = 59\ 150$. Reject option D. The answer is B.

Method 2

For a perpetuity investment, the interest earned per period is equal to the amount paid to the

investor per period. The period in this question is 1 year. So $5850=65\ 000 \times \frac{r}{100}$ Solving for *r* gives r = 9%. In the recurrence relation $P_0 = 65\ 000$, $P_{n+1} = RP_n - 5850$, $R = 1 + \frac{r}{100 \times p}, \text{ where } p \text{ is the number of compounding periods per year i.e. 1}$ $= 1 + \frac{9}{100 \times 1}$ =1.09The answer is B.

Using finance solver:

N: 120 I(%):? PV: - 320 000 Pmt: 3750 FV: 0 Ppy: 12 Cpy: 12 I(%):7.2088...

The interest rate per annum is 7.2088...%

In the recurrence relation $B_0 = 320\ 000$, $B_{n+1} = RB_n - 3750$

The interest on the account is calculated monthly, so

$$R = 1 + \frac{7.2088...}{100 \times 12}$$
$$= 1.0060...$$
The closest answer is 1.006
The answer is A.

Question 23

Interest rate per month (using payment 178) is $\frac{81.13}{15702.97} \times 100\% = 0.5166...\%$. Alternatively, the interest rate per month (using payment 179) is $\frac{48.48}{9384.10} \times 100\% = 0.5166...\%$.

Interest for payment 180 is $3032.58 \times 0.5166...\% = 15.67$ to the nearest cent. The last payment must include payment of the balance owing plus the interest on that balance for the month i.e. 3032.58 + 15.67 = 3048.25The answer is C.

The total of the monthly payments over the three years that Elisse would have paid without the \$5000 gift is $1516.57 \times 36 = 54596.52$.

To begin finding the total of the monthly payments with the \$5000 gift, use finance solver to calculate the amount still owing at the end of the first year.

N: 12 I(%): 5.8 PV: 50 000 Pmt: - 1516.57 FV: ? Ppy: 12 Cpy: 12 FV: - 34 287.841...

With the gift of \$5000, the new amount owing at the end of the first year is $34\ 287.84 - 5000 = \$ 29 287.84.

Use finance solver to find the adjusted monthly payment for the final two years.

Pmt: - 1295.417...

Adjusted monthly payment is \$1295.42.

The total of the monthly payments over the three years that Elisse would have paid with the 5000 gift is $1516.57 \times 12 + 1295.42 \times 24 = 49288.92$

The difference is \$ 54 596.52 - \$ 49 288.92 = \$ 5307.60 The answer is C.

Matrices

Question 25

An identity (or unit) matrix has 1's down its leading diagonal. Reject option A.

A binary matrix contains only 0's and 1's. Reject option B.

The matrix is a diagonal matrix because all the elements that aren't on the leading diagonal are zero.

A permutation matrix cannot contain a 2. Reject option D. The answer is C.

Question 26

Starting with Zoe, part of a possible communication tree is given by



Z and V are linked via the top branches.

The shortest sequence of possible communications links that enable Zoe to communicate with Val is Z - W - X - V.

The answer is D.

Question 27

 $(M \times O^T)^T \times N$

Matrix *O* is a 2×3 matrix, so matrix O^T is a 3×2 matrix.

 $M \times O^T$ $1 \times 3 \times 3 \times 2$

gives a 1×2 matrix

So
$$(M \times O^T)^T$$
 is a 2×1 matrix.

So $(M \times O^T)^T \times N$

 $2 \times 1 \times 1 \times 2$

gives a 2×2 matrix.

The answer is B.

Method 1

The element '1' which occurs on the leading diagonal of the transition matrix indicates that all (i.e. 100%) of the wallabies at location R this evening, go back there the next evening. Also, 10% of wallabies at location P this evening, go to location R the next evening and 30% of wallabies at location Q this evening go to location R the next evening.

In the long term, all the wallables will end up at location R. Therefore, there will be no wallables at locations P or Q.

Only option B describes this situation.

The answer is B.

Method 2

Since we are looking at the wallaby population in the long term, we can consider the steady

		0.9	0.1	0	100	0	0	0]
state.	For example, $T^{100} =$	0	0.6	0	=	0	0	0
		0.1	0.3	1		1	1	1

So in the long term, all the wallabies will end up at *R*. The answer is B.

Question 29

From the diagram, those fish in age group 0-1 do not produce offspring (i.e. there is no loop at the 0-1 age group vertex). So the element in the first row and first column must be zero. Reject option C.

The birth rates for the other three age groups, i.e., 3.2, 5.6, and 1.4 should appear in the first row. Reject option B.

The survival rates for the three groups shouldn't appear along the leading diagonal because survival rate refers to one age group surviving to the next age group, not remaining in the same age group which the entries along the leading diagonal represent. Reject option D. The answer is A.

Question 30

Since matrix A has a determinant, then it must be a square matrix, i.e. it cannot be a row matrix. The first statement is not true.

The inverse matrix A^{-1} does exist because the determinant of A does not equal zero. The second statement is not true.

Matrix A does not have to be an identity (or unit) matrix. For example, it could be the matrix $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ which has a determinant of $3 \times 2 - 1 \times 5 = 1$.

The third statement is not true.

If matrix A was a 2×2 identity matrix, i.e. if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then det(A) = 1×1-0×0=1.

Also,
$$A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$$
.

The fourth statement is true. So one of the four statements is true. The answer is B.

The two-step dominances for each of the players can be found using the matrix

0	0	0	0	1	2	0	1	1	1	0	E
1	0	0	1	0		1	0	0	0	1	F
1	1	0	1	0	=	2	0	0	1	1	G
1	0	0	0	0		0	0	0	0	1	H
0	1	1	1	0		3	1	0	2	0	Ι

The sum of each row gives the sum of the two-step dominance for each player. Erin's sum is 3, Fred's is 2, Georgie's is 4, Harry's is 1 and Ivo's is 6.

The correct player ranking is given by

Ranking	Player				
1	Ivo				
2	Georgie				
3	Erin				
4	Fred				
5	Harry				

The answer is D.

Question 32

$$S_{1} = \begin{bmatrix} 80 \\ 40 \\ 10 \\ 50 \end{bmatrix} \begin{bmatrix} 0.6 & 0.3 & 0.4 & 0.1 \\ 0.1 & 0.5 & 0.1 & 0.5 \\ 0.2 & 0.1 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} 0.6 \\ B \\ C \\ D \end{bmatrix}$$

Note that the columns of T must add to give 1 because the total number of guests dining each night doesn't change (i.e. these are the only places they can eat dinner on the ship).

A

$$S_{2} = TS_{1}$$

$$S_{3} = T^{2}S_{1}$$

$$\begin{bmatrix} 69\\54 \end{bmatrix} B$$

$$\begin{bmatrix} 72\\50.8 \end{bmatrix} B$$

$$B$$

$$= \begin{bmatrix} 29 \\ 28 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 33.6 \\ 23.6 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

72 guests dined at the Ark Restaurant on the third night.

69 guests dined at the Ark Restaurant on the second night, and from T, 0.6 or 60% of them will dine there again on the third night i.e. 60% of 69 = 41.4

 $So\left(\frac{41.4}{72} \times 100\right)\% = 57.5\%$ of guests who dined at the Ark Restaurant on the third night of

the cruise had dined there on the second night of the cruise. The answer is A.

Networks and decision mathematics

Question 33

For option A, there are existing connections between vertices A, B and C, so option A is a subgraph.

For option B, there are existing connections between vertices A, C and D, so option B is a subgraph.

For option C, there are existing connections between vertices A, B, C, D and E, so option C is a subgraph.

For option D, there exists two edges joining vertices C and E whereas on the original graph, only one of these exists. Option D is not a subgraph. The answer is D.

Question 34

There is a loop connecting vertex H to itself, therefore there should be a 1 in the first row and first column of the adjacency matrix. Reject options B and C.

There are two direct path connections between vertex H and vertex I, therefore there should be a '2' in the first row and second column of the adjacency matrix. Reject option D. The answer is A.

Question 35

The graph is not in planar form. When redrawn in planar form it may appear as the graph shown below. Note there are many alternative ways that it may be redrawn in planar form.



The faces are numbered on the graph above. There are three faces. The answer is A.

Question 36

The minimum spanning tree is indicated by the heavy line (not the dotted line) on the graph below.

Note that you start the minimum spanning tree along the edge with the weight of 3, i.e. along the edge with the lowest weight.

The total weight is 3+4+6+6+6+7+4+6=42. The answer is D.



The minimum cut across the network gives us the maximum flow.

The cuts across the network that **don't** cut through the edge with the *x* on it are shown below, together with their respective capacities.



The minimum cut for these cuts has a capacity of 10 litres per minute. Since we are told that the maximum possible flow is 9 litres per minute, then the minimum cut cannot be one of these and therefore must include the edge with the x on it.

The cuts across the network that **do** include the edge with the *x* on it are shown below.



The minimum cut will pass through the edge with the 5 on it (rather than with the 6 or 7) and therefore x must equal 4 in order to have a minimum cut with a capacity of 9 litres per minute.

The answer is C.

An example of a connected graph with seven vertices and six edges which is bipartite is shown to the right so the first statement is true.

An example of a connected graph with seven vertices and six edges is shown to the right.

A path is a walk with no repeated edges and no repeated vertices on it so there are many paths possible so the second statement is true.

Because the graph is planar, Euler's formula is applicable

7+f=6+2 so f=1. This tells us that the graph will have one face. It is therefore not possible to have a cycle on the graph because that will mean that the graph has two faces as shown in the example to the right. Also, this graph is not connected.

The third statement is not true.



There are many graphs that have seven vertices and six edges and contain two or more bridges. Two examples are shown below. So the fourth statement is true.



Three of the four statements are true. The answer is C.

Draw a directed network.



In drawing this network, it becomes apparent that a dummy activity needs to be drawn from the end of activity B to the start of activity G. The answer is D.

Question 40

Use a forward scan to find the earliest start times (EST) for those activities that are predecessors of activity *K* and for activity *K* itself.



Note that since activity K has a float time, then it is not on the critical path.

The earliest start time for activity K is 11 days.

Since the float time for activity *K* is 5 days, then the latest start time for activity *K* is 11 + 5 = 16 days.

The minimum completion time for the project is 17 days so the duration of activity K is 1 day. The answer is A.