1994

MATHEMATICAL METHODS TRIAL CAT 3

Based on the Victorian Certificate of Education Mathematics Study Design.

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STUDENT NUMBER LETTER

figures									
words									

Victorian Certificate of Education
Mathematics 1994

MATHEMATICAL METHODS
1994 TRIAL CAT 3
Analysis Task

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOKLET

Directions to students

Materials
Question and answer booklet of 10 pages including 2 blank pages for rough working.
There is a detachable sheet of miscellaneous formulas.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
You may use an approved calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from this booklet during reading time.
Ensure that you write your **student number** in the space provide on the cover of this booklet.
Answer **all** questions.
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 60 marks available for the task.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task.
Hand in this question and answer booklet.

Question 1

The position x of a particle at time t is given by the equation $x = 2\cos 2(t - \frac{\pi}{2}) + 1$.

(i) Find the position of the particle when $t = 0$.

(2 marks)

(ii) What is the maximum value of x ?

(1 mark)

(iii) What is the minimum value of x ?

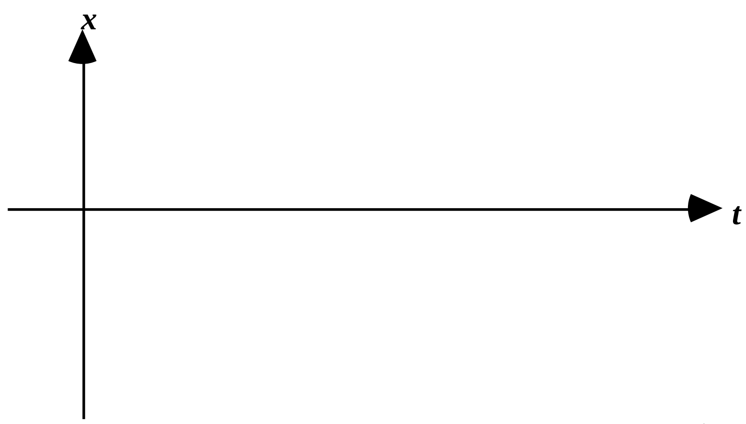
(1 mark)

Question 1 (continued)

(iv) At what time will the particle first reach the position $x = 0$.
Give your answer to one decimal place.

(5 marks)

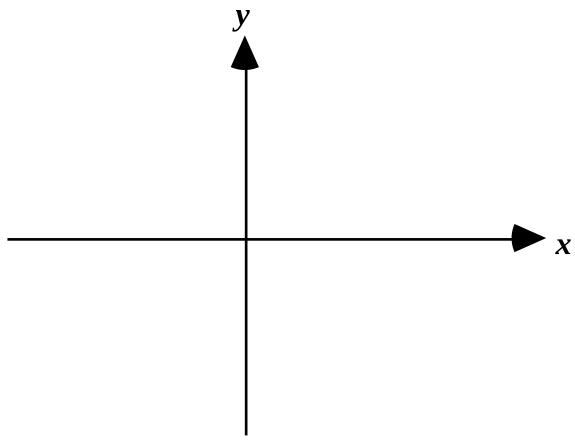
(v) Sketch the graph of x for $0 \le t \le 2\pi$.



(4 marks)

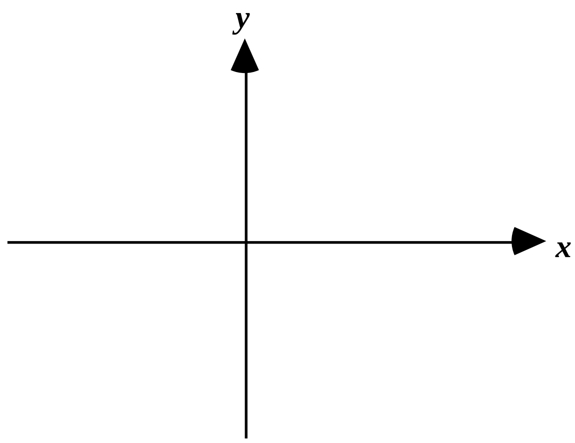
Question 2

(i) Sketch the graph of $y = 4 - x^2$ showing turning points and intercepts with the axes.



(4 marks)

(ii) A rectangle with base on the x axis has its upper vertices on the curve $y = 4 - x^2$
Show this rectangle in a sketch if the base co-ordinates are $(-a,0)$ and $(a,0)$.



(1 mark)

(iii) Show that the area of this rectangle equals $8a - 2a^3$.

(2 marks)

Question 2 (continued)

(iv) For what value of a is the area a maximum?
Give your answer to two decimal places.

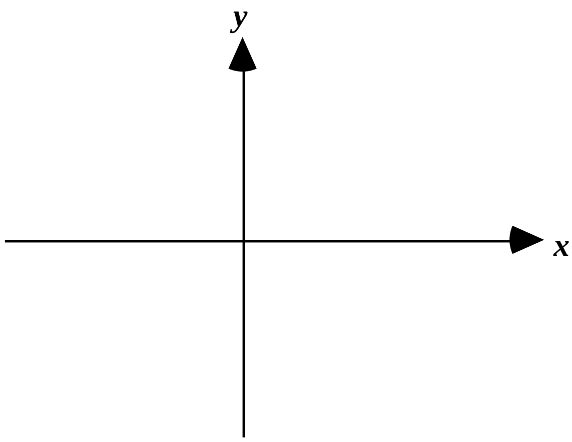
(4 marks)

(v) What is this maximum area? Give your answer to two decimal places.

(1 mark)

Question 3

(i) Sketch the graph of $y = x^3 - 3x^2$, locating the turning points of the graph and giving their co-ordinates.



(5 marks)

(ii) Calculate the area of the region enclosed by the ***x*** axis and the graph.

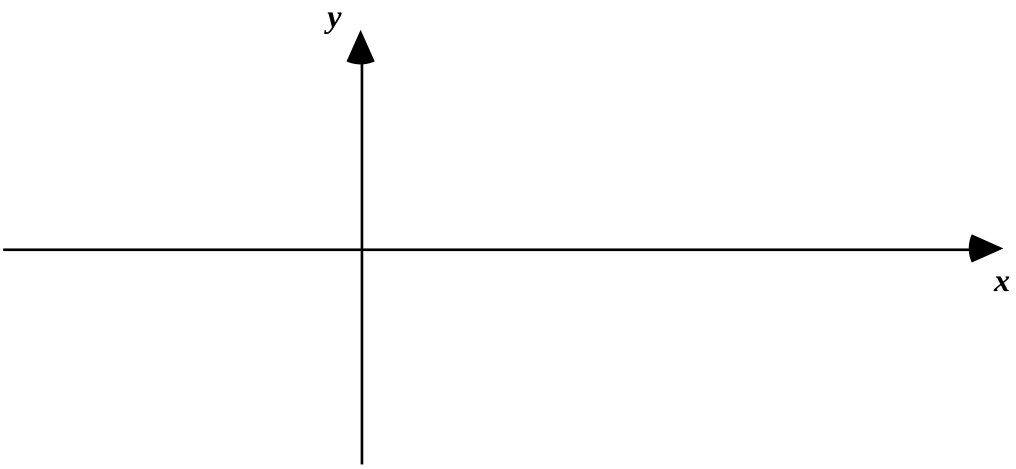
(4 marks)

Question 3 (continued)

(iii) Find the equation of the tangent to the graph at the point where $x = 1$.

(4 marks)

(iv) On a separate set of axes, sketch the graph $y = \dfrac{1}{x^3 - 3x^2}$.



(4 marks)

Question 4

Cumulative Normal Distribution

$$F(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-0.5t^2}\, dt$$

a	$F(a)$	a	$F(a)$	a	$F(a)$
0.0	0.500				
0.1	0.540	1.1	0.864	2.1	0.982
0.2	0.579	1.2	0.885	2.2	0.986
0.3	0.618	1.3	0.903	2.3	0.989
0.4	0.655	1.4	0.919	2.4	0.992
0.5	0.692	1.5	0.933	2.5	0.994
0.6	0.726	1.6	0.945	2.6	0.995
0.7	0.758	1.7	0.955	2.7	0.996
0.8	0.788	1.8	0.964	2.8	0.997
0.9	0.816	1.9	0.971	2.9	0.998
1.0	0.841	2.0	0.977	3.0	0.999

A lathe turns out brass cylinders with a mean diameter of 2.000 cm and standard deviation 0.002 cm. Assuming that the diameters are normally distributed

(i) Find the probability that a randomly selected cylinder will have a diameter less than 1.998 cm.

(5 marks)

Question 4 (continued)

(ii) If specifications require that acceptable cylinders must have a diameter between 1.999 cm and 2.001 cm, find the probability that a randomly selected cylinder is acceptable.

(5 marks)

(iii) The profit on an acceptable cylinder, that is, one whose diameter lies between the specified limits, is \$3.00 while unacceptable cylinders incur a loss of \$1.00. If P dollars is the profit on a randomly selected cylinder produced on the lathe, find the mean and variance of P.

(8 marks)

ROUGH WORKING

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End Of Questions 1994 Mathematical Methods Trial Cat 3

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Question 1

(i)

When $t = 0$, $x = 2\cos 2(-\frac{\pi}{2}) + 1$

$= 2\cos(-\pi) + 1$

$= 2\cos\pi + 1$

$= 2 \times -1 + 1$

$= -2 + 1$

$= -1$ **ANS**

(ii)
The maximum value of x occurs

when $2\cos 2(t - \frac{\pi}{2}) = 2$.

That is, $x = 2 + 1 = 3$ **ANS**

(iii)
The minimum value of x occurs

when $2\cos 2(t - \frac{\pi}{2}) = -2$.

That is, $x = -2 + 1 = -1$ **ANS**

(iv) $2\cos 2(t - \frac{\pi}{2}) + 1 = 0$

$2\cos 2(t - \frac{\pi}{2}) = -1$

$\cos 2(t - \frac{\pi}{2}) = -\frac{1}{2}$

$2(t - \frac{\pi}{2}) = \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$(t - \frac{\pi}{2}) = \frac{\pi}{3}$

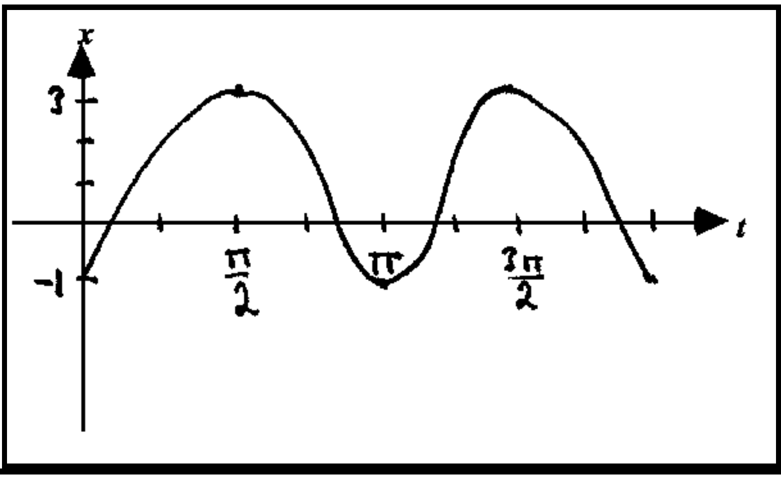
$t = \frac{\pi}{3} + \frac{\pi}{2}$

$= \frac{2\pi + 3\pi}{6}$

$= \frac{5\pi}{6}$

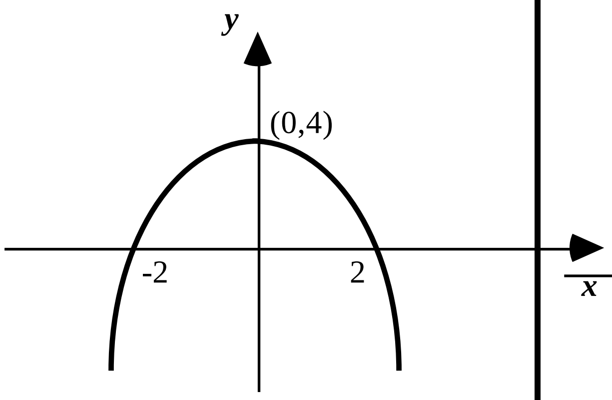
$t = 2.6$ **ANS**

(v)

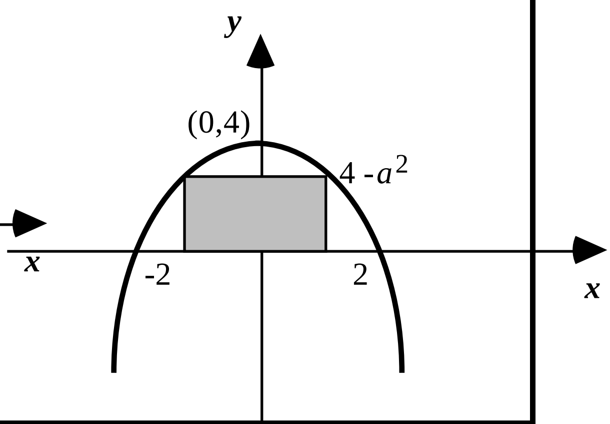


Question 2

(i)



(ii)



(iii) $A = L \times W$

$= 2a(4 - a^2)$

$= 8a - 2a^3$

(iv)

$\frac{dA}{dx} = 8 - 6a^2 = 0$ for a turning point.

Hence, $6a^2 = 8$

$a^2 = \frac{4}{3}$

therefore, $a = \pm \frac{2}{\sqrt{3}}$

$= \pm 1.1547$

When $a > 1.1547$, say 2, $\frac{dA}{dx} = 8 - 24 < 0$

When $a < 1.1547$, say 1, $\frac{dA}{dx} = 8 - 6 > 0$.

Therefore, a maximum occurs when $a = 1.15$ to two decimal places. **ANS**

(v)

$A = 8a - 2a^3$

$= 8 \times \frac{2}{\sqrt{3}} - 2 \times \frac{8}{3\sqrt{3}}$

$= \frac{16}{\sqrt{3}} - \frac{16}{3\sqrt{3}}$

$= \frac{48 - 16}{3\sqrt{3}}$

$= \frac{32}{3\sqrt{3}}$

= 6.16 sq units. **ANS**

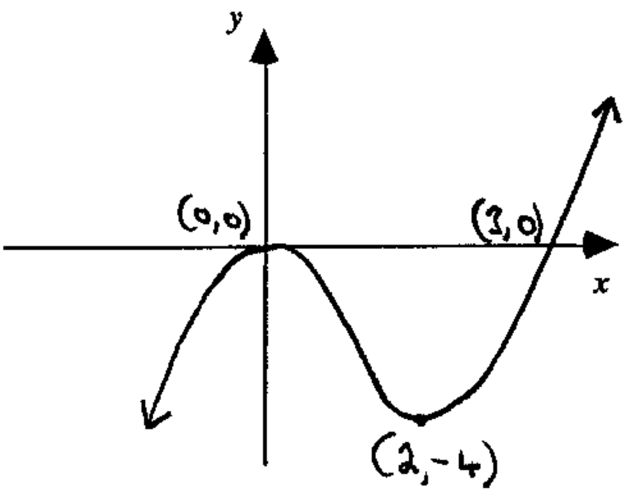
Question 3

(i)

$y = x^2(x - 3)$. The x intercept occurs when $y = 0$.
Hence, $x = 0$ or 3.
The y intercept occurs when $x = 0$. Hence, $y = 0$.

$y = x^3 - 3x^2$

Hence, $\frac{dy}{dx} = 3x^2 - 6x = 0$ for a turning point.
Hence, $3x(x - 2) = 0$.
Therefore, $x = 0$ or 2.
When $x = 0$, $y = 0$.
When $x = 2$, $y = 8 - 12 = -4$.
Therefore, (0,0) and (2,-4) are turning points.
Hence, the graph is



(ii)

$$A = \int_0^3 x^3 - 3x^2 \, dx$$

$$= \left[\frac{x^4}{4} - x^3\right]_0^3$$

$$= \frac{81}{4} - 27$$

$$= \frac{81 - 108}{4}$$

$$= -\frac{27}{4}$$

Area = $6\frac{3}{4}$ sq units under the ***x*** axis. **ANS**

(iii)

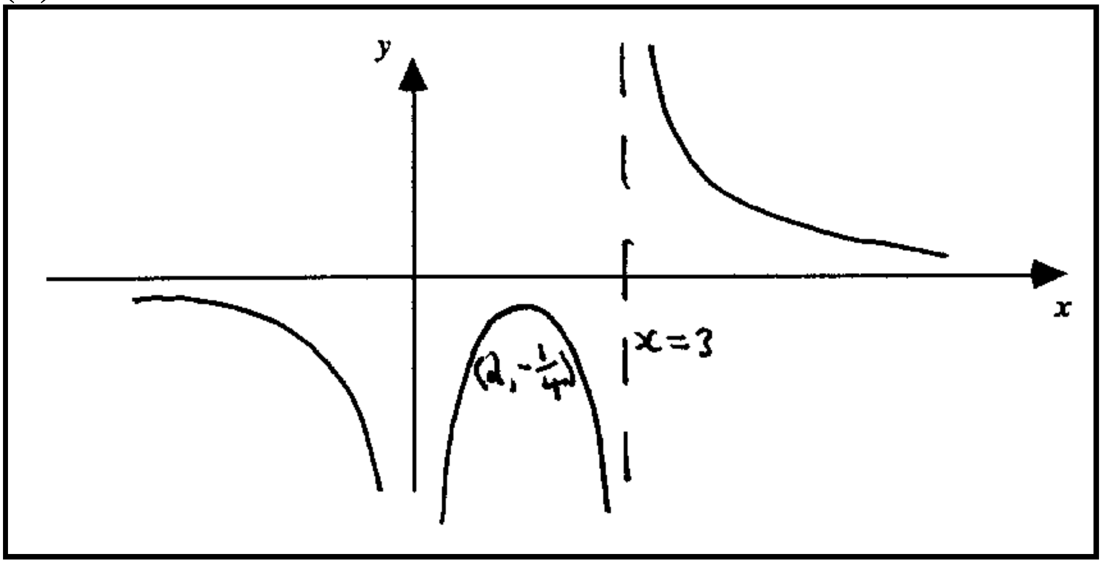
Equation of the tangent is $y = mx + c$.

$m = \frac{dy}{dx} = 3x^2 - 6x$

When $x = 1$, $m = 3 - 6 = -3$.
Hence, $y = -3x + c$.
On the curve, when $x = 1$, $y = 1 - 3 = -2$.
Hence, on the line, when $x = 1$, $y = -2$.
Hence, $-2 = -3 + c$.
Therefore, $c = 1$.
Hence, the equation of the tangent to the curve at the point where $x = 1$ is
$y = -3x + 1$ or $y = 1 - 3x$. **ANS**

Question 3 (continued)

(iv)



Question 4

(i) $Z = \frac{X - \mu}{\sigma} = \frac{1.998 - 2.000}{0.002} = -1$

Pr (X < 1.998)
= Pr (Z < -1)
= Pr (Z > 1)
= 1 - Pr (Z < 1)
= 1 - 0.840
= 0.160 **ANS**

(ii) $Z = \frac{X - \mu}{\sigma} = \frac{2.001 - 2.000}{0.002} = 0.5$

Pr (1.999 < X < 2.001)
= Pr (X < 2.001) - Pr (X < 1.999)
= Pr (Z < 0.5) - Pr (Z < -0.5)
= Pr (Z < 0.5) - Pr (Z > 0.5)
= Pr (Z < 0.5) - [1 - Pr (Z < 0.5)]
= 2 Pr (Z < 0.5) - 1
= 2 x 0.692 - 1
= 0.384 **ANS**

(iii)

P	3	-1
Pr ($P = p$)	0.384	0.616

$\mu = \Sigma\, x \Pr(X = x) = 1.152 - 0.616 =$ \$0.536 = \$0.54 **ANS**

$\sigma^2 = E(X^2) - \mu^2 = 9 \times 0.384 + 0.616 - (0.536)^2 = 4.072 - 0.287296 =$ \$3.78 **ANS**

End Of Suggested Solutions 1994 Mathematical Methods Trial Cat 3

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