IARTV '95
CAT 2

Specific Instructions for Section A

Section A consists of 33 questions.
Answer all questions in this part on the multiple-choice answer sheet provided.
A correct answer scores 1, an incorrect answer scores 0. No credit will be given for a question if two or more letters are marked for that question. Marks will not be deducted for incorrect answers and you are urged to attempt every question.

Question 1

In the expansion of $(3x - 1)^4$, the coefficient of x^2 is equal to

A 6 **B** 12 **C** 18
D 54 **E** 108

Question 2

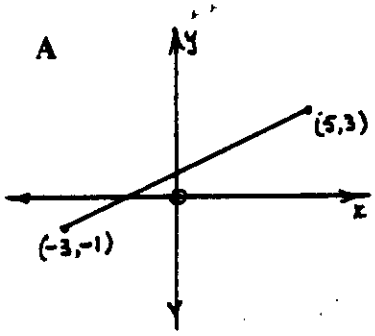
The graph of the function $f : \mathbf{R} \to \mathbf{R}$, $f(x) = x^4 - x$ has

A no x-intercepts
B one x-intercept
C two x-intercepts
D three x-intercepts
E four x-intercepts

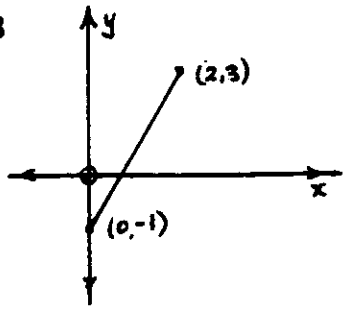
Question 3

Which one of the following graphs best represents the **inverse function** g^{-1}, where g is the function $g : [-1,3] \to \mathbf{R}$, $g(x) = 2x - 1$?

A



B



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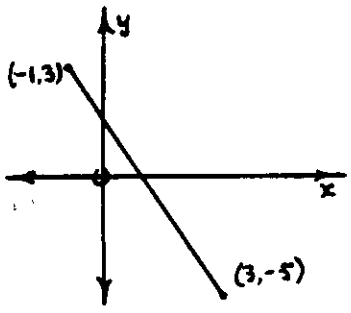

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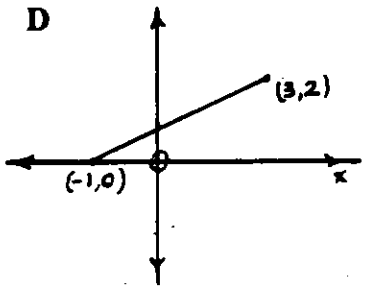
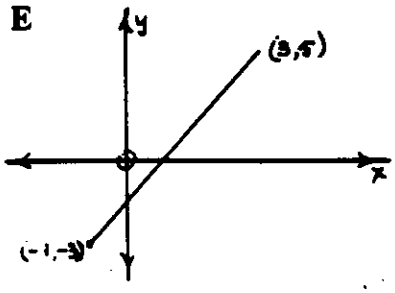


E

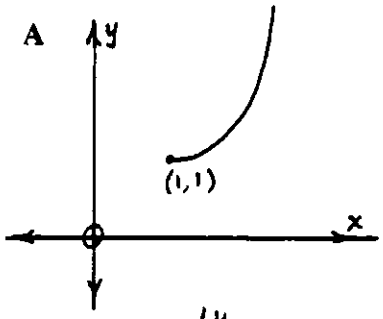


Questions 4 and 5 refer to the function defined by the rule $y = 1 - \sqrt{x - 1}$

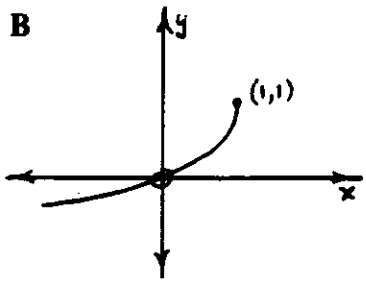
Question 4

Which one of the following graphs best represents the function?

A



B



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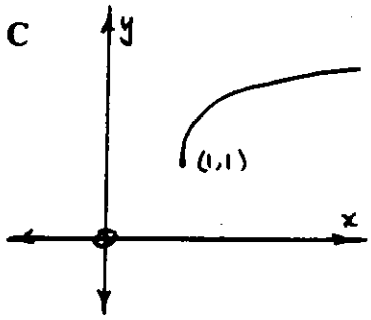

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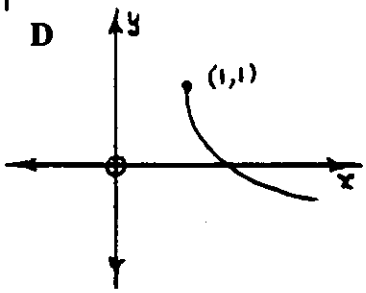
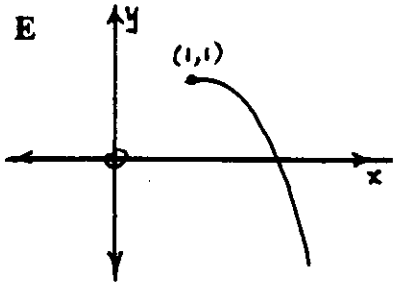


E



Question 5

The **inverse function** is defined by

A $\{(x,y) : y = (x - 1)^2 + 1, x \geq 1\}$
B $\{(x,y) : y = (x + 1)^2 + 1, x \geq 1\}$
C $\{(x,y) : y = 1 - (x - 1)^2, x \geq 1\}$
D $\{(x,y) : y = (x + 1)^2 + 1, x \leq 1\}$
E $\{(x,y) : y = (x - 1)^2 + 1, x \leq 1\}$

Questions 6 and 7 refer to the function $h : \mathbf{R} \rightarrow \mathbf{R}$, $h(x) = 1 - 2e^{x-3}$

Question 6

The ***y*-intercept** of the graph of h has a value of approximately

A 0.90 **B** 0.73 **C** −1.00
D −4.44 **E** −39.17

Question 7

The ***x*-intercept** of the graph of h has a value of approximately

A 0.6931 **B** 0.9004 **C** 0.9163
D 2.3069 **E** 2.6931

Question 8

x	−1	0	1	2	3	4
y	2	1	0	5	22	57

The data in the table above may be modelled by a rule of the form

A $y = Ax^n \ \ (A, n \in \mathbf{R})$
B $y = Ae^{kx} \ \ (A, k \in \mathbf{R})$
C $y = mx + c \ \ (m, c \in \mathbf{R})$
D $y = ax^2 + bx + c \ \ (a, b, c \in \mathbf{R})$
E $y = ax^3 + bx^2 + cx + d \ \ (a, b, c, d \in \mathbf{R})$

Question 9

For the function defined by the rule $y = 2\cos(\pi x)$, the **period** and **amplitude** are, respectively,

A π and 2 **B** 2 and π **C** 1 and 2
D π and π **E** 2 and 2

Question 10

d is the function with rule $d(t) = 8 + 0.5 \sin\left(\frac{\pi t}{4}\right)$ and domain $\{t : t \geq 0\}$.

The **range** of function d is

A $\mathbf{R}^+ \cup \{0\}$ **B** [−8,8] **C** [7.5,8.5]
D [0,16] **E** [7,9]

Question 11

Within the restricted domain $\{x : 0 \leq x \leq 2\pi\}$, the equation $\sin 3\left(x - \frac{\pi}{2}\right) + 1 = 0$ has

A two solutions **B** three solutions **C** four solutions
D five solutions **E** six solutions

Question 12

If $g(x) = x^3 e^x$, then $g'(1)$ has a value of

A e **B** 2e **C** 3e
D 4e **E** 5e

Question 13

If $y = \dfrac{2x}{\sin x}$, then $\dfrac{dy}{dx} = \dfrac{\phi(x)}{\sin^2 x}$ where $\phi(x)$ is equal to

A	$2(\sin x + x \cos x)$	**D**	$-2(\sin x - x \cos x)$
B	$-2(\sin x + x \cos x)$	**E**	$2(\sin x - x \cos x)$
C	$2(x \sin x - \cos x)$		

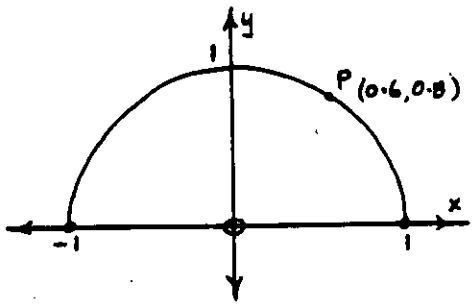
Question 14

If $g(x) = \log_e(\cos x)$, then $g'(x)$ is equal to

A	$-\tan x$	**D**	$\dfrac{\cos x}{x} - (\log_e x)(\sin x)$
B	$\dfrac{\sin x}{\cos x}$	**E**	$-(\sin x)\log_e(\cos x)$
C	$\dfrac{-\sin(\log_e x)}{x}$		

Question 15

The diagram at right represents the semi-circle defined by the equation $y = \sqrt{1 - x^2}$.
The **gradient** of the tangent to the circle at the point P : (0.6,0.8) is equal to



A	-0.6	**D**	-1
B	-0.75	**E**	$-1\frac{1}{3}$
C	-0.8		

Questions 16 and 17 refer to an object being thrown vertically upwards so that its height above the ground h m, at time t seconds $(0 \le t \le 10)$ is given by $h = 50t - 5t^2$.

Question 16

The **average** rate at which the height is increasing over the first four seconds is

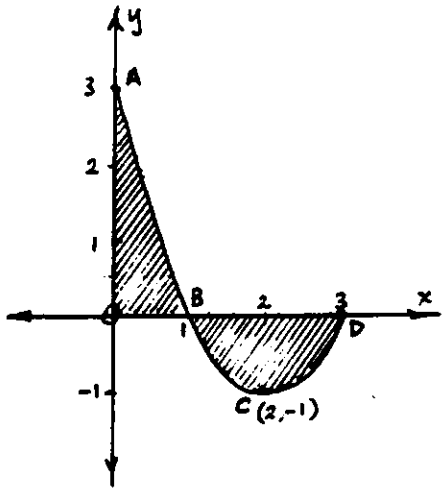
A	10 m/s	**B**	20 m/s	**C**	25 m/s
D	30 m/s	**E**	40 m/s		

Question 17

The object is thrown with an **initial velocity** of

A	100 m/s	**B**	50 m/s	**C**	40 m/s
D	25 m/s	**E**	20 m/s		

Questions 18 and 19 refer to the function
$f : [0,3] \to \mathrm{R}, f(x) = x^2 - 4x + 3 = (x - 2)^2 - 1$
whose graph is sketched at right.



Question 18

The area of the shaded region (bounded by the graph of f and the x and y-axes) may be approximated by the sum of the areas of the triangles OAB and BCD.
Using this approximation, the area is

A ½ square unit
B 1 square unit
C 1½ square units
D 2 square units
E 2½ square units

Question 19

$\int_0^3 f(x)\,dx$ is exactly equal to

A 0 **B** 1⅓ **C** 1⅔
D 2 **E** 2⅔

Question 20

The value of $\int_0^1 (2x + 1)^3\,dx$ is

A 10 **B** 10⅛ **C** 13⅓
D 20 **E** 80

Question 21

$\int_2^3 \frac{2}{x - 1}\,dx = \log_e k$, where k is equal to

A 1 **B** 2 **C** 3
D 4 **E** 8

Questions 22, 23 and 24 refer to the following table, which represents a discrete probability distribution for a random variable X:

x	0	1	2	3
$\Pr(X=x)$	k	$2k$	$3k$	$4k$

Question 22

The value of k is

A 0 **B** 0.1 **C** 0.2
D 0.4 **E** 1

Question 23

The **expected value** of X is

A 1.0 **B** 1.8 **C** 2.0
D 2.2 **E** 3.0

Question 24

The **variance** of X is

A 0.6 **B** 1.0 **C** 1.6
D 2.0 **E** 5.0

Question 25

A target shooter fires 16 bullets at a target. The probability of hitting the bullseye is estimated to be ¼. The number of bullseyes hit has an expected value of 4 and a standard deviation of

A $\sqrt{3}$ **B** 2 **C** $2\sqrt{3}$
D 4 **E** 12

Question 26

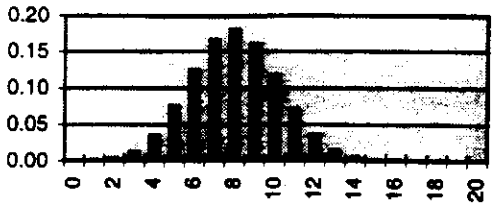
A fair die is tossed 180 times. The expected number of sixes is 30. On 95% of repetitions of this experiment, we would expect the number of sixes to be between

A 25 and 35 **B** 20 and 40 **C** 15 and 45
D 10 and 50 **E** 5 and 55

Question 27

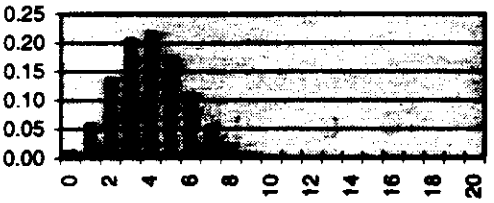
The following graphs show $\Pr(X=x)$ for five Binomial probability distributions, each with $n = 20$. Which distribution has the smallest value of p?

A

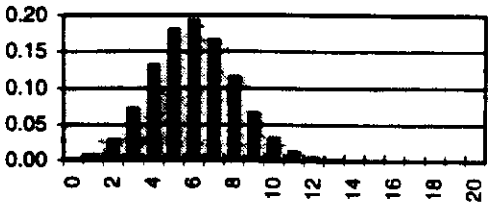


B

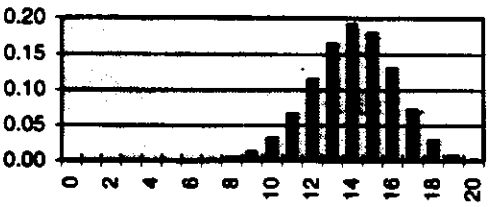
C



D

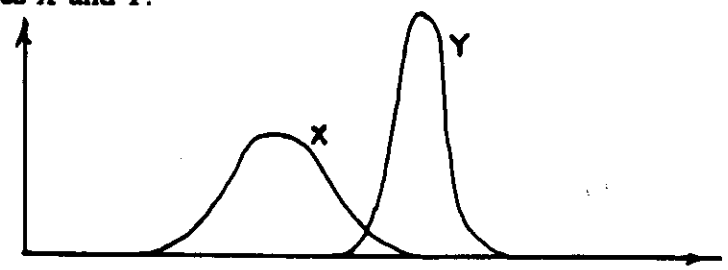


E



Question 28

The following diagram represents the probability distributions for two **normally distributed random variables** X and Y.



Which of the following observations about these distributions is correct?

A X has a lower mean and lower variance than Y.
B X has a higher variance and mean than Y.
C X has a lower mean, but higher variance than Y.
D X has a lower variance, but higher mean than Y.
E X has a lower mean than Y, but an equal variance.

Question 29

If Z has a standard normal distribution, then $\Pr(Z<1.5) = 0.9332$.
$\Pr(-1.5 < Z < 1.5)$ is equal to

A	0.0668	**B**	0.1336	**C**	0.4332
D	0.8664	**E**	0.9332		

Question 30

A fair coin is tossed 100 times. If X is the number of heads which turn up, then X has a Binomial distribution. Using the fact that the standard normal distribution Z may be used as an approximation for X, $\Pr(X > 60)$ is approximated by

A	$\Pr(Z > 0.4)$	**D**	$\Pr(Z > 3)$
B	$\Pr(Z > 1)$	**E**	$\Pr(Z > 4)$
C	$\Pr(Z > 2)$		

Questions 31 to 33 refer to a researcher who is trying to find the proportion (p) of Australian residents who oppose the testing of nuclear weapons.

Question 31

The researcher chooses a random sample of 100 Australian residents, and finds the percentage of the sample who oppose the testing of nuclear weapons. This percentage is an example of a

A	sample	**D**	population
B	statistic	**E**	sample parameter
C	parameter		

Question 32

If 80% of this random sample of 100 Australian residents oppose the testing of nuclear weapons, then the **standard error of the estimate** of p is

A	0.0016	**D**	0.04
B	0.004	**E**	0.16
C	0.016		

Question 33

On the basis of finding that 80% of the sample of 100 oppose testing of nuclear weapons, the researcher can be 95% confident that the percentage of Australian residents who oppose testing of nuclear weapons is between

A	78% and 82%	**D**	72% and 88%
B	76.8% and 83.2%	**E**	70% and 90%
C	76% and 84%		

END OF SECTION A
PLEASE TURN OVER

SECTION B

Specific Instructions for Section B

Section B consists of 6 questions. There are a total of 17 marks available.

Answer all six questions neatly on the lined paper provided.

Please number each question clearly.

Full marks may not be given for answers which do not show appropriate working, or do not state answers clearly.

Question 1

Sketch the graph of the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^4 - 2x^3$, clearly showing all important features, labelling coordinates of stationary points and coordinates of intercepts with the coordinate axes.

[3 marks]

Question 2

Consider the function $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 1 - e^{-x}$.

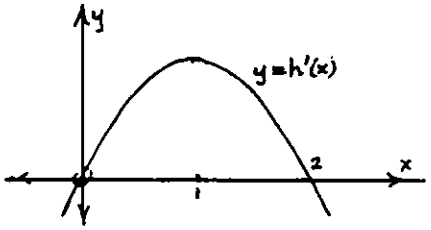
(a) On the same set of Cartesian axes, and using the same scale on both axes, sketch the graphs of (i) the function g, and
(ii) the **inverse function** g^{-1}.

(b) State the **domain** and **rule** for the **inverse function** g^{-1}.

[2 + 2 = 4 marks]

Question 3

The following graph represents a **derivative function** h':



Sketch a possible graph of the function h given that $h(0) = 0$ and $h(3) = 0$.

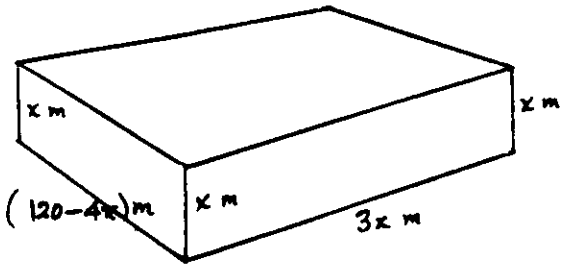
[2 marks]

Question 4

An airline has the regulation that pieces of luggage must have the sum of their dimensions (length + width + depth) less than or equal to 120 cm.

If a piece of luggage is designed so that the length is three times the depth, that is, if depth = x cm and length = $3x$ cm, then the volume $V(x)$ cm^3 is given by

$$V(x) = 3x^2(120 - 4x)$$



(a) State the **domain** for the volume function V,

(b) Find the dimensions of the piece of luggage (with length three times depth) which has **maximum volume**.

[1 + 2 = 3 marks]

Question 5

If a drawing pin is tossed on a table top, it is estimated that the probability it will land "point up" will be ⅔. If **five** drawing pins are tossed,

(a) what is the probability that exactly two pins will land "point up" ?

(b) what is the probability that at least one pin lands "point up" ?

[1 + 1 = 2 marks]

Question 6

An automatic drink dispenser fills cups with drink when money is inserted. The amount of drink in each cup is normally distributed with a mean of 150 ml and a standard deviation of 5 ml.

(a) If a cup has a capacity of 165 ml, what is the probability that the drink will overflow?

You may use a calculator or the following cumulative standard normal distribution function table:

z	Pr(Z<z)
0.0	0.5000
0.5	0.6915
1.0	0.8413
1.5	0.9332
2.0	0.9772
2.5	0.9938
3.0	0.9987
3.5	0.9998

Question 6 (continued)

(b) If 90% of cups contain more than x ml of drink, find the value of x.

You may use a calculator or the following inverse cumulative standard normal distribution function table:

$\Pr(Z<z)$	z
0.50	0.0000
0.55	0.1257
0.60	0.2533
0.65	0.3853
0.70	0.5244
0.75	0.6745
0.80	0.8416
0.85	1.0364
0.90	1.2816
0.95	1.6449

[1 + 2 = 3 marks]

END OF BOOKLET