**YEAR 12**
**IARTV TEST — OCTOBER 1995**
**MATHEMATICAL METHODS CAT 3**
**ANSWERS & SOLUTIONS**

Question 1.

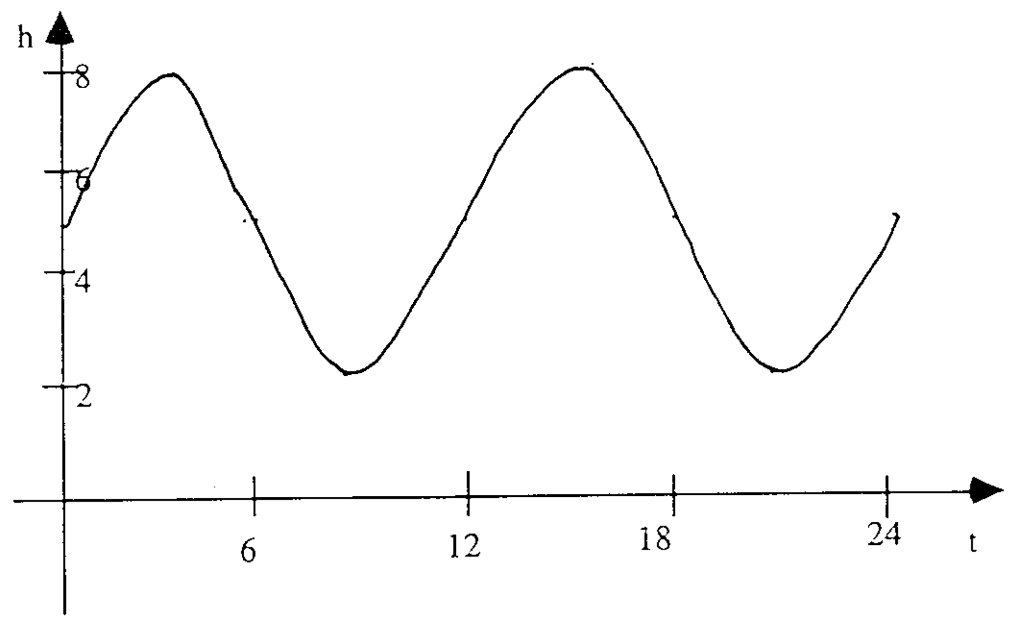
(a) $h(0) = 5$ (m)

(b) $h(6) = 5$ (m)

(c) solve $h(t) = 2$ , $3\sin\frac{\pi t}{6} + 5 = 2$ , $\sin\frac{\pi t}{6} = -1$ , $t = 9 \Rightarrow$ 9:00am

(d) for maximum height $\sin\frac{\pi t}{6} = 1$ , $\frac{\pi t}{6} = \frac{\pi}{2}$ , $\frac{5\pi}{2}$ , $t = 3$ , $15 \Rightarrow$ 3am, 3pm

(e)




(f) $\frac{dh}{dt} = \frac{\pi}{2}\cos\frac{\pi t}{6} = -\frac{\pi}{2} m/s$ when t = 6

(g) falling

Question 2.

(a) $\Pr(X < 83) = \Pr(z < -1) = 0.15866$

(b) $\Pr(X > 86) = \Pr(z > 2) = 1 - \Pr(z < 2) = 0.02275$

(c) $\Pr(83 < X < 86) = 1 - (0.15866 + 0.02275) = 0.81859$

(d) Y = profit on pin ,

$E(Y) = 0.818598 * 5 - 0.18141 * 2 = 3.7301$

$\Rightarrow$ expected profit on 10000 pins is $37300

(e) 1] $^{10}C_2 0.1^2 0.9^8 = 0.1937$

2] $1 - 0.9^{10} = 0.6513$

Question 3

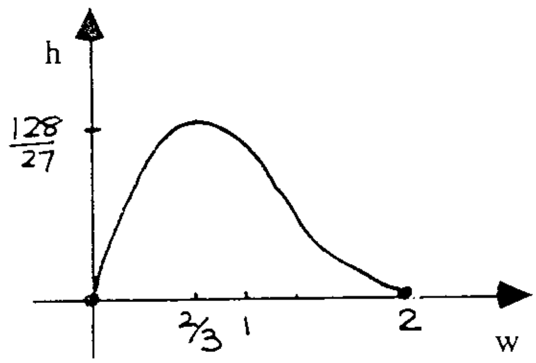
(a) $\frac{dh}{dw} = 4(w-2)^2 + 4w * 2(w-2) = 4(w-2)(w-2+2w)$

$= 4(w-2)(3w-2)$

(b) The maximum height occurs when $\frac{dh}{dw} = 0$

$i.e.\ w = \frac{2}{3}.\quad h(\frac{2}{3}) = \frac{128}{27}$ so the point is $(\frac{2}{3}, \frac{128}{27})$

(c)




(d)

The gradient is a minimum at a point of inflexion

$h'' = 4(3w-2) + 4*3(w-2) = 24w - 32 = 0$ when $w = \frac{4}{3}$

$h(\frac{4}{3}) = \frac{64}{27}$ so the point is $(\frac{4}{3}, \frac{64}{27})$

(e)

As $w \to 0, \frac{dh}{dw} \to 16$ when $w = \frac{4}{3}, \frac{dh}{dw} = -\frac{16}{3}$

so the boundary will be steepest when $w \to 0$.

(f) $Area = \int_0^2 4w(w-2)^2 dw = 4[\frac{w^4}{4} - \frac{4w}{3} + 2w^2]_0^2 = \frac{16}{3} m^2$

Question 4.

(a) $\log_e y = 27$ when $x=12$, and $x=8.5$ when $\log_e y=20$

$(b) gradient = 2$

$(c) A = 2, B = 3$

$(d) \log_e y = 2x+3 \Rightarrow y = e^{2x+3} = e^3 e^{2x} \Rightarrow D = e^3, C = 2$

$(e) y = e^{2x+3} = 0.013$ when $x = -3.67$

$(f) \log_{10} \frac{y}{F} = Gx \Rightarrow \frac{y}{F} = 10^{Gx} \Rightarrow y = (10^G)^x$ and $y = e^3(e^2)^x$

so that $F = e^3 = 20.09$, and $G = \log_{10} e^2 = 0.869$