IARTV '95
CAT 3

QUESTION ONE (15 marks)

The height, h metres, of a tidal river on September 23rd varied with the time t, measured in hours after midnight, according to the rule:

$$h(t) = 3\sin\frac{\pi t}{6} + 5$$

(a) State the height of the river at midnight.

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(1 mark)

(b) State the height of the river at 6:00am.

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(1 mark)

(c) Find the first time after midnight when the river had a height of 2 metres.

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(3 marks)

(d) At what times on September 23rd was the river at a maximum height?

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(3 marks)

(e) Sketch the graph of h(t) for $0 \leq t \leq 24$ on the axes below.

(4 marks)

(f) Find $\frac{dh}{dt}$ and hence find the rate at which the height of the river is changing at 6:00am.

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(g) Is the tide rising or falling at 6:00am?.....

(1 mark)

QUESTION TWO (15 MARKS)

A machine produces cylindrical pins for the "Holdon Fastening Company". Its output is normally distributed with mean diameter 84mm and standard deviation 1mm.

(a) What proportion of pins are less than 83mm in diameter?

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(1 mark)

(b) What proportion of pins are greater than 86mm in diameter?

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(2 marks)

Pins within the range 83mm to 86mm are acceptable whereas others are defective.

(c) If a pin is chosen at random find the probability that it is acceptable?

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(3 marks)

An acceptable pin gives a profit of \$5 whereas a defective pin gives a loss of \$2.

(d) How much profit should the company expect from the production of 10 000 pins?

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(3 marks)

The machine is adjusted so that the probability of producing an acceptable pin is 0.9. A batch of 10 pins is chosen at random from the output.

(e) Find the probability that this batch contains:

1] exactly two defective pins

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(3 marks)

2] at least one defective pin.

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(3 marks)

QUESTION THREE (15 MARKS)

A vertical cross section is taken through a sculpture. The height of this section is given by the formula

$$h(w) = 4w(w-2)^2 \text{ for } 0 \le w \le 2,$$

where

h is the height of the sculpture(in metres)

w is the distance from the eastern edge (in metres)

(a) Find an expression for the gradient of h. (i.e. $\frac{dh}{dw}$)

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(3 marks)

(b) Give the coordinates of the maximum height of this cross section.

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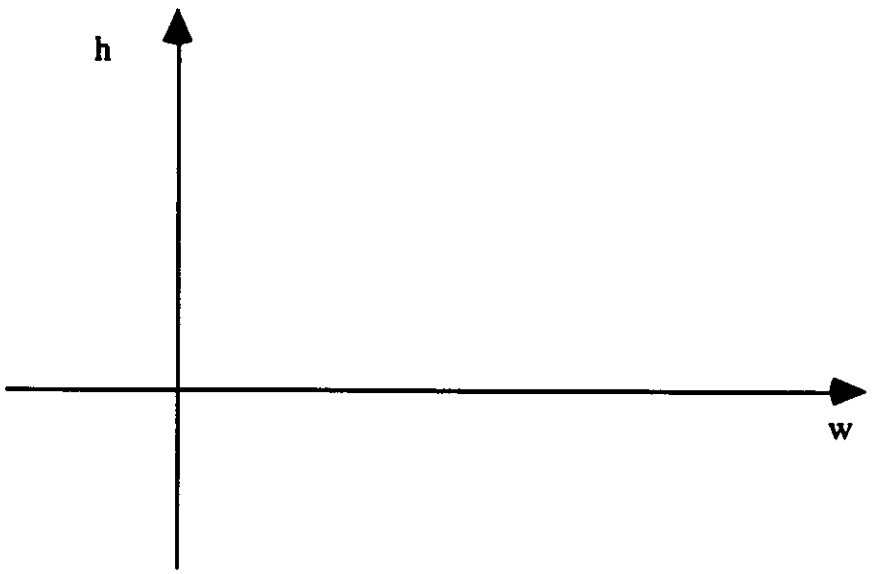
(2 marks)

(c) If the bottom of the cross section of the sculpture is the line $h = 0$ for $0 \le w \le 2$, and the sculpture is solid, sketch the cross section on the axes below.



(3 marks)

(d) State the coordinates of the point on the boundary of this section where the gradient is a minimum.

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(3 marks)

(e) Where is the steepest part of the boundary of this cross section?

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(1 mark)

(f) Calculate the area of this cross section.

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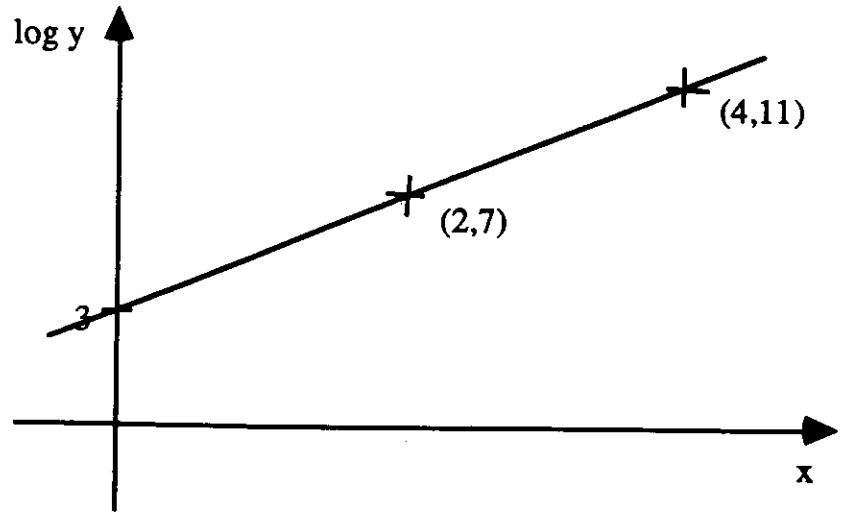
(3 marks)

QUESTION FOUR (15 MARKS)

A scientist records results from an experiment which involves taking measurements of the two variables x and y. He then plots his results and his plot of $\log_e y$ against x is shown below:



Assume that the above graph is a straight line graph.

(a) Using the above information complete the table:

x	2	4	12
$\log_e y$	7	11	20

(2 marks)

(b) State the gradient of the line shown.

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(1 mark)

The equation of this line can be written in the form

$$\log_e y = Ax + B$$

(c) Use the graph or the above values to find the values of A and B.

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(2 marks)

(d) Show that the above equation can be rearranged to give $y = De^{Cx}$, and find the values of D and C.

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(e) Find the value of y when x is -3.67.

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(2 marks)

Another scientist states that she has found a formula connecting the same variables.Her result was of the form:

$$\log_{10} y = \log_{10} F + Gx$$

(f) If both scientists have discovered the same result what are the values of F and G.

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(5 marks)

END OF BOOKLET