Victorian Certificate of Education
1995

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination (Facts, skills and applications task)

Friday 10 November 1995: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOKLET

Directions to students

This task has two parts: part I (multiple-choice questions) and part II (short-answer questions).
Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this booklet.

At the end of the task
Place the answer sheet for multiple-choice questions (part I) inside the front cover of the question and answer booklet (part II) and hand them in.
You may retain this question booklet.

Specific instructions to students

This part consists of 33 questions.

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions.

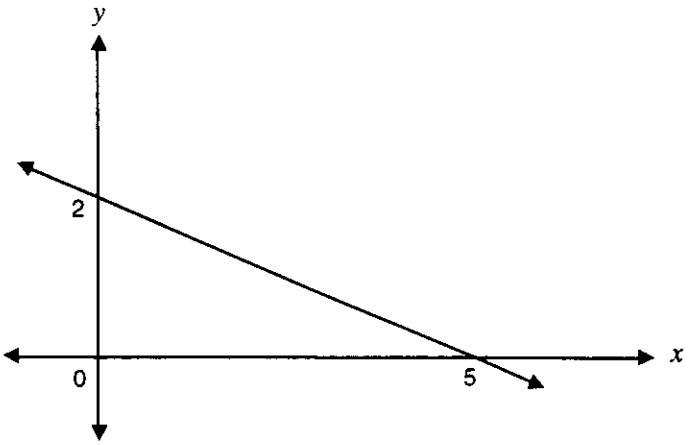
A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No credit will be given for a question if two or more letters are marked for that question.

Question 1

The graph shown is that of the linear function



A. $f: R \to R,\ f(x) = 5x + 2$

B. $f: R \to R,\ f(x) = 2 - \frac{2x}{5}$

C. $f: R \to R,\ f(x) = \frac{2x}{5} + 2$

D. $f: R \to R,\ f(x) = -\frac{5x}{2} + 5$

E. $f: R \to R,\ f(x) = 2x + 5y$

Question 2

If the graph of $f: R \to R$ crosses the x–axis exactly three times, which one of the following rules could **not** be the rule for f?

A. $f(x) = x(x^2 - 4)$

B. $f(x) = x(x - 2)(x + 4)(x^2 + 1)$

C. $f(x) = (3 - x)(x^4 - 16)$

D. $f(x) = (x^2 - x - 6)(x - 4)$

E. $f(x) = (x^2 - x - 6)(x^2 - x - 12)$

Question 3

The range of the function with graph as shown is

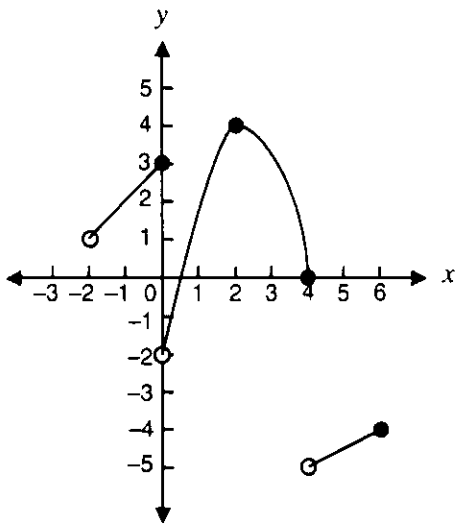
A. $(-2, 6]$

B. $(-5, -4] \cup (-2, 4]$

C. $(-5, 4]$

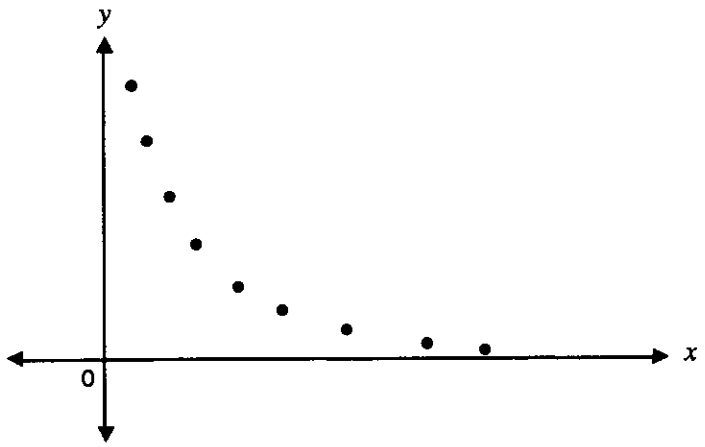
D. $(-5, -4] \cup (-2, 3]$

E. $(-2, 4] \cup (5, 6]$



Question 4

Data about the relationship between quantities x and y is represented graphically as shown below.



If a and b are positive constants, the equation relating x and y is most likely to be of the form

A. $y = ax^2$

B. $y = \frac{a}{x^2}$

C. $y = a\log_{10}(bx)$

D. $y = ae^{bx}$

E. $y = a\cos(bx)$

Question 5

The five graphs shown below are graphs of the relations

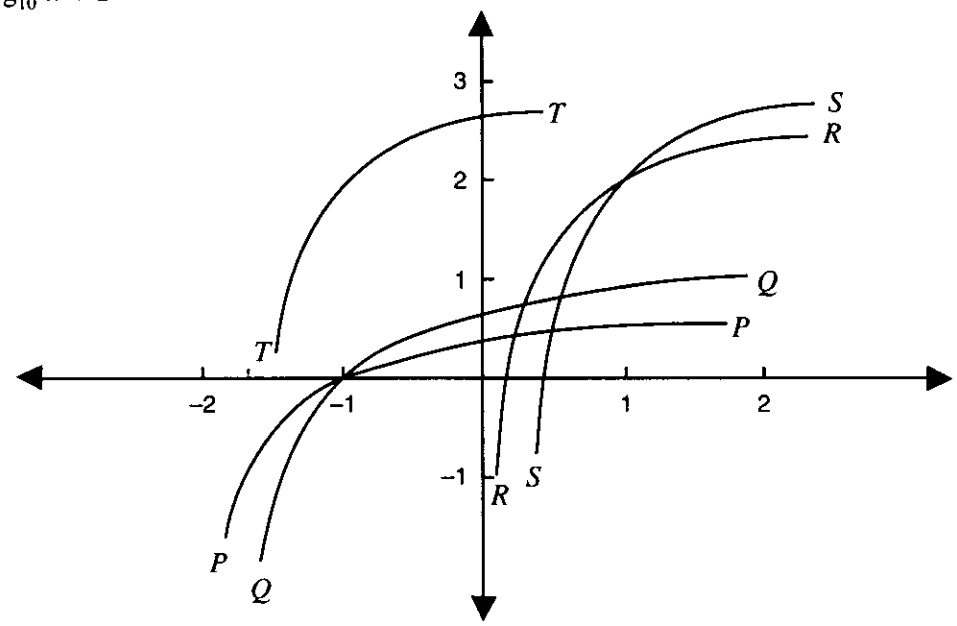
$y = \log_e(x + 2)$

$y = \log_e x + 2$

$y = \log_e(x + 2) + 2$

$y = \log_{10}(x + 2)$

$y = \log_{10} x + 2$



Which one is the graph of the relation $y = \log_{10}(x + 2)$?

A. P

B. Q

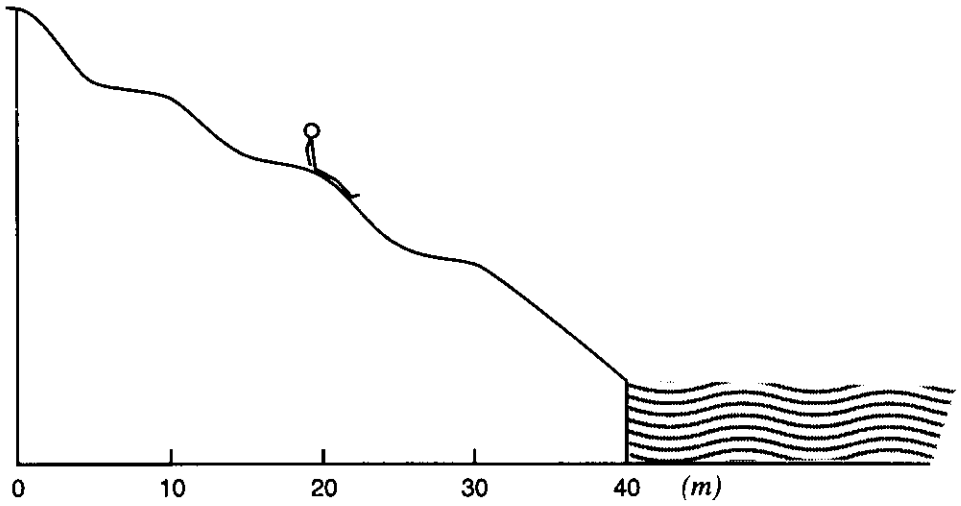
C. R

D. S

E. T

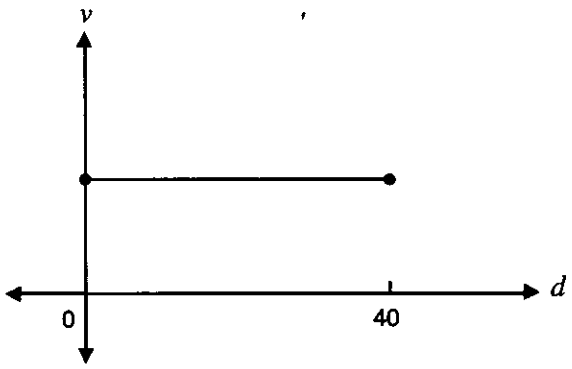
Question 6

At the swimming pool, Vinh goes as fast as he can down a water slide. He starts from rest at the top of the slide.

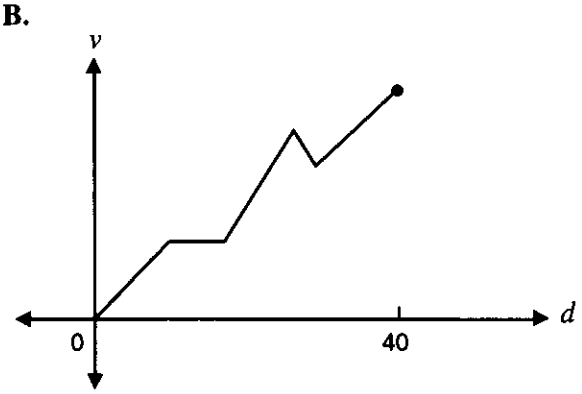


Which one of the following best represents Vinh's speed (v) on the slide as a function of the horizontal distance travelled (d)?

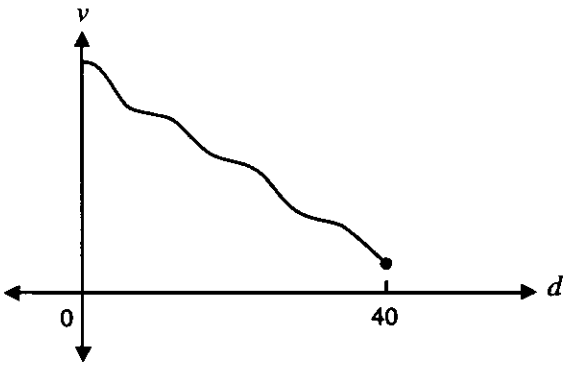
A.



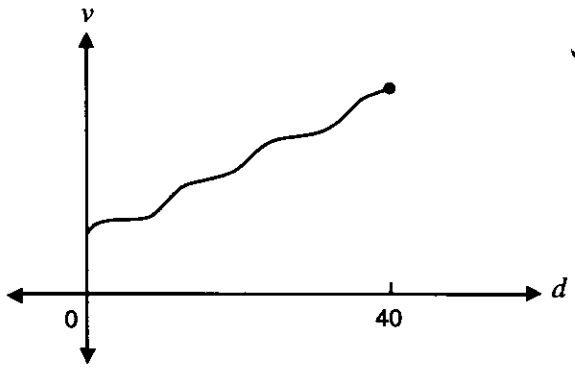
B.



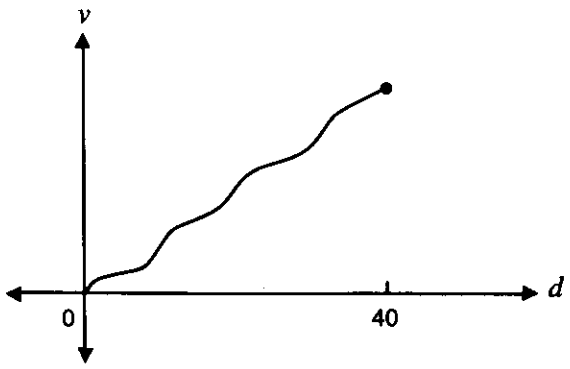
C.



D.



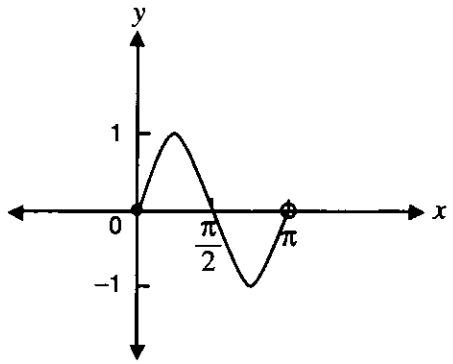
E.



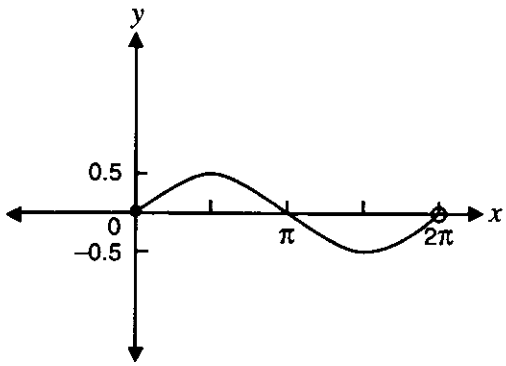
Question 7

Which one of the following best represents one cycle of the graph with equation $y = \sin(0.5x)$?

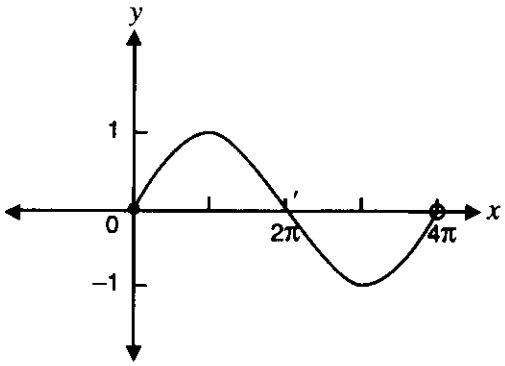
A.



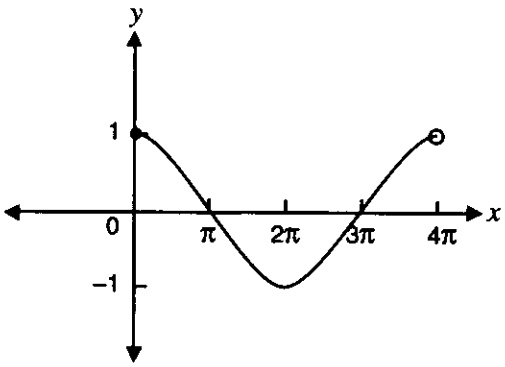
B.



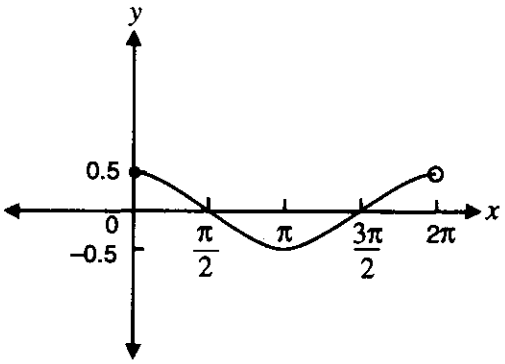
C.



D.



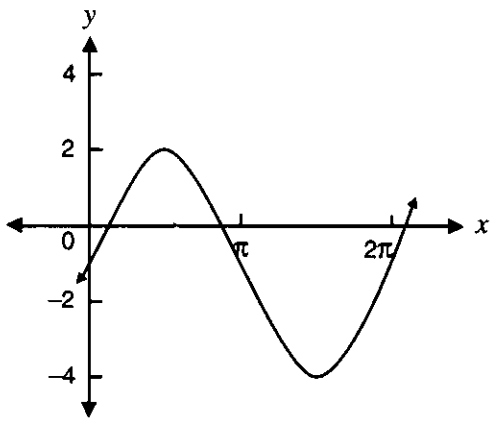
E.



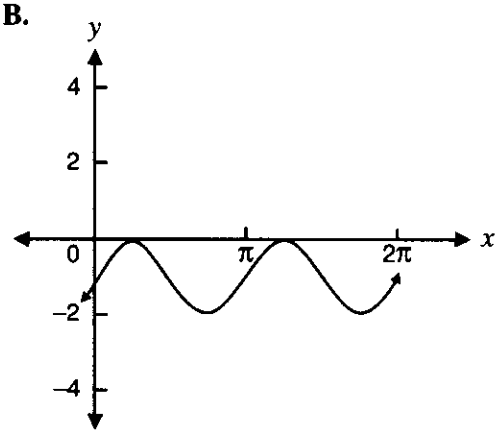
Question 8

Which one of the following graphs best represents the graph with equation $y = 3\sin(2x) - 1$?

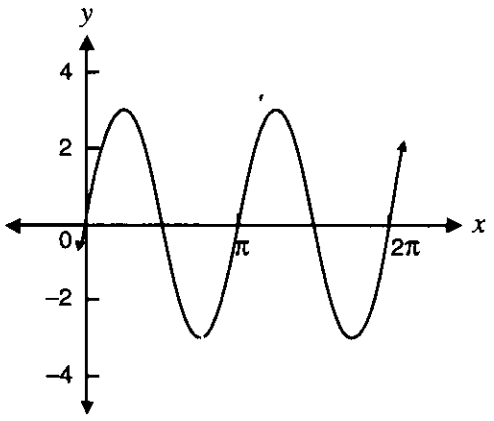
A.



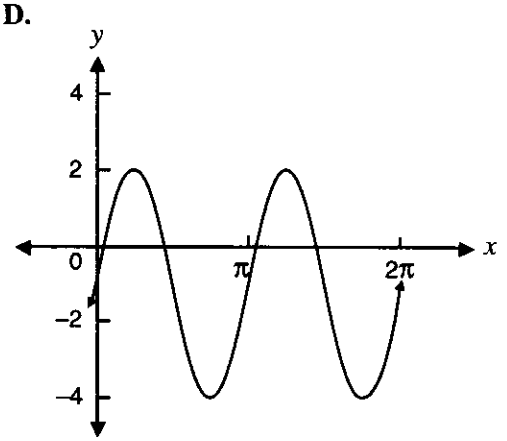
B.



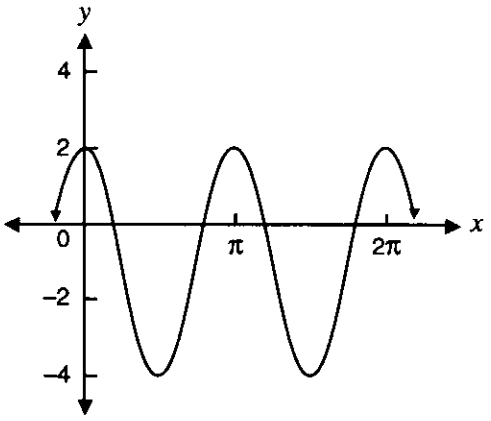
C.



D.



E.



Question 9

The number of solutions of the equation $\cos(3x) = -0.5$ between $x = 0$ and $x = 2\pi$ is equal to

A. 0
B. 1
C. 2
D. 3
E. 6

Question 10

A trigonometric function is given by

$$f: R \to R, f(x) = 3\cos(2(x - \pi)) + 1.$$

The amplitude, period and range, respectively, of the function are

	amplitude	period	range
A.	3	π	R
B.	2	$\frac{2\pi}{3}$	[–4, 4]
C.	2	$\frac{2\pi}{3}$	R
D.	π	3	[–2, 4]
E.	3	π	[–2, 4]

Question 11

If $f(x) = a\cos x + c$, where a is a positive real number, then $f(x) < 0$ for all real values of x if

A. $c > a$
B. $c < -a$
C. $c = 0$
D. $-a < c < a$
E. $c > -a$

Question 12

The derivative of $2\sqrt{x}$ is equal to

A. $\frac{1}{2\sqrt{x}}$

B. $\frac{1}{4\sqrt{x}}$

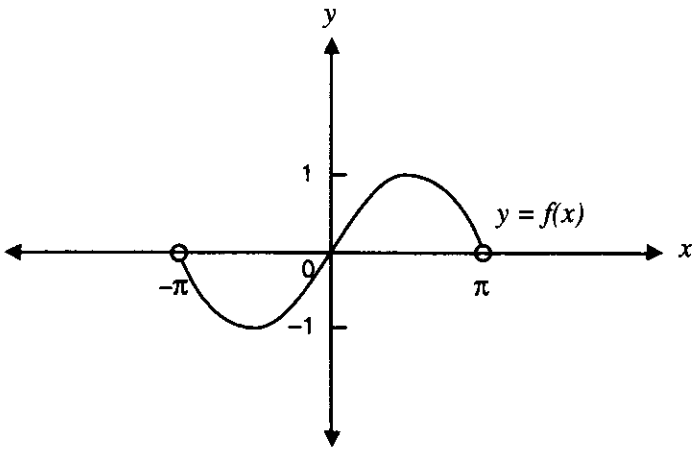
C. $\frac{1}{\sqrt{x}}$

D. $\frac{4x^{\frac{3}{2}}}{3}$

E. 2

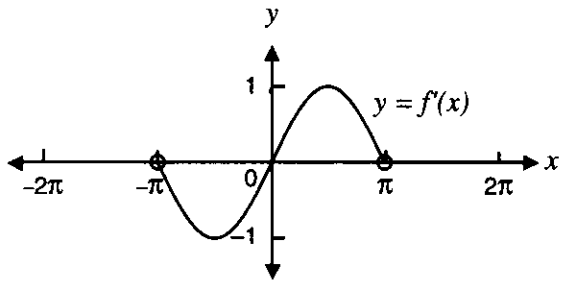
Question 13

The graph of the function $f:(-\pi, \pi) \rightarrow R$ is shown below.

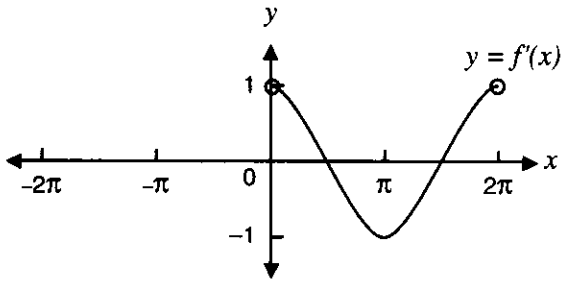


The graph of the derived function f' is most likely to be

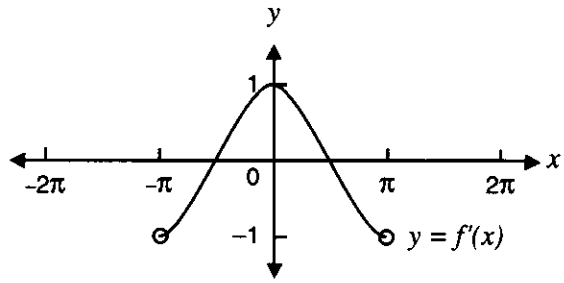
A.



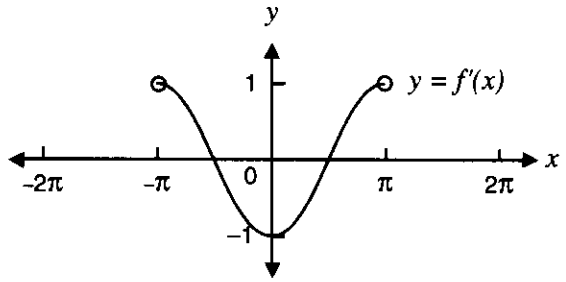
B.



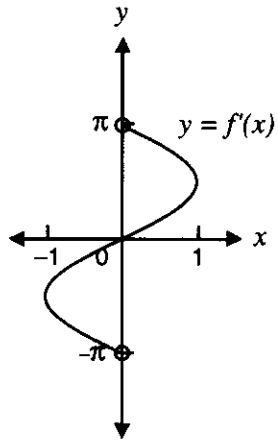
C.



D.



E.



Question 14

The derivative of $e^{\sin x}$ is equal to

A. $(\cos x).e^{\sin x}$

B. $e^{\sin x}$

C. $e^{\cos x}$

D. $(\cos x).e^{\cos x}$

E. $(\cos x).e^{x}$

Question 15

The gradient of the normal to the curve $y = \log_e x$ at the point where $x = 2$ is equal to

A. -2

B. $\frac{-1}{\log_e 2}$

C. -0.5

D. 0.5

E. 2

Question 16

If $f'(x) = \frac{1}{x^2}$, then $f(x)$ could be

A. $\frac{1}{x}$

B. $\frac{-2}{x^3}$

C. $\frac{-1}{x}$

D. $\log_e(x^2)$

E. $\log_e x$

Question 17

If $y = \frac{\sin x}{e^x}$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

A. -1

B. 0

C. $\frac{1}{e}$

D. $\frac{1}{2}$

E. 1

Question 18

An anti-derivative of $\frac{1}{(2x+5)^4}$ is

A. $4\log_e(2x+5)$

B. $\frac{-1}{6(2x+5)^3}$

C. $\frac{-6}{(2x+5)}$

D. $\frac{-1}{3(2x+5)^3}$

E. $\frac{10}{(2x+5)^5}$

Question 19

An anti-derivative of $e^{3x} + \sin(3x)$ is

A. $\frac{e^{3x}}{3} - \frac{\cos(3x)}{3}$

B. $3e^{3x} + 3\cos(3x)$

C. $e^{3x} - \frac{\cos(3x)}{3}$

D. $\frac{e^{3x}}{3} - \cos(3x)$

E. $e^{3x} - \cos(3x)$

Question 20

The total area of the shaded rectangles can be used as an approximation for the area between the curve with equation $y = \frac{1}{x}$, the x-axis, and the lines with equations $x = 1$ and $x = 5$. The value of this approximation is equal to

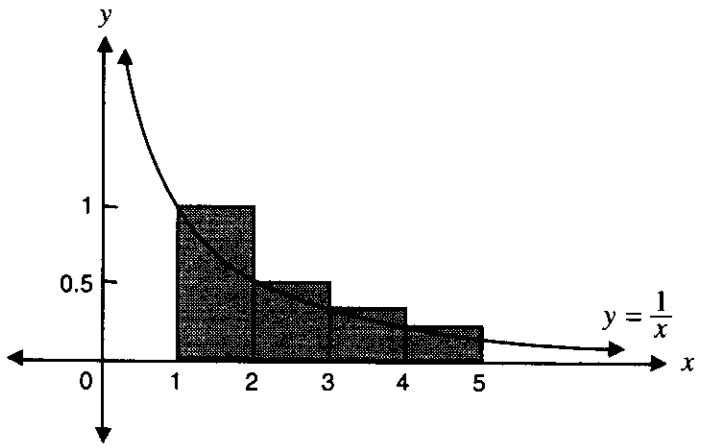
A. 1

B. 1.61

C. 2

D. 2.08

E. $2\frac{1}{12}$



Question 21

The area between the curve $y = e^x + 1$, the x-axis and the lines with equations $x = 0$ and $x = 1$ is equal to

A. $e - 1$

B. 1

C. e

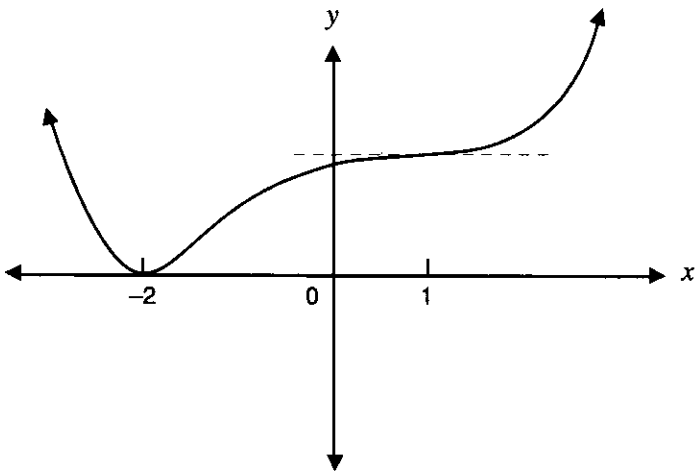
D. $e + 1$

E. $e + 2$

Question 22

The graph shown is of the function with rule

$$y = (x + 2)^2 (x^2 - 4x + 6)$$



Which one of the following is **not** true?

A. The gradient of the tangent to the graph at $x = 1$ is zero.

B. $\frac{dy}{dx} = 0$ when $x = 1$ and when $x = -2$ and at no other point.

C. There is only one turning point on the graph.

D. There is only one stationary point on the graph.

E. $y \geq 0$ for all values of x.

Question 23

$\log_e(x^2) + 2\log_e x + \log_e(4x)$ is equal to

A. $\log_e(2x^2 + 4x)$

B. $\log_e(x^2 + 6x)$

C. $\log_e(4x^5)$

D. $5\log_e(4x)$

E. $8\log_e x$

Question 24

Let a be the coefficient of x^2 and b be the coefficient of x in the expansion of the polynomial $(2x-1)^4$. Then

A. $a = b$

B. $a = 3b$

C. $a = 4b$

D. $a = -3b$

E. $a = -2b$

Question 25

The inverse of the function

$$f:[0,\infty) \to R, f(x) = 2\log_e(x+1)+1 \quad \text{is}$$

A. $f^{-1}:[0,\infty) \to R, f^{-1}(x) = 2\log_e(x+1)+1$

B. $f^{-1}:[0,\infty) \to R, f^{-1}(x) = e^{\frac{x-1}{2}} - 1$

C. $f^{-1}:R \to R, f^{-1}(x) = e^{\frac{x-1}{2}} - 1$

D. $f^{-1}:[1,\infty) \to R, f^{-1}(x) = 2\log_e(x+1)+1$

E. $f^{-1}:[1,\infty) \to R, f^{-1}(x) = e^{\frac{x-1}{2}} - 1$

Question 26

Which one of the following random variables is **not** discrete?

A. The number of runs scored by a team in a one-day cricket match.

B. The time, to the nearest minute, that it takes a student to walk to school.

C. The volume of milk consumed by a family in one week.

D. The number of customers served in one day at a milk bar.

E. The number of baseball caps owned by a student.

The following information relates to questions 27 and 28.

Over a twenty-five-day period Police Sergeant Bob Cryer kept a record of the number of motorists per day that PC Datta booked for speeding. His results are given in the table below.

Number of motorists booked (x)	0	1	2	3	4	5	6
Number of days on which x motorists are booked (f)	2	0	4	3	2	6	8

Question 27

During the twenty-five-day period, the proportion of days on which PC Datta booked fewer than 4 motorists for speeding is equal to

A. $\frac{2}{25}$

B. $\frac{9}{25}$

C. $\frac{11}{25}$

D. $\frac{14}{25}$

E. $\frac{16}{25}$

Question 28

During the twenty-five-day period, the mean number of motorists booked per day for speeding by PC Datta is equal to

A. 3.5

B. 4

C. 4.12

D. 5

E. 6

Question 29

Angie notes that 2 out of 10 peaches on her peach tree are spoilt by birds pecking at them. If she randomly picks 30 peaches the probability that exactly 10 of them are spoilt is equal to

A. 0.2

B. $(0.2)^{10}(0.8)^{20}$

C. $(0.2)^{20}(0.8)^{10}$

D. ${}^{30}C_{10}(0.2)^{10}(0.8)^{20}$

E. ${}^{30}C_{20}(0.2)^{20}(0.8)^{10}$

Question 30

A random variable X has probability distribution as shown in the table below.

x	0	1	2	3
Pr (X = x)	0.2	0.4	0.3	0.1

The expected value of x is equal to

A. 0

B. 0.25

C. 1.0

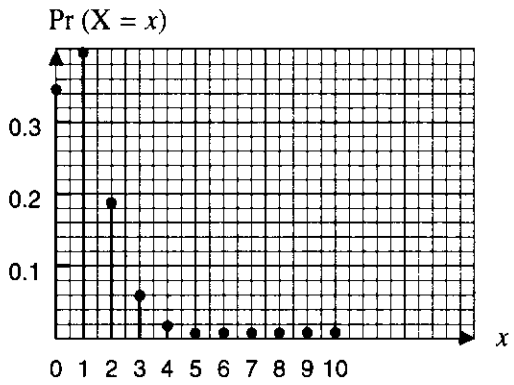
D. 1.3

E. 1.5

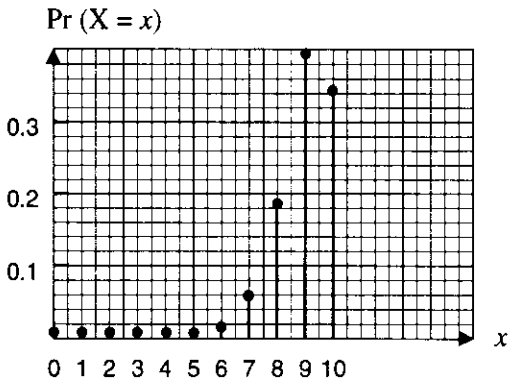
Question 31

Which one of the following graphs best represents the shape of a binomial distribution of the random variable X with 10 independent trials and probability of success for each trial equal to 0.1?

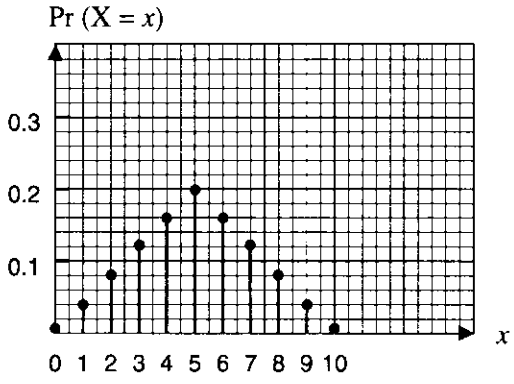
A.



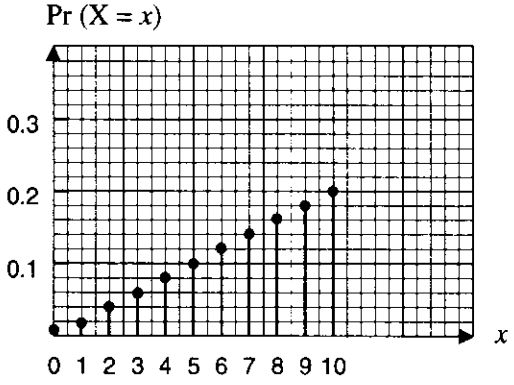
B.



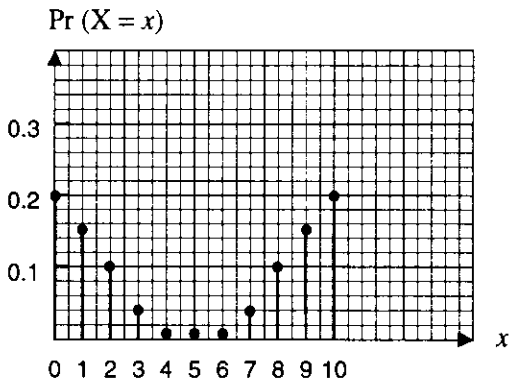
C.



D.



E.



The following information refers to questions 32 and 33

A large group of students is given a fitness test rated on a scale from 0.0 ('very unfit') to 50.0 ('extremely fit'). The test results follow a normal distribution with mean 30.0 and standard deviation 7.0.

Question 32

If a student has a score of 38 what proportion of the population will be less fit than this student?

A. 0.1265
B. 0.2800
C. 0.7200
D. 0.8735
E. 1.1430

Question 33

From the test, the lower 40 per cent of the population are defined to be unfit. What is the minimum score for a fit person?

A. 20.0
B. 23.5
C. 25.9
D. 28.2
E. 31.8

Total 33 marks