SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures									
Words									

Victorian Certificate of Education 1995

MATHEMATICAL METHODS

Common Assessment Task 3: Written examination (Analysis task)

Monday 13 November 1995: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOKLET

Structure of booklet

Number of questions	*Number of questions to be answered*
4	4

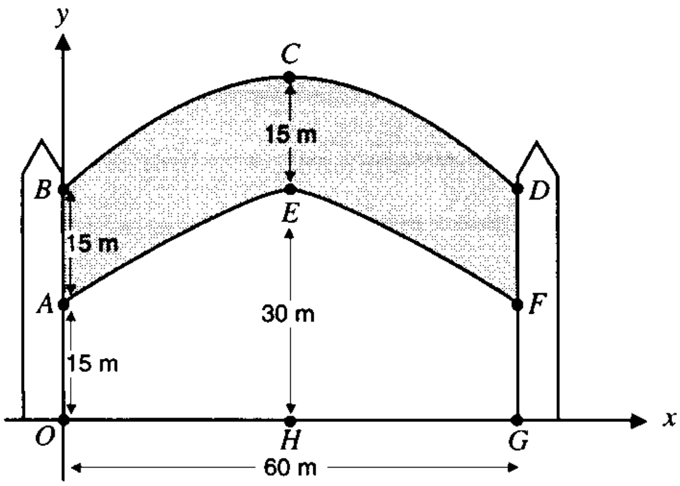
Directions to students

Materials
Question and answer booklet of 14 pages, including one blank page for rough working. There is a detachable sheet of miscellaneous formulas in the centrefold.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
You may use an approved calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the centre of this booklet during reading time.
Ensure that you write your **student number** in the space provided on the front cover of this booklet.
Answer **all** questions.
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 70 marks available for the task.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task
You should hand in this question and answer booklet.

Question 1



A newly designed bridge is to be built across the 60 metre wide Yarra River. The architect draws on a set of axes a cross-section of the bridge, which is symmetrical about the line CH, as shown in the diagram above. The water surface is along OG. The architect introduces an x, y coordinate system for calculation purposes as shown where, for example, point F has coordinates (60, 15).

a. State the coordinates of the points B, C, and D.

1 mark

b. If curve BCD is a parabola with equation

$$y = a\,(x - 30)^2 + k,$$

find the values of a and k.

3 marks

c. State the coordinates of the points A and E.

1 mark

d. If curve AE has equation

$$y = m\log_e(x + 10) + n$$

find the values of m and n, correct to two decimal places.

4 marks

The architect plans to have the span of the bridge painted (as shaded in the diagram on page 3).
To find the area to be painted the architect approximates the bridge section by a hexagon, $ABCDFE$.

e. Find the area of the hexagon $ABCDFE$.

2 marks

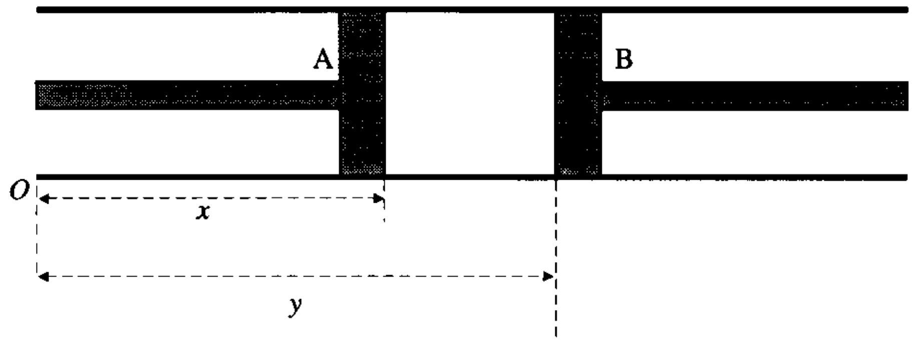
f. If the area of the region between the arch AEF and the water level OG is 1451 m^2, find the exact area of the bridge section $ABCDFE$.

4 marks

Total 15 marks

Question 2

Two pistons A and B move backwards and forwards in a cylinder as shown.



The distance x centimetres of the right hand end of piston A from the point O at time t seconds is modelled by the formula

$$x = 3 \sin (2t) + 3$$

and the distance y centimetres of the left hand end of piston B from the point O at time t seconds is modelled by the formula

$$y = 2 \sin \left(3t - \frac{\pi}{4}\right) + 8.$$

The pistons are set in motion at time $t = 0$.

a. **i.** State the amplitude of the motion of piston A.

ii. Show that the maximum and minimum values of x are 6 and 0 respectively.

iii. Show that when $t = \frac{\pi}{4}$, the right hand end of piston A is at its maximum distance from O.

iv. Find the next four t values $\left(t > \frac{\pi}{4}\right)$ for which $x = 6$.

5 marks

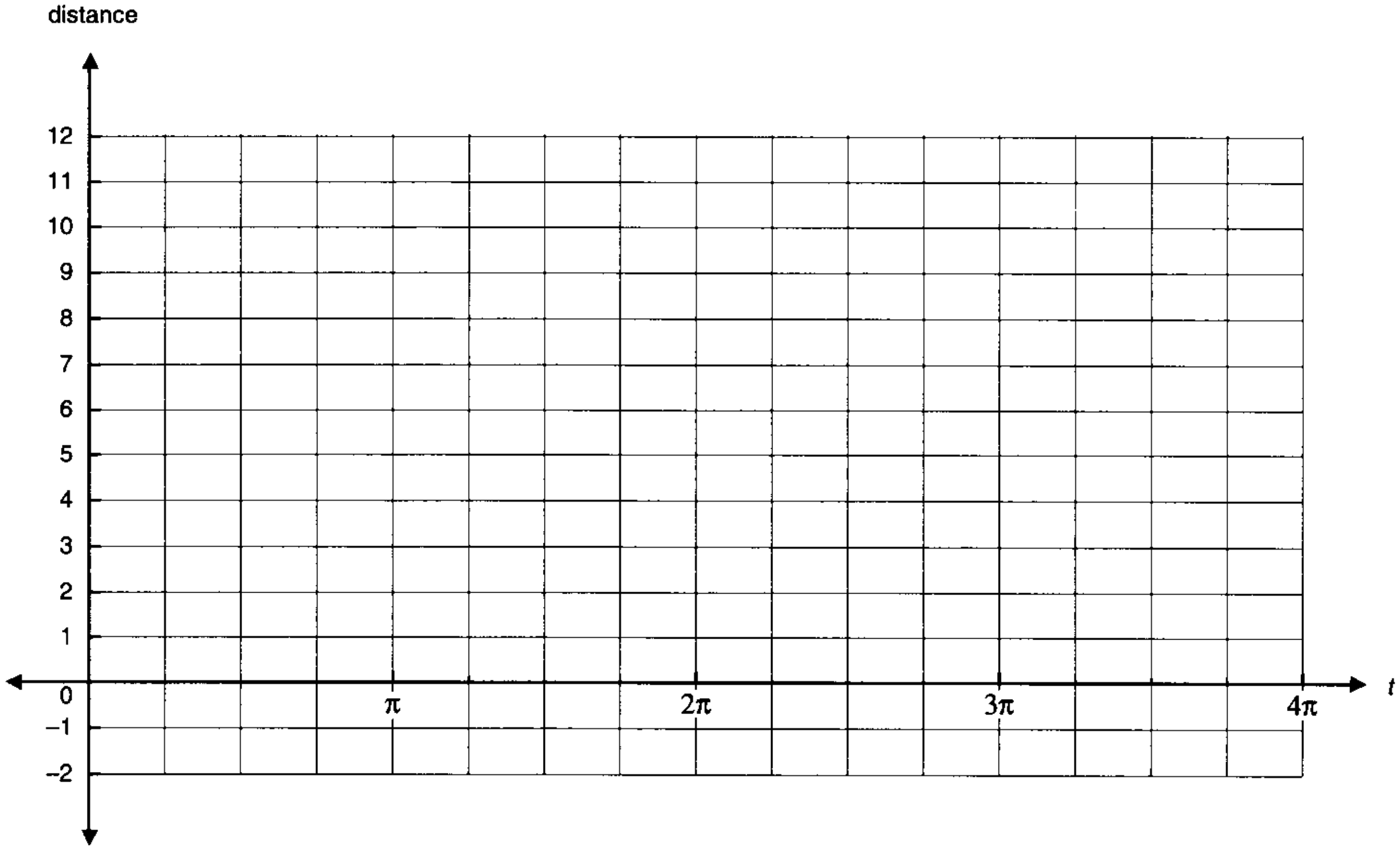
b. **i.** The maximum value of y is 10. State the minimum value of y.

ii. Show that y attains its minimum value when $t = \frac{7\pi}{12}$.

iii. Find all other values of t, $0 \le t \le 4\pi$, for which y attains its minimum value.

4 marks

c. On the one set of axes below, draw graphs of $x(t)$ and $y(t)$.



3 marks

d. i. State the time when the pistons first touch each other.

ii. How many seconds are there between the first and second times the pistons touch?

2 marks

e. Let T_n seconds be the time at which the pistons meet for the nth time. Then

$$T_n = a\pi + bn\pi,$$

where a and b are constants.

Find the values of a and b.

2 marks

f. At what time is the right hand end of piston A first 4 centimetres from O? Give your answer to the nearest one-hundredth of a second.

2 marks

g. **i.** Find the average rate of change of position with respect to time (average speed) of piston B in the first 0.2 seconds.

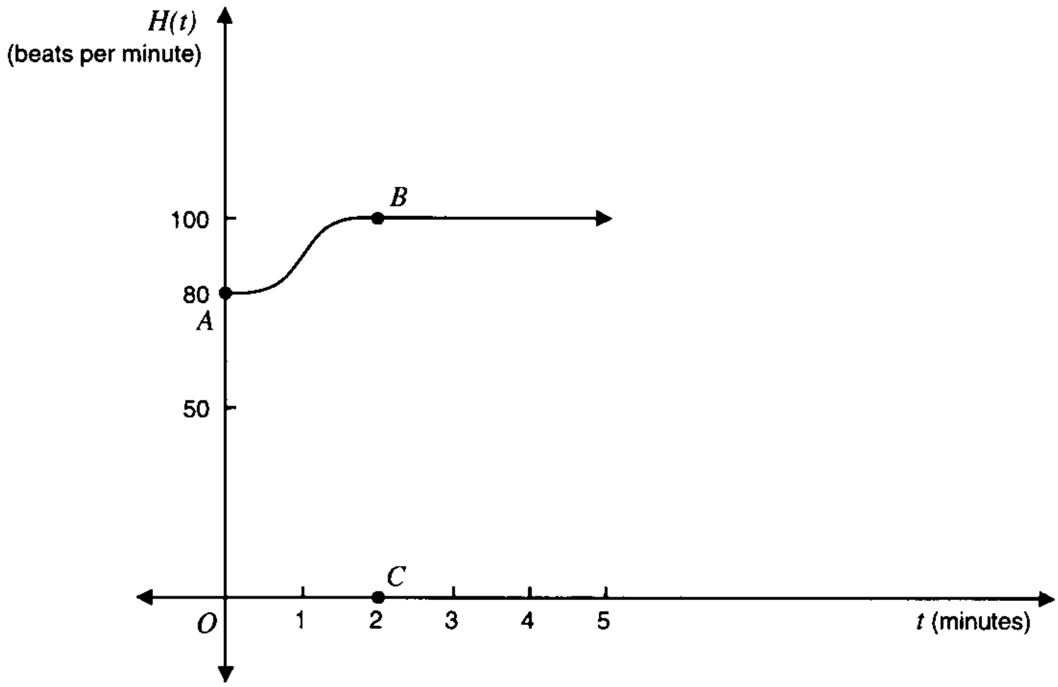
ii. Use calculus to find the speed of piston B at time $t = 0.2$.

4 marks

Total 22 marks

Question 3

During a mild five minute exercise sequence a student's heartrate $H(t)$, in beats per minute, at time t minutes, is monitored and graphed as shown below. The student's heart pumps a volume of 70 millilitres of blood with each beat.



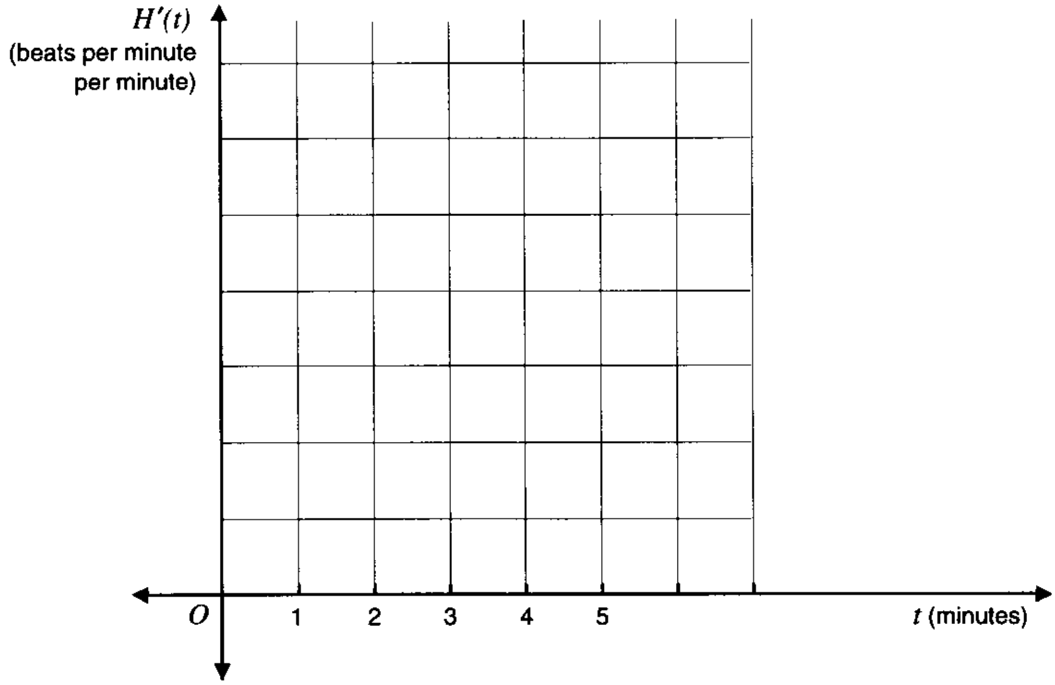
a. From the graph estimate

i. the student's heartrate when $t = 0$ and when $t = 1$ and when $t = 2$.

ii. the student's rate of change of heartrate (in beats per minute per minute) when $t = 0$ and when $t = 1$ and when $t = 2$.

3 marks

b. On the set of axes below, sketch the graph of the rate of change of heartrate, $H'(t)$, over the five-minute exercise program, taking care to put a scale on the vertical axis.



2 marks

c. On the original graph, on page 9, draw in the straight line segment from A to B.
Calculate the area of the trapezium $OABC$, and hence estimate the volume of blood pumped by the student's heart in the first two minutes of the exercise program.

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2 marks

The function $f: R \rightarrow R$ where $f(t) = at^3 + bt^2 + ct + d$ has a local minimum at (0,80) and a local maximum at (2,100).

For parts **d.**, **e.** and **f.** below, assume that the student's heartrate **for the first two minutes** is modelled by the function f.

d. **i.** Show that $c = 0$ and $d = 80$.

ii. Find the values of a and b.

4 marks

e. **i.** Write down a rule for the rate of change of heartrate, $f'(t)$ beats per minute per minute, for $0 \leq t \leq 2$.

ii. Find when the heartrate is increasing most quickly.

3 marks

f. Write down and evaluate a definite integral which gives the total number of heartbeats during the first two minutes, and hence calculate the volume of blood pumped during this time.

3 marks

Total 17 marks

Question 4

Victoria Jones is a contestant in the long-jump event at the world championships.

In a particular jump, Victoria jumps X metres. X is a normally distributed random variable with mean 7.2 and standard deviation 0.2.

a. For any jump, find the probability that Victoria jumps

i. more than 7.5 metres.

ii. less than 7.0 metres.

iii. between 7.0 and 7.5 metres.

3 marks

During the championships each competitor in the long jump event has five jumps.

b. In her five jumps, find the probability, stated to three significant figures, that

i. Victoria's first three jumps are all less than 7.5 metres and both her last two jumps are more than 7.5 metres.

ii. at least three of Victoria's jumps are more than 7.5 metres.

iii. Victoria's final jump is more than 7.5 metres, given that at least three of her five jumps are more than 7.5 metres.

6 marks

c. Between what two distances, symmetrically placed about the mean, would 95 per cent of Victoria's jumps be expected to lie?

1 mark

d. If 75 per cent of Victoria's jumps are greater than a metres, what is the value of a?

2 marks

e. During training for the championships, Victoria had sixty practice jumps. How many of these jumps would be expected to be more than 7.5 metres?

1 mark

After her five world championship jumps, Victoria's best jump was 7.8 metres. She was leading the event with Valda Bos's final jump still to come. In a particular jump, Valda jumps Y metres; where Y is a normally distributed random variable with variance 0.16.

f. If the probability that Valda's final jump is more than 7.8 metres is 0.21, find the mean of the distances that Valda Bos jumps, correct to two decimal places.

3 marks
Total 16 marks