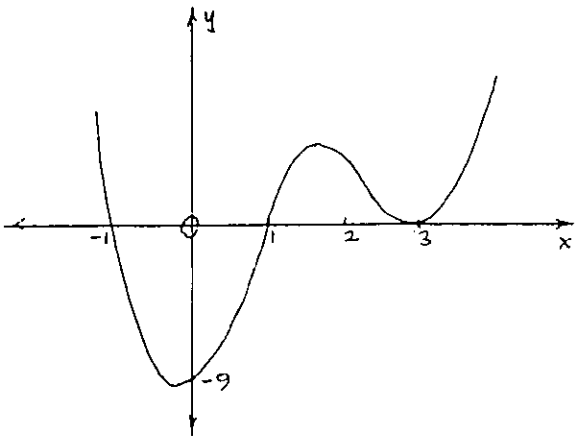
SECTION A

Specific Instructions for Section A

Section A consists of 33 questions.
Answer all questions in this Section on the multiple-choice answer sheet provided.
A correct answer scores 1, an incorrect answer scores 0. No credit will be given for a question if two or more letters are marked for that question. Marks will not be deducted for incorrect answers and you should attempt every question.

Question 1

The equation which defines the graph at right could be



A $y = (x - 1)(x + 1)(x^2 - 9)$
B $y = (x^2 - 1)(x - 3)^2$
C $y = (x - 1)(x + 1)(x - 3)(x + 3)$
D $y = (x^2 - 1)(x^2 + 9)$
E $y = (x - 1)(x + 1)(x + 3)^2$

Question 2

For the function $g : \mathbb{R}^+ \to \mathbb{R}$, $g(x) = \log_{10} x$, which of the following statements is **NOT** true?

A The function has range $\mathbb{R}$.
B The domain of the derivative function g' is $\mathbb{R}^+$.
C The x-axis is an asymptote to the graph of g.
D The graph of g has an x-intercept at (1,0).
E The y-axis is an asymptote to the graph of g.

Question 3

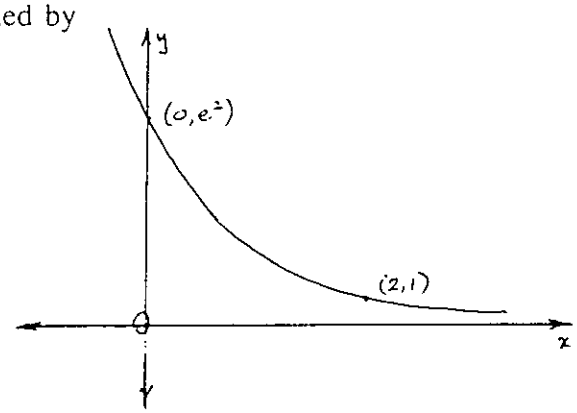
The graph of the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \sin x$ is transformed into the graph of the function $h : \mathbb{R} \to \mathbb{R}$, $h(x) = \sin 2x$ by a **dilation**

A from the x-axis (in the y direction) with scale factor 2
B from the y-axis (in the x direction) with scale factor 2
C from the origin (0,0) with scale factor 2
D from the y-axis (in the x direction) with scale factor ½
E from the x-axis (in the y direction) with scale factor ½

Question 4

The graph at right represents the function defined by

$y = e^{a(x-b)}$ where

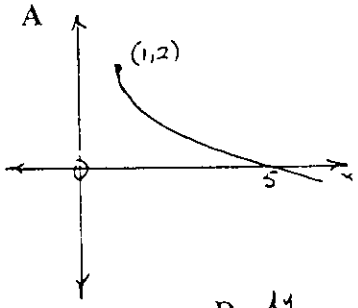


A $a = 1$ and $b = 2$
B $a = 1$ and $b = -2$
C $a = 2$ and $b = 1$
D $a = -1$ and $b = -2$
E $a = -1$ and $b = 2$

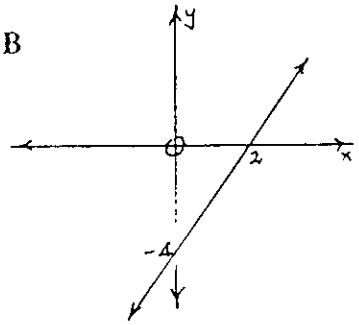
Question 5

The function p is defined by $p : [2,\infty) \to \mathbb{R}$, $p(x) = (x-2)^2 + 1$.
Which one of the following graphs best represents the **inverse function** p^{-1}?

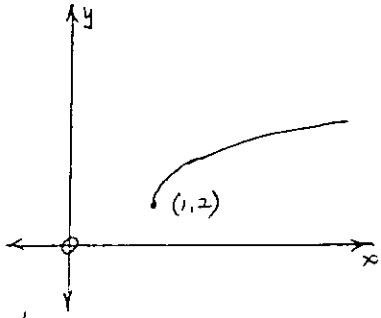
A



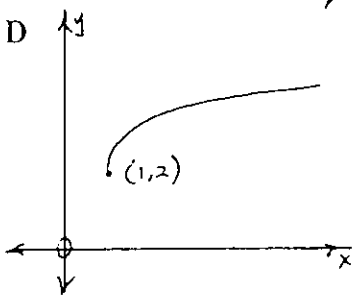
B



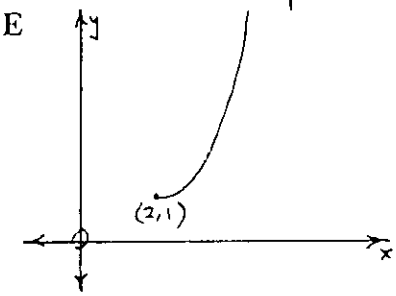
C



D



E



Question 6

x	1	2	3	4
y	8	12	18	27

The data in the table above may be modelled by a rule of the form

A $y = mx + c$ $(m,c \in \mathbb{R}^+)$
B $y = Ax^n$ $(A,n \in \mathbb{R}^+)$
C $y = Ae^{kx}$ $(A,k \in \mathbb{R}^+)$
D $y = A \log_e(kx)$ $(A,k \in \mathbb{R}^+)$
E $y = A \sin(kx)$ $(A,k \in \mathbb{R}^+)$

Questions 7 and 8 refer to the equation $y = 3\cos(4x)$

Question 7

The graph defined by the equation has

A a period of $\frac{\pi}{2}$ and an amplitude of 3

B a period of $\frac{2\pi}{3}$ and an amplitude of 4

C a period of 8π and an amplitude of 3

D a period of 6π and an amplitude of 4

E a period of $\frac{\pi}{2}$ and an amplitude of 12

Question 8

On the restricted domain $[0,\pi]$, the graph defined by the equation has

A no x-intercepts B one x-intercept C two x-intercepts
D three x-intercepts E four x-intercepts

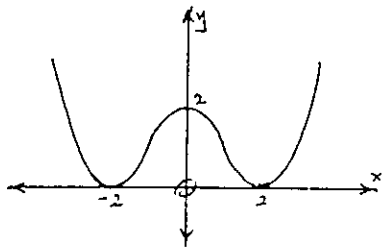
Question 9

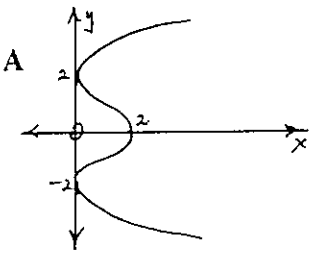
For the function $g : [0, \pi] \to \mathbb{R}, \quad g(x) = 3\sin\left(\frac{x}{2}\right) + 2$ the **range** is

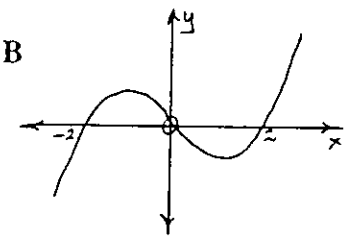
A $[-3,3]$ B $[-1,5]$ C $[0,5]$
D $[1,5]$ E $[2,5]$

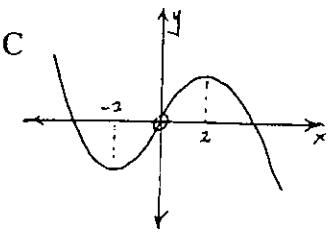
Question 10

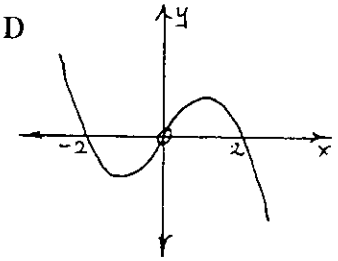
The graph at right represents a function $g : \mathbb{R} \to \mathbb{R}$. Which one of the following graphs best represents the **derivative function** g'?

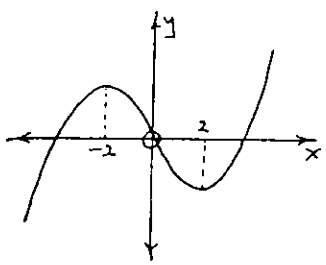


A


B


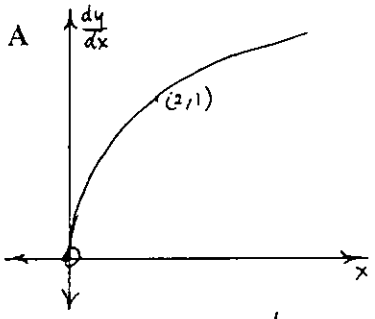
C


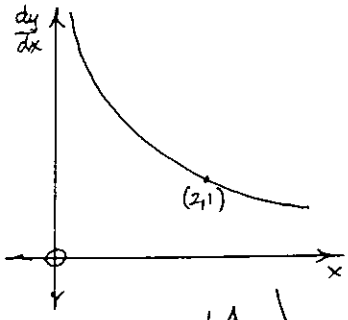
D


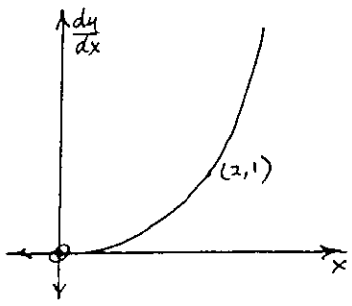
E


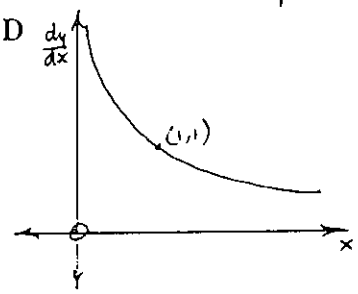
Question 11

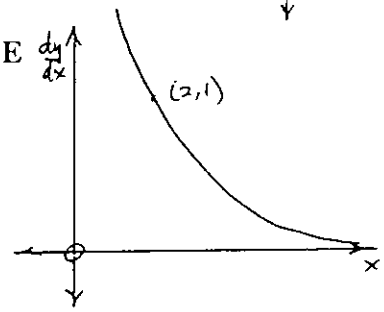
If $y = 2\sqrt{x}$ then which one of the following best represents the graph of $\frac{dy}{dx}$?

A 

B 

C 

D 

E 

Question 12

The derivative of $\frac{e^{3x} - e^{x}}{e^{x}}$ is

A	$2e^{2x}$	B	$4e^{2x} - 2$	C	$3e^{3x}$
D	$3e^{2x} - 1$	E	$\frac{1}{2}e^{2x}$		

Question 13

If $g(x) = x \sin x$, then $g'(\frac{\pi}{2})$ is equal to

A	0	B	1	C	0.5π
D	π	E	2		

Question 14

If $y = \sqrt{x^2 - 4}$ then $\frac{dy}{dx}$ is equal to

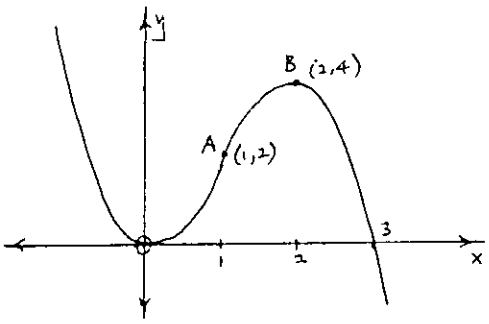
A	1	B	$4xy$	C	$\frac{x}{y}$
D	xy	E	$\frac{2x}{y}$		

Question 15

The graph defined by $y = x^4 - x^3 - 3x^2 + 5x - 2$ has a stationary point of inflexion at (1,0) and another x-intercept at (−2,0). The graph has a turning point (a local minimum) when x is equal to

A	−2	B	−1¼	C	−1
D	−½	E	0		

Questions 16 and 17 refer to the graph of $y = x^2(3 - x)$, part of which is sketched at right. O, A and B are the points with coordinates (0,0), (1,2) and (2,4) respectively.



Question 16

The gradient of the line OB is

A	0	**B**	1	**C**	2
D	3	**E**	4		

Question 17

The gradient of the curve at the point A is

A	0	**B**	1	**C**	2
D	3	**E**	4		

Question 18

The water depth d m in a harbour at t hours after 12.00 noon on 1 October 1996, is given by the formula: $d = 10 + 4 \sin\left(\frac{\pi t}{4}\right)$

At exactly 3.00 pm on 1 October 1996, the water depth is

A rising at a rate of approximately 0.7 m per hour
B falling at a rate of approximately 0.7 m per hour
C neither rising nor falling
D falling at a rate of approximately 2.2 m per hour
E rising at a rate of approximately 2.2 m per hour

Question 19

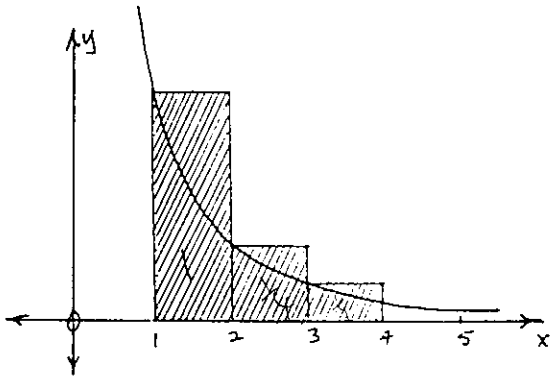
A **normal** (perpendicular) is drawn to the graph of the equation $y = 1 - e^{-x}$ at the point where $x = 1$.
This normal has a **gradient** of

A	$-e$	**B**	-1	**C**	$-e^{-1}$
D	e^{-1}	**E**	e		

Questions 20 and 21 refer to the function

$f: \mathbf{R}^+ \to \mathbf{R}, \quad f(x) = \dfrac{1}{x^2}$

whose graph is sketched at right.



Question 20

The total area of the shaded rectangles can be used as an approximation to the area between the curve, the x-axis, and the ordinates defined by $x = 1$ and $x = 4$.

The value of this approximation is equal to

A $\frac{13}{36}$ square units B $\frac{61}{144}$ square units C $\frac{49}{72}$ square units

D $\frac{49}{36}$ square units E $\frac{11}{6}$ square units

Question 21

$\int_1^4 f(x)\,dx$ is exactly equal to

A $\frac{61}{192}$ B $\frac{1}{2}$ C $\frac{3}{4}$

D $\frac{15}{16}$ E 1

Question 22

The value of $\int_1^2 (3x + 1)^2\,dx$ is

A 18 B 22 C 26

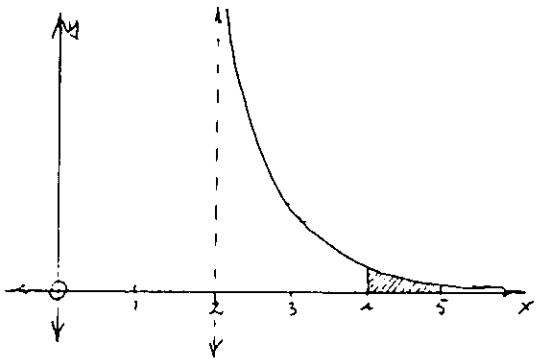
D 31 E 38

Question 23

The curve defined by the equation $y = \dfrac{1}{x - 2}$

is sketched at right for the domain $x > 2$.

The shaded region, bounded by the curve, the x-axis, and the ordinates $x = 4$ and $x = 5$, has an area of $\log_e k$ square units. The value of k is



A 1 B 1.5 C 2

D 3 E 6

Question 24

If $(4x - 1)^3 = ax^3 + bx^2 + cx + d$, then which one of the following statements is **FALSE**?

A $b = 4c$ B $a = 64$ C $c = 12$
D $a = 16c/3$ E $d = -1$

Question 25

The graph of $y = 2 \log_e(x + 3) - 5$ cuts the x-axis at approximately

A -2.8028 B -2.0837 C -1.5082
D 4.4918 E 9.1825

Question 26

Which of the following represents a **discrete probability distribution** for the random variable X?

A $$\Pr(X = x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } 0 \le x < 0 \\ 0 & \text{if } x > 1 \end{cases}$$

B

x	0	1	2	3
$\Pr(X = x)$	0.2	0.3	-0.1	0.6

C $\Pr(X = x) = 1/x$ for $x \in \{1, 2, 3, 4, 5, 6, 7, \ldots\}$

D

x	-2	-1	0	1	2
$\Pr(X = x)$	0.1	0.2	0.3	0.2	0.1

E $$\Pr(X = x) = \frac{{}^3C_x}{8} \quad \text{for} \quad x \in \{0, 1, 2, 3\}$$

Questions 27 to 29 refer to the probability distribution for random variable X arising from a binomial experiment with four trials and a probability of success on each trial of 0.2

Question 27

The **expected value** of X is

A 0.2 B 0.64 C 0.8
D 1 E 1.2096

Question 28

The **variance** of X is

A	0.16	**B**	0.64	**C**	0.8
D	1	**E**	1.44		

Question 29

On 95% of repetitions of the binomial experiment, we would expect values of the random variable X to be

A	1 only	**B**	0 or 1	**C**	0, 1 or 2
D	0, 1, 2 or 3	**E**	0, 1, 2, 3 or 4		

Questions 30 and 31 refer to the following information: "Study scores" for VCE Units 3/4 are "normalised" (and thus may be assumed to be normally distributed) with a mean of 30 and a standard deviation of 7. The highest score awarded is 50.

Question 30

The percentage of students of VCE Mathematical Methods Units 3/4 who will obtain a study score over 44 is approximately

A	0.6%	**B**	1.1%	**C**	2.3%
D	7.9%	**E**	15.9%		

Question 31

If 12,000 students study VCE Mathematical Methods Units 3/4 in 1996, the number expected to obtain the maximum score of 50 would be approximately

A	10	**B**	25	**C**	50
D	100	**E**	200		

Questions 32 and 33 refer to the following information: In July 1996, the Melbourne *Age* newspaper published the results of a survey conducted with 2057 enrolled Australian electors. On a two-party preferred basis, 54% of the sample expressed their intent to vote for the Coalition (Liberal and National Parties).

Question 32

The **standard error** of the proportion of the sample who would support the Coalition is approximately

A	0.00012	**B**	0.00024	**C**	0.0055
D	0.011	**E**	0.018		

Question 33

On the basis of the research published in *The Age*, we could be 95% confident that the proportion of the Australian electors who would have voted for the Coalition in July 1996 is approximately between

A	0.53 and 0.55	B	0.52 and 0.56	C	0.51 and 0.57
D	0.50 and 0.58	E	0.49 and 0.59		

SECTION B

Specific Instructions for Section B

Section B consists of 6 questions. There are a total of 17 marks available.

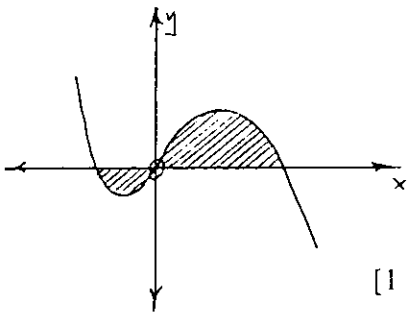
Answer all six questions neatly on the lined paper provided.

Please number each question clearly.

Full marks may not be given for answers which do not show appropriate working, or do not state answers clearly.

Question 1

The graph defined by the equation $y = 2x + x^2 - x^3$ is sketched at right.



Find (a) the x-intercepts, and
(b) the area of the shaded region.

[1 + 2 = 3 marks]

Question 2

Consider the function $g : \mathbb{R}^+ \to \mathbb{R}$, $g(x) = 1 + \log_e(2x)$

(a) On the same set of Cartesian axes, and using the same scale on both axes, sketch the graphs of (i) the function g, and
(ii) the **inverse function** g^{-1}.
Carefully label **asymptotes** and **intercepts** with the coordinate axes.

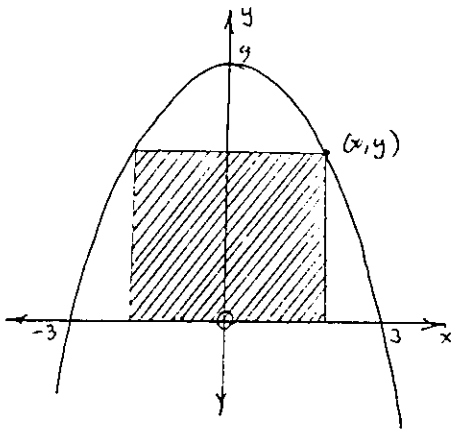
(b) State the **domain** and **rule** for the **inverse function** g^{-1}.

[2 + 2 = 4 marks]

Question 3

The area of a rectangle bounded by the x-axis and the graph of $y = 9 - x^2$ is given by the rule

$$A(x) = 2x(9 - x^2)$$



(a) State the **domain** for the area function A,

(b) Find the dimensions (width and height) of the rectangle which has **maximum area**.

[1 + 2 = 3 marks]

Question 4

As part of her training for the 400 metre event, Cathy Freeman runs a series of **five** 400 m time trials with a two minute recovery between each. She finds that her probability of running under 55 seconds on at least one of her five trials is 0.99968.

Assuming that her probability of running under 55 seconds remains constant from trial to trial, what is her probability of running under 55 seconds on any **one** trial 400 m run?

[2 marks]

Question 5

In a particular Victorian school in 1995, it was found that the standard deviation of study scores for their students of VCE Mathematical Methods Units 3/4 was 5. A student in this school obtained a study score of 45, and was told that he had a percentile rank of 95 for the subject within his school.

Assuming that the study scores for the subject were normally distributed within this school (as well as the State), calculate the mean study score for VCE Mathematical Methods in this school.

[3 marks]

Question 6

A researcher suspects that the question of whether or not Australia should become a republic is controversial, and that approximately 50% of the Australian population would be in favour of Australia becoming a republic. She proposes to ask a randomly selected sample of Australian residents: "Are you in favour of Australia becoming a republic - Yes or No?". She wishes to be 95% confident that the sample proportion in favour reflects the population proportion, to an accuracy within 1%.

Calculate the minimum size of the sample she should choose.

[2 marks]

END OF BOOKLET